

Finite Element Analysis

(MCEN 4173/5173)

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What is Finite Element Analysis (FEA)?

- A numerical method.
- Traditionally, a branch of Solid Mechanics.
- Nowadays, a commonly used method for multiphysics problems.

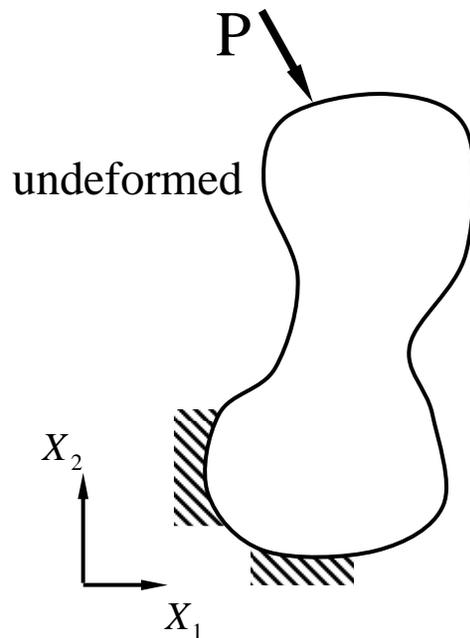
What areas can FEA be applied?

- Structure analysis: a cantilever, a bridge, an oil platform...
- Solid mechanics: a gear, a automotive power train ...
- Dynamics: vibration of Sears Tower, earthquake, bullet impact...
- Thermal analysis: heat radiation of finned surface, thermal stress brake disc...
- Electrical analysis: piezo actuator, electrical signal propagation...
- Biomaterials: human organs and tissues...
- ...

What is Finite Element Analysis (FEA)?

FEA is originally developed for solving solid mechanics problem.

Solid Mechanics:



Mr. Potato

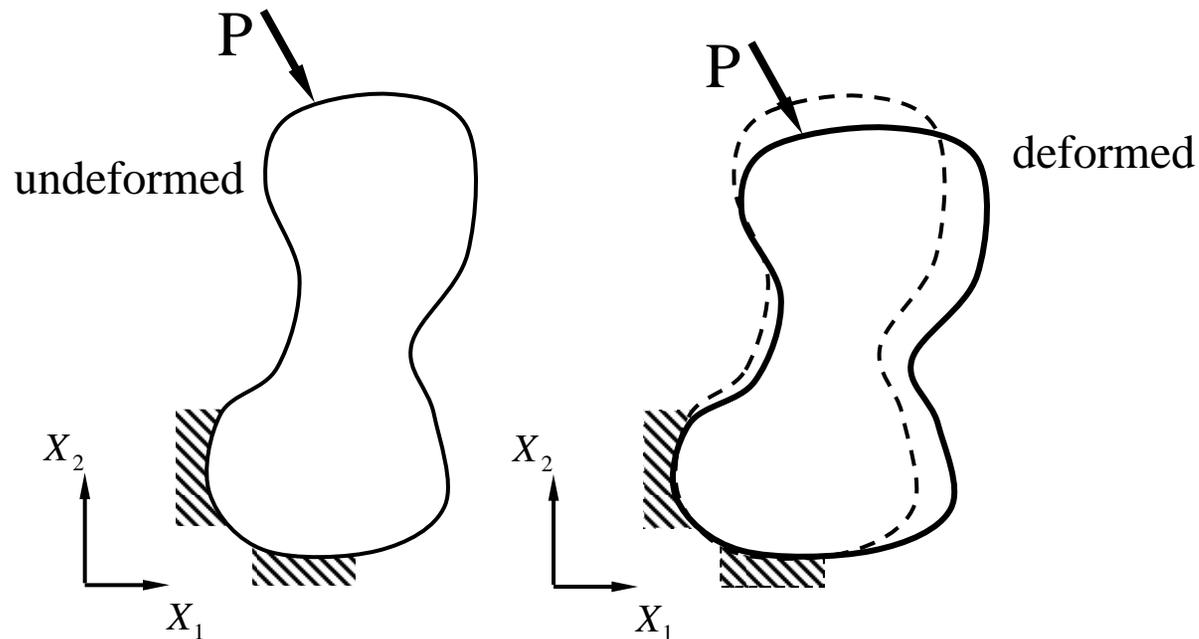
Object: A Solid with known mechanical properties. (Mr. Potato; a shaft; human tissue.....)

Concepts:

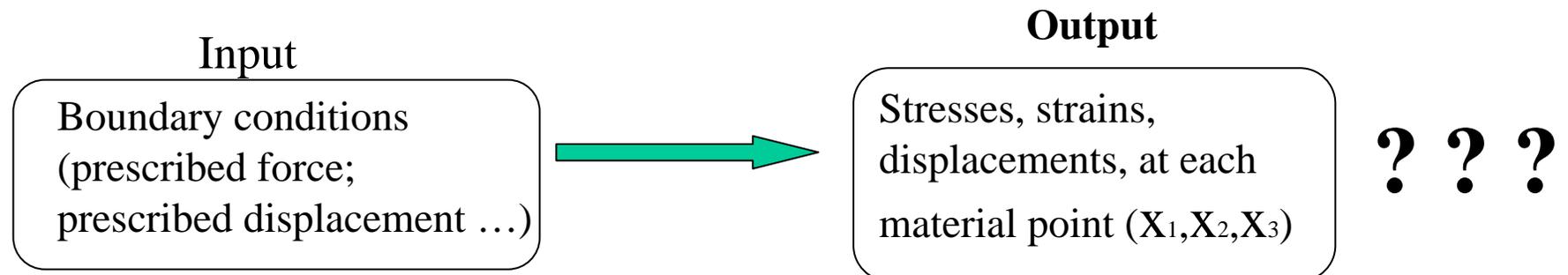
- Boundary: The surface enclosing the geometry
- Solid: Interior + Boundary
- Boundary conditions: Any prescribed quantities, such as prescribed displacements and prescribed tractions on the boundary

What is Finite Element Analysis (FEA)?

