

# SPATIAL EFFECTS AND ECOLOGICAL INFERENCE

Luc Anselin\*

Wendy K. Tam Cho

University of Illinois, Urbana-Champaign

## Abstract

This paper examines the role of spatial effects, both spatial autocorrelation and spatial heterogeneity, in ecological inference. Both formally and through simulation experiments, we consider the problems associated with ecological inference and ecological inference methods in the presence of various forms of spatial effects. Although assessments of spatial autocorrelation in aggregate data analysis have focused on one-dimensional processes, these processes are not very representative of real-world data. Thus, they do not accurately portray how realistic spatial dependence affects cross-level inference. We focus on how realistic forms of spatial autocorrelation and spatial heterogeneity affect ecological inference, in particular, King's EI estimator.

For at least half a century, it has been known that making accurate individual-level inferences from aggregate data is extremely difficult (Robinson 1950, Goodman 1953, 1959). The problem, commonly called the ecological inference problem, is complex and multi-faceted, so our understanding of the nuances involved in modeling aggregate data are continually evolving. With any set of aggregate data—and with real data, in particular—it is difficult to ascertain whether the conditions under which micro-level or individual-level data will aggregate consistently and without bias are met. Indeed, making such determinations often proves sufficiently difficult to render the desired data analysis infeasible.

To make progress on the ecological inference problem, we approach the estimation from an under-traveled path. In particular, we focus on insights that may be gained from an explicit consideration of spatial effects (i.e., spatial autocorrelation and spatial heterogeneity) and how they affect the information contained and observable in aggregate units. We proceed by first defining spatial heterogeneity in the context of the ecological inference problem. Next, we discuss the inter-relationship between spatial effects and aggregation bias. We follow this with a brief outline of King's "solution" to the ecological inference problem (hereafter referred to as "EI") before then proceeding to classify different types of spatial heterogeneity and discussing how they pertain to the problem of making cross-level inferences. We then demonstrate the role of different forms of

---

\* Anselin's research was supported in part by NSF Grant BCS-9978058 to the Center for Spatially Integrated Social Science (CSISS).

spatial heterogeneity and spatial autocorrelation through a series of Monte Carlo simulations. We conclude by outlining some directions for future research.

#### EXTREME SPATIAL HETEROGENEITY

Thus far, much of the work on the ecological inference problem in the mainstream social science literature has focused on the condition of aggregation bias, while other definably troublesome characteristics of aggregate data have received less attention (e.g., Ansolabehere and Rivers 1997, King 1997, Cho 1998). Something of an exception to this general pattern is the literature in geography, where ecological inference is often included under the study of the “modifiable areal unit problem” or MAUP (Openshaw and Taylor 1979). Here, in addition to concern over aggregation bias, considerable emphasis is also placed on the issue of “zoning,” in the sense that one is mindful of selecting the spatial arrangement and the size of the aggregate units of observation themselves. In this paper, we abstract from this particular geographic perspective, and approach the ecological inference problem differently, as an example of *spatial heterogeneity*, or the phenomenon where a model (parameters, functional specification, error specification) is not constant across observations (Anselin 1988, 1990).

For ease of exposition, we will use the same notation as King (1997) and will couch our examples in the same terminology, that of racial voting patterns in geographic units called “precincts.” We begin with the basic accounting identity that relates total turnout by precinct,  $T_i$ , to the composition of the precinct’s population in terms of two mutually exclusive and exhaustive subgroups, say, proportion non-white,  $X_i$ , and proportion white,  $(1 - X_i)$ . The parameters in the model are the unknown (and usually unobservable) proportions of non-white turnout,  $\beta_i^b$ , and of white turnout,  $\beta_i^w$ . The relationship is

$$T_i = \beta_i^b X_i + \beta_i^w (1 - X_i).$$

This accounting identity holds for each of the  $p$  precincts in the data set, yielding a system with  $p$  equations in  $2p$  unknowns.

From a classical (non-Bayesian) perspective, estimation of the parameters in this model is a special case of the *incidental parameter* problem. More precisely, no consistent estimator can be constructed for the individual parameters, since no informational gain results from obtaining additional observations (each additional observation creates two more parameters to estimate). The standard approach to deal with this issue is to treat the incidental parameters as “nuisance parameters” and condition the estimation process on their values in order to obtain consistent estimators for the other (non-nuisance) parameters of interest (for a review of the technical issues, see Lan-

caster 2000, and the classic paper by Neyman and Scott 1948). However, this approach does not apply in the ecological inference context, since the parameters of interest are in fact the incidental parameters. To be clear, in a classical framework, there is no consistent estimator that can be constructed for the individual parameters,  $\beta_i^b$  and  $\beta_i^w$ . In the spatial econometric literature, this situation is referred to as *extreme spatial heterogeneity* (Anselin 2000) and the “solution” is to impose spatial or geographical structure on the nature of the variation of the individual coefficients across observations. However, this is only a partial solution, in the sense that the parameters to be estimated cannot be incidental, and thus must be constant in some manner across observations (or subsets of the observations). King applies this logic in his model: the rationale underlying King’s EI estimator is to assume a constant  $\beta^b$  and  $\beta^w$  across all observations in the data set, and to treat any heterogeneity as random variation around the constant means. In essence, the constant means (and the covariance matrix for the errors around the means) are the parameters that are estimated. The random coefficient paradigm then yields a statistical model to construct “optimal” predictors for the individual  $\beta_i^b$  and  $\beta_i^w$ , based on the estimates of the overall mean and the associated covariance matrix (e.g., Griffiths 1972). We return to this point in more detail below.