Solid Mechanics:



Question:
If we apply a force on a solid, what are the values of the displacements, stresses, and strains at **EACH MATERIAL POINT**?



What is Finite Element Analysis (FEA)?

To answer this question, we need to solve the following equations:

$$e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} \right)$$

Equations for solving
linear solid mechanics
problem

$$\sigma_{ij} = 2Ge_{ij} + \lambda e_{kk} \delta_{ij}$$

$$\frac{\partial \sigma_{i1}}{\partial X_1} + \frac{\partial \sigma_{i2}}{\partial X_2} + \frac{\partial \sigma_{i3}}{\partial X_3} + f_i = 0$$

We need to solve a problem consisting of total 15 equations, among which 9 equations are partial differential equations!!

Finding an exact solution: **MISSION IMPOSSIBLE !!!**

Then: Mission changes to find a solution that **APPROXIMATES** the exact solution.

FEA is a numerical method that offers a means to find this **Approximate Solution**.

How does FEA work?

Before we start to look at how FEA works, let's first review some calculus

$$\int_0^1 x dx = \frac{1}{2} x^2 \Big|_0^1 = \frac{1}{2} (1^2 - 0) = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{2}} \sin x dx = -\cos x \Big|_0^{\frac{\pi}{2}} = -\cos\left(\frac{\pi}{2}\right) + \cos(0) = 1$$

$$\int_0^{\frac{\pi}{2}} (\sin x)^2 dx \quad ???$$

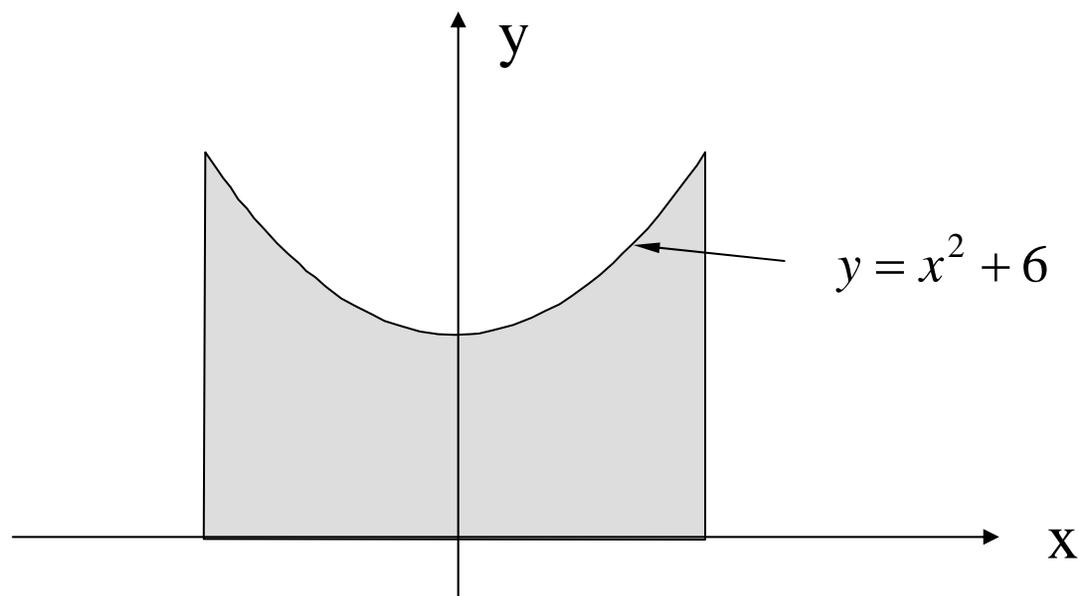
We can use numerical method to find the answer.

How does FEA work?

Integration using numerical methods:

Example: $F = \int_{-1}^1 (x^2 + 6) dx$

Exact solution: $F = \int_{-1}^1 (x^2 + 6) dx = \left(\frac{1}{3} x^3 + 6x \right) \Big|_{-1}^1 = \frac{38}{3} \approx 12.667$



The integration represents the area under the curve

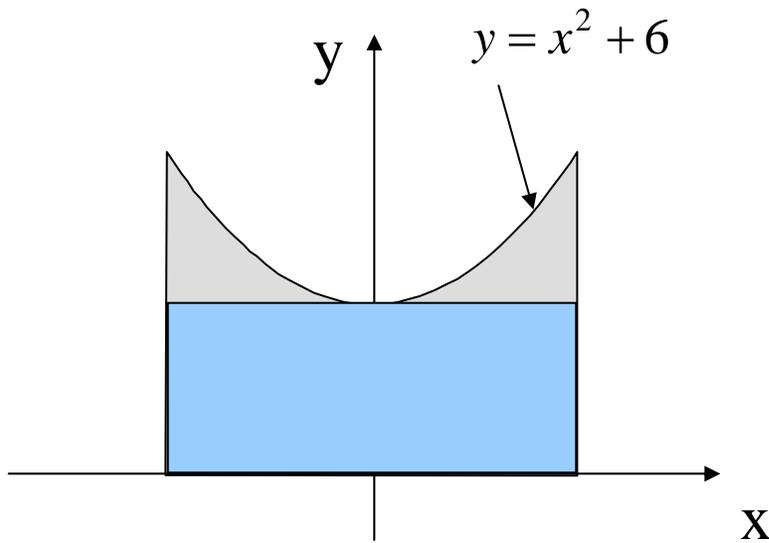
How does FEA work?

Integration using numerical methods:

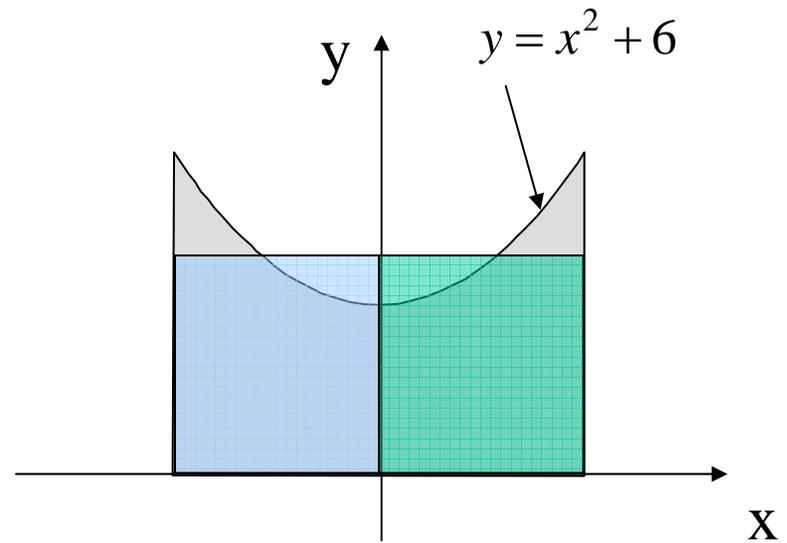
Numerical integration

Scheme 1:

1. Divide the interval of integration into N section;
2. Choose a function to approximate the variation of $f(x)$ in each section; the simplest such function is a constant function that equals to the value of $f(x)$ at the mid-point of each section.
3. The product of this constant function and the length of the section approximates the integration of $f(x)$ over this section.
4. Summing the products for all sections gives an approximate answer to the integration of $f(x)$ over $(-1,1)$



N=1, F=12, Error= -5.26%

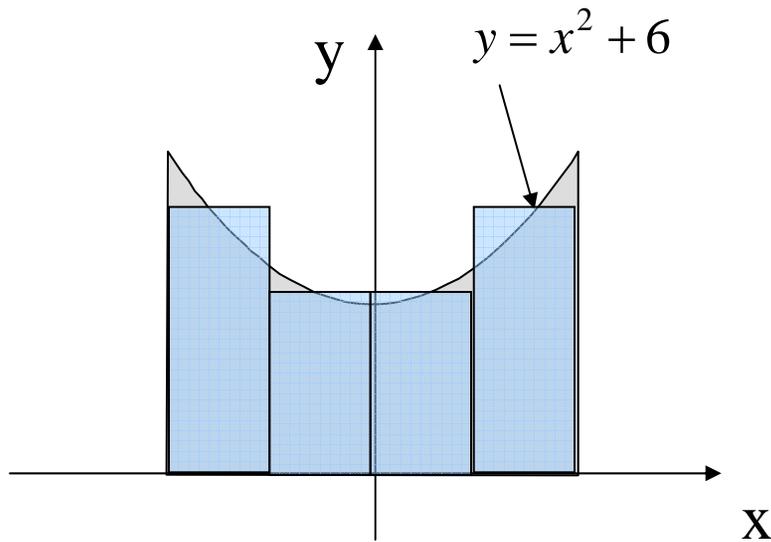


N=2, F=12.5, Error= -1.32%

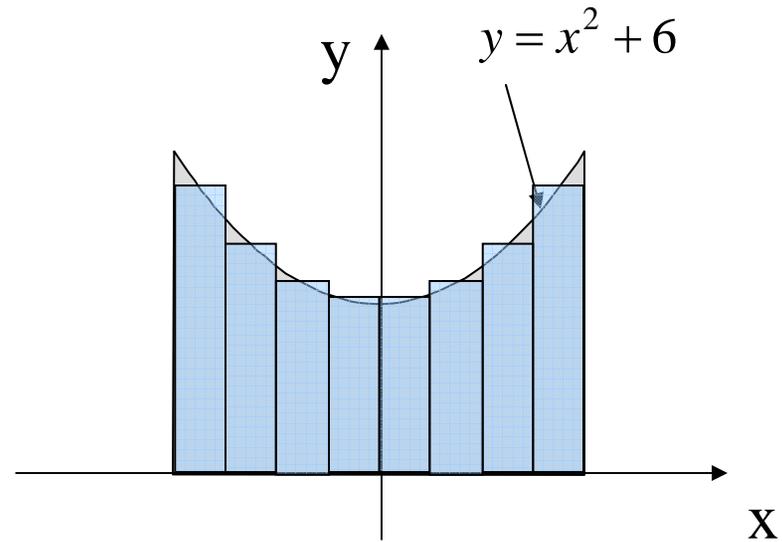
How does FEA work?

Integration using numerical methods:

Numerical integration



$N=4, F=12.625, \text{Error}=-0.33\%$



$N=8, F=12.656, \text{Error}=-0.08\%$

As the number of sections increases, the error decreases.

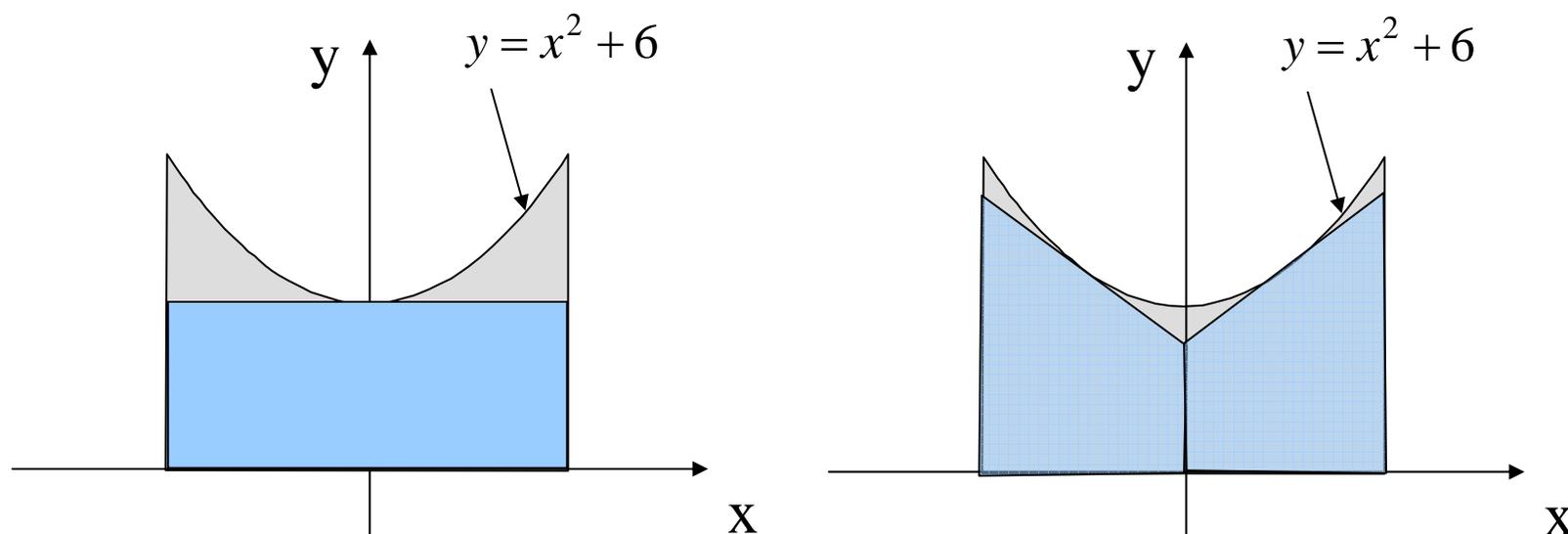
How does FEA work?

Integration using numerical methods:

Numerical integration

Scheme 2:

Same as Scheme 1, except that we choose a linear function in each section to approximate the variation of $f(x)$. This linear function takes the same value and slope of $f(x)$ at the mid-point of that section.



Different functions can be chosen to approximate $f(x)$.

How does FEA work?

Integration using numerical methods:

Two key steps:

1. Divide the interval of integration.
2. In each sub-interval, choose proper simple functions to approximate the true function.

Two key features:

1. The numerical result is an **approximation** to exact solution.
2. The accuracy of numerical result depends on the number of sub-interval and approximate function.

How does FEA work?

Two key steps in numerical integration:

1. Divide the interval of integration.
2. In each sub-interval, choose proper simple functions to approximate the true function.



A quarter of Mr. Potato

Discretize the solid



Element

Node

Mesh of the 3D solid



Using a simple function to approximate the displacements in each element



Formulate a set of linear equations with displacements at each node as unknowns

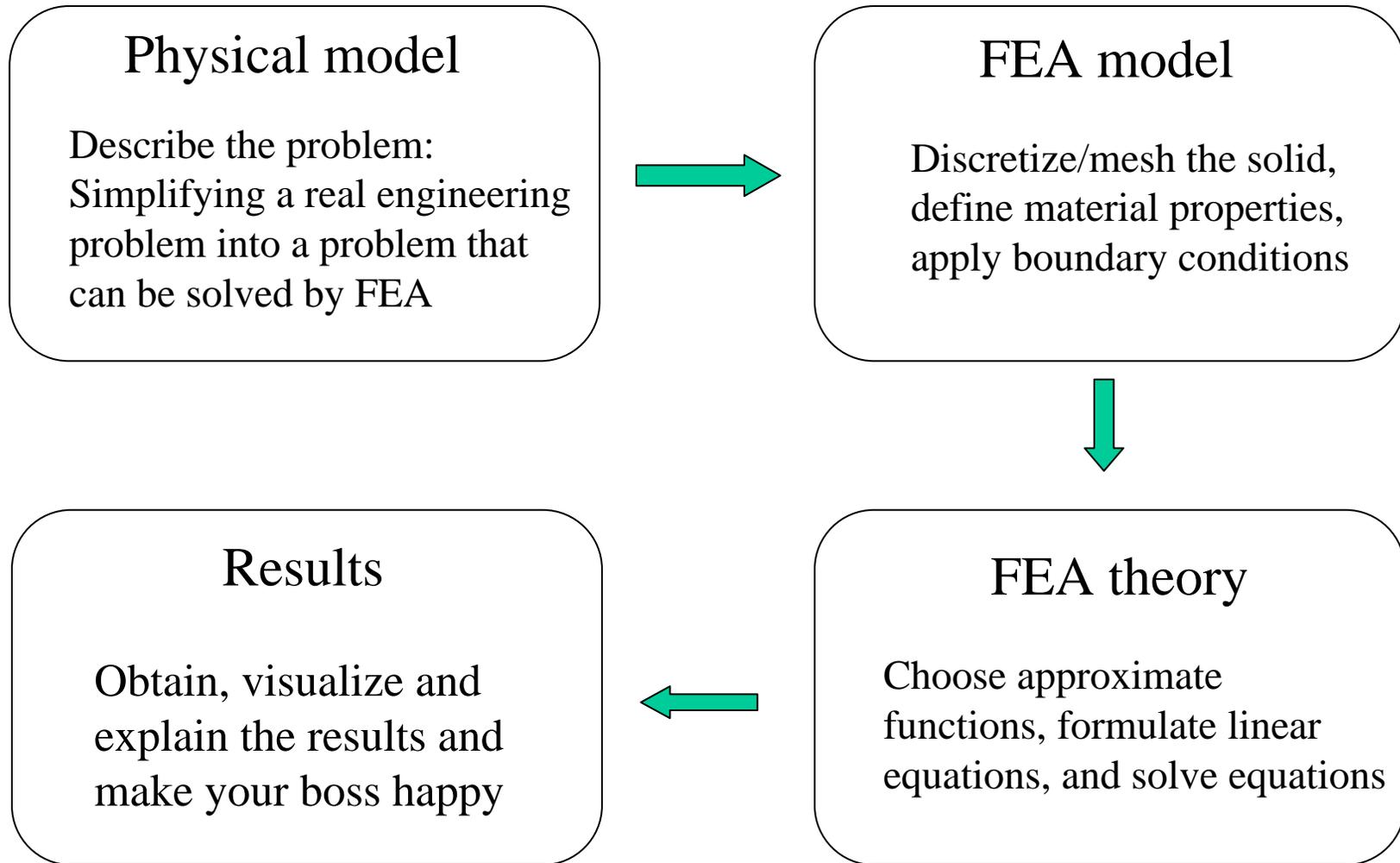


Solve linear equations

How does FEA work?

General Procedure

Pre-processing **Fun!**

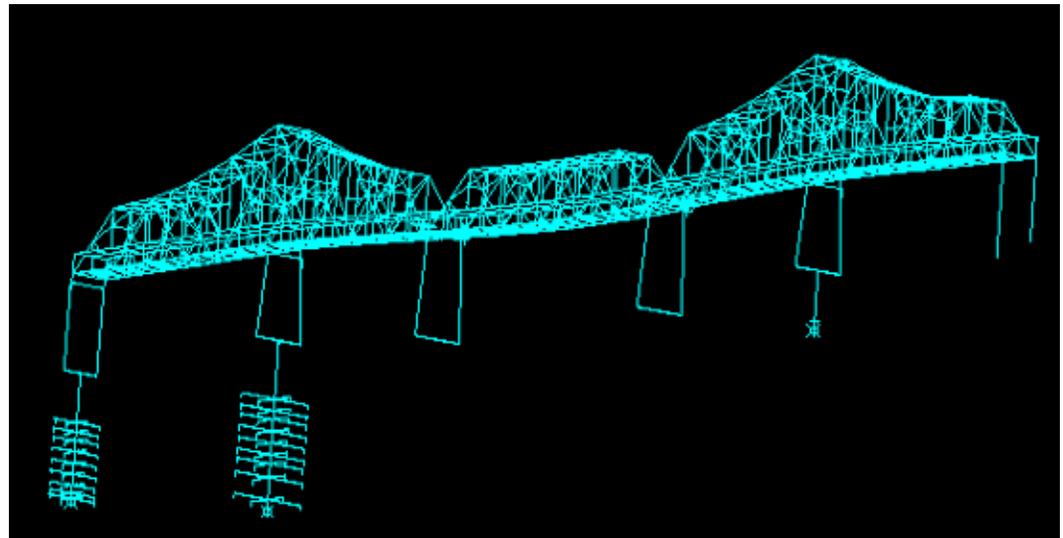


Post-processing **Fun!**

FEA core **Math!!**

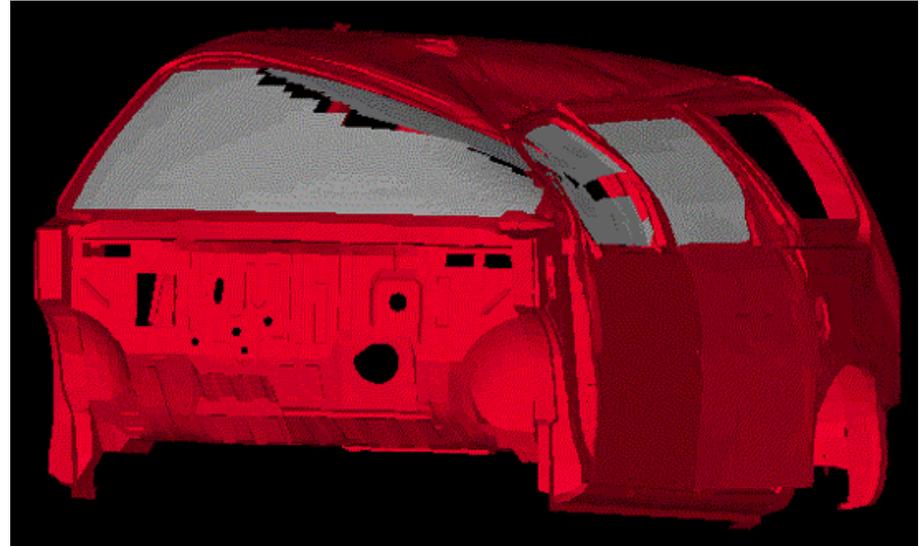
Examples of FEA

FEM simulation of the damage of San Francisco Oakland Bay Bridge caused by the 1989 Loma Prieta earthquake. (From Adina R & D, Inc.)



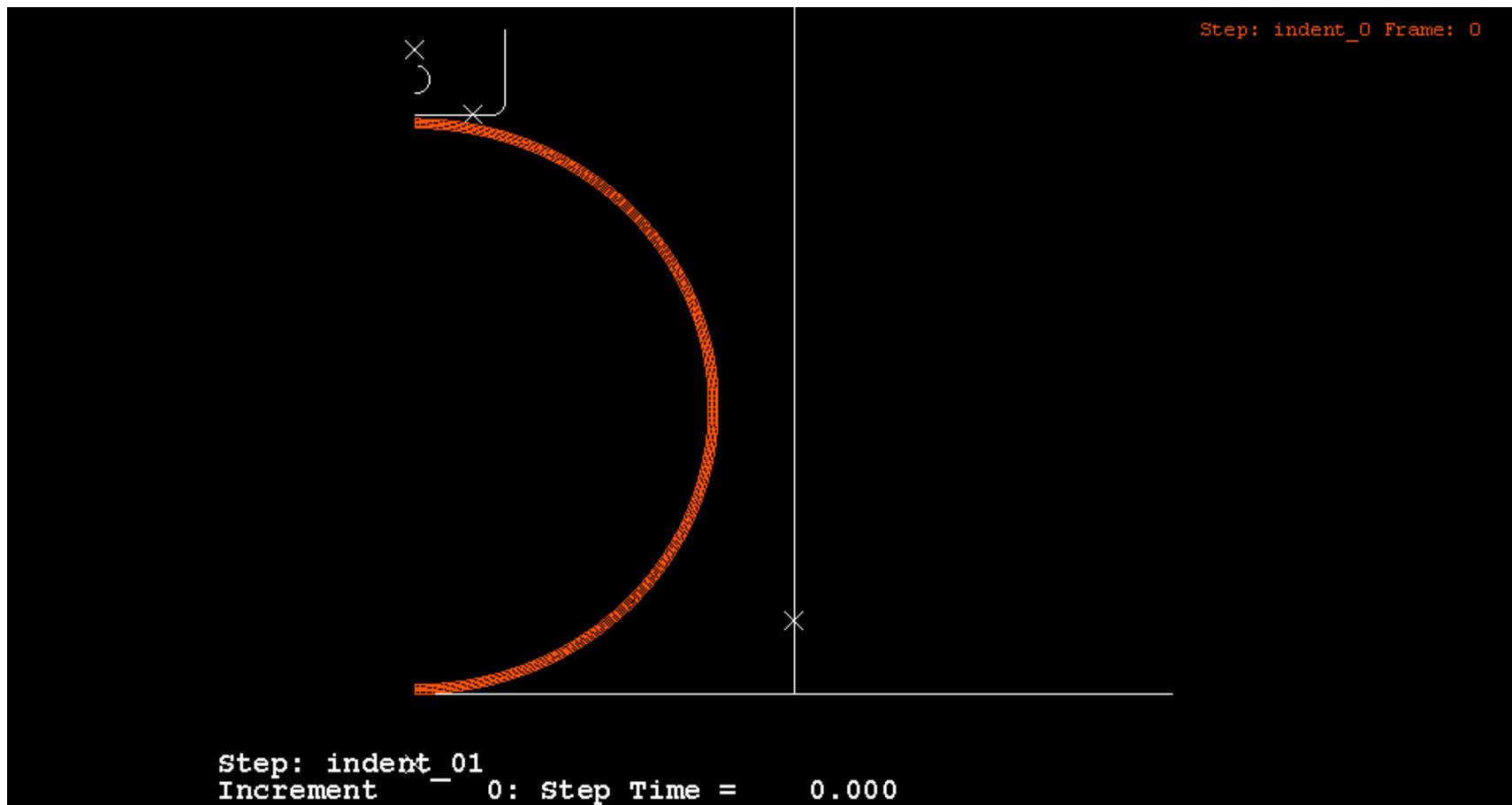
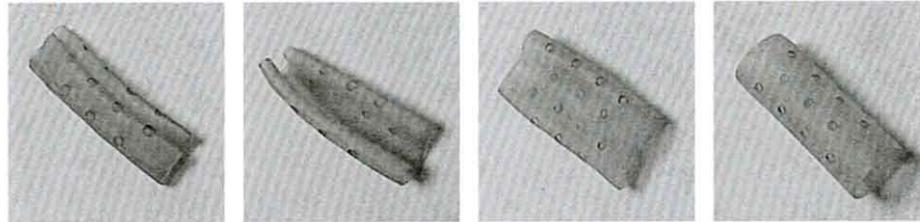
Examples of FEA

FEM simulation of crush of a car in roll-over situation. (*From Adina R & D, Inc.*)

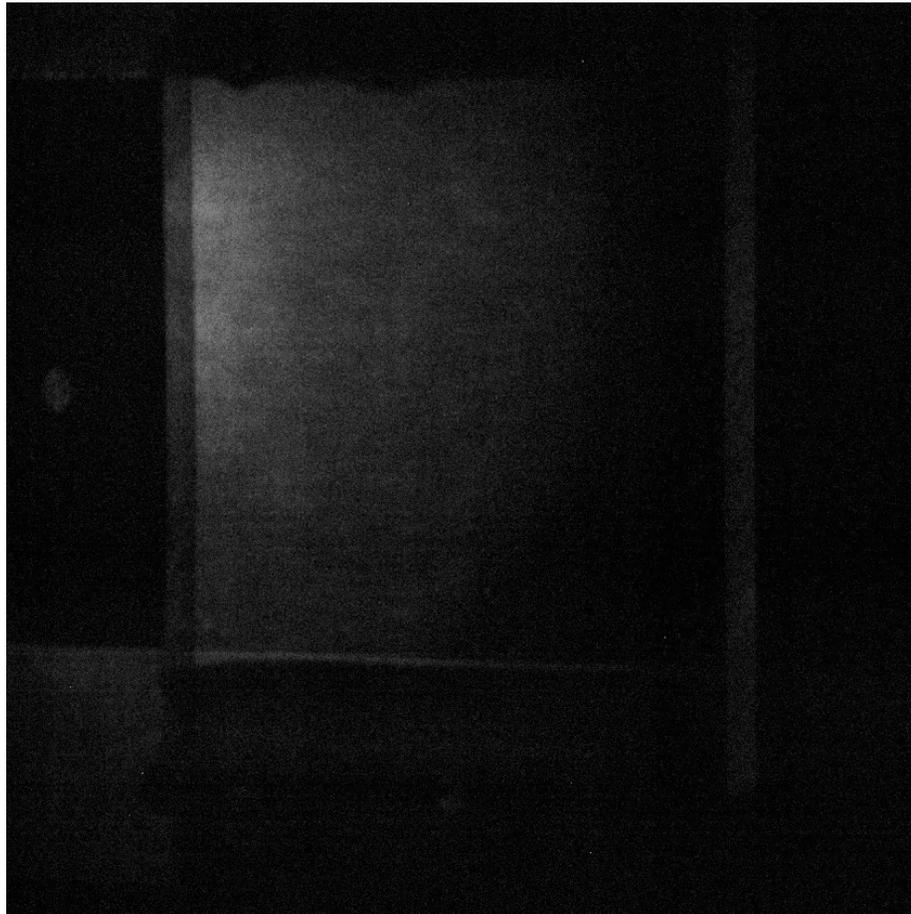


Examples of FEA

FEA simulation of shape memory polymer tube

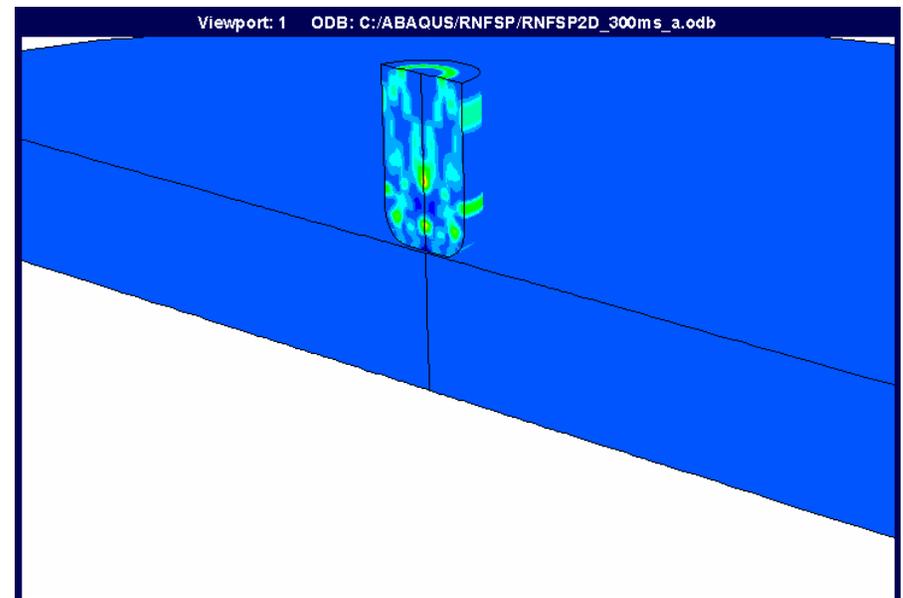
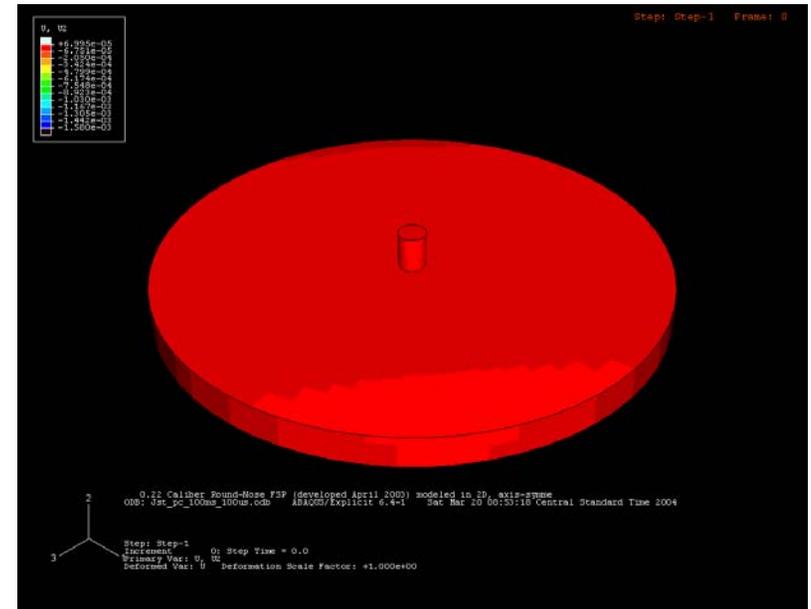


Examples of FEA



PC, 500m/s

High Speed Bullet Impact



Courtesy of A. D. Mulliken, S. Sarva @ MIT

New Developments in FEA

- **Integrating FEA into CAD design software**

Do analysis as you design

- **Self-adaptive analysis**

Change the mesh during the analysis

- **Analysis of problem of huge size**

Analyze a model with millions of nodes; Parallel computing

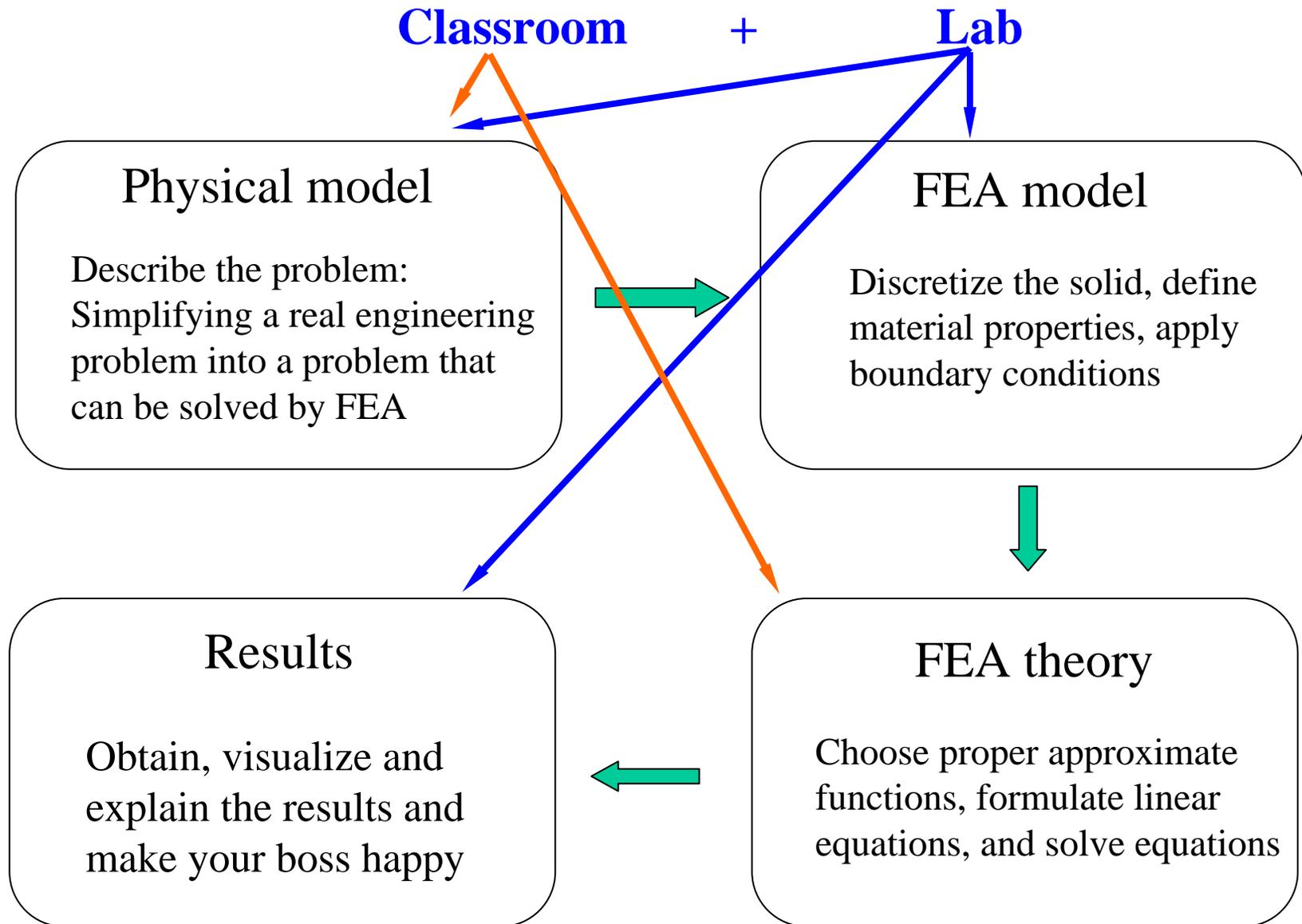
- **Multi-scale analysis**

Analyze physical problem ranging from atomistic level to macroscopic level; Combine FEA with molecular dynamics simulations

- **Multi-physics analysis**

Mechano-electrical coupling (MEMS); mechano-chemical coupling (Chemical-mechanical polishing (CMP))

What is this class about?



Things before we start...

Website: <http://www.colorado.edu/MCEN/MCEN4173/MCEN4173.html>

Mail list: mcen4173_06@lists.colorado.edu

How to subscribe the mail list:

**Send an e-mail to listproc@lists.colorado.edu with
Subject line empty and
SUBSCRIBE MCEN4173_06 Your Name as the content.**

Things before we start...

Instructor

Dr. H. “Jerry” Qi

Office: ECME 124

Telephone: 2-1270

E-mail: qih@colorado.edu

TA

Mr. Kevin Long

Office:

E-mail: kevin.long@colorado.edu

Mr. Philip Kao

Office:

E-mail: philip.kao@colorado.edu

Office Hours

Instructor: M: 4:00-6:00PM

W: 4:00PM-6:00PM

TA (KL about lecture):

Tu: 3:00PM-4:00PM (GS),

Th: 3:00PM-4:00PM (GS).

TA (PK: @ about lab)

Th: 10:00AM-11:00AM

For other time, by appointment.

Things before we start...

Textbook

A first course in finite element method (3rd Edition). Daryl L. Logan. Brooks/Cole, 2002.
ANSYS Tutorial, K.L. Lawrence. SDC Publications, 2003.

References

The finite element methods: Linear static and dynamic finite element analysis. T.J.R. Hughes. Dover Publications, 1987.
Finite element procedures. K.J. Bathe. Prentice Hall, 1996.

Homework:

Discussions are encouraged but your work has to be finished by you own.
Due on Wednesday before the class.

Be on time!!!

Things before we start...

Grade:

Your final grade depends on the overall performance of the class.

Homework	15%
Lab Report	25%
Exam 1:	30%
Exam 2:	30%
Total:	100%

Important Days:

No Class Days:

09/04, Labor Day
11/20, 11/22, Fall Break
11/24, Thanksgiving

Exam Days (Tentative):

10/13 (F), First Exam
12/15 (F), Second Exam
Exams will be in ECCE 141

Having fun for the semester!