While the random coefficient model may properly capture heterogeneity in some situations, it ignores other more definite forms of spatial structure that may lead to the heterogeneity. The purpose of this paper is to assess the extent to which such spatial structure may aid in the process of specifying and estimating the underlying parameter. A critical issue here is determining the extent to which the assumed spatial structure can be verified solely through observations of the aggregate data. This is an area where research is sparse, at best.

#### SPATIAL EFFECTS AND AGGREGATION BIAS

Before we turn to specific models, we first review the connection between spatial effects and the more familiar notion in the ecological inference context, aggregation bias. Recent work on the ecological inference problem has emphasized that the origin of the cross-level inference problem is aggregation bias. Aggregation bias occurs when the parameters in the model are correlated with the regressors. For instance, for the model outlined above, if the parameters,  $\beta_i^b$  or  $\beta_i^w$ , are correlated with the regressors,  $X_i$ , then aggregation bias exists and making cross-level inferences is not straightforward. In terms of the random coefficient model that we consider in more detail below, aggregation bias amounts to a correlation between the random variation around the common mean and the regressor  $X_i$ . As long as the latter is assumed to be exogenous, there cannot be any aggregation bias. However, the assumption of exogeneity is suspect and not typically reasonable. Indeed, one can conceive of many instances where the variation in  $\beta_i^b$  (or in  $\beta_i^w$ ) would, in fact, be

a function of  $X_i$ . Hence, in order to produce accurate estimates of the micro-level behavior, one must re-specify the model in such a way that the parameters will be mean independent of the regressors. This can, for example, be accomplished by incorporating the functional relation between the parameter and the  $X_i$  explicitly.

Since problems of ecological inference typically involve geographic units, the link between spatial patterns and ecological inference should be evident. The nature of the effect, whether one exists, and how different forms of spatial effects affect the analysis, however, are not clear. In their book on cross-level inference, Achen and Shively (1995) focus much of their discussion of aggregate data analysis on a problem they term “intraconstituency spatial autocorrelation—the unmeasured similarity of voters in the same district” (See Achen and Shively 1995, Chapter 4). Their claim is that a “properly specified” aggregate data model must control intraconstituency spatial autocorrelation. In making this claim, they may seem to deviate from the literature that places aggregation bias at the forefront of conditions that must be controlled. While the Achen and Shively claim seems somewhat unorthodox at first, however, a closer reading reveals that they consider spatial autocorrelation and aggregation bias to be virtually one and the same. For instance, they state that “Different constituencies will exhibit different loyalty and defection rates, and it is only by quirk that these differences will fail to correlate with the aggregate independent variable” (1995, 106). The implication here is clearly that spatial autocorrelation is symptomatic of aggregation bias. Achen and Shively’s discussion of a solution—“what is needed for most applications is strong substantive knowledge of how individuals group themselves into constituencies and how best one might control for the resulting differences in mean disturbances” (1995, 114)—while framed in terms of spatial autocorrelation, clearly implies that fixing the autocorrelation problem will simultaneously aid in alleviating aggregation bias.

A key observation, then, is that the problems of spatial autocorrelation and aggregation bias, while separate and fundamentally different conditions, are, paradoxically, inter-related. Violation of one assumption is often accompanied by a violation in the other assumption (King 1997; Cho 1998). Hence, it is somewhat artificial and unhelpful to separate the discussion of these two assumptions, even though one can, of course, concoct data in which only one of the conditions occurs. In some of these special situations where the assumptions are separated, it is clear that aggregation bias is the more important assumption (King 1997, Cho 1998). However, even apart from the question of whether separating the assumptions is reasonable, King’s Monte Carlo evidence is quite limited and far less general than he implies. King’s rendition of spatial autocorrelation in his Monte Carlo simulations is carefully conceived and not generalizable to all, or even most, forms of spatial autocorrelation. And, since these conditions virtually never occur separately in real data, even if spatial autocorrelation were the less consequential assumption, insight into spatial autocor-

relation would remain important for any serious researcher interested in aggregate data. Hence, we explore the consequences of spatial autocorrelation in the context of aggregate data and discuss the link between these two obviously related assumptions in aggregate data analysis.

### KING'S ECOLOGICAL INFERENCE SOLUTION

The ecological inference solution proposed by King can be viewed as a combination of a random coefficients approach and the familiar method of bounds, couched primarily in a Bayesian framework.<sup>1</sup> It is useful for expository purposes to characterize King's EI primarily as a random coefficient model and to set aside the role of the bounds for now.<sup>2</sup> A basic assumption is that the heterogeneity in the model is due to random variation around an underlying common mean, which yields a regression model with heteroskedastic error terms.<sup>3</sup> Formally,  $\beta_i^b = \beta^b + \varepsilon_i^b$  and  $\beta_i^w = \beta^w + \varepsilon_i^w$ , such that

$$T_i = \beta^b X_i + \beta^w (1 - X_i) + u_i,$$

with  $E(u_i) = E[\varepsilon_i^b X_i + \varepsilon_i^w (1 - X_i)] = 0$ , provided that  $E(\varepsilon_i X_i) = 0$  (which is satisfied as long as  $X_i$  is exogenous, i.e., as long as there is no aggregation bias as defined above). The variance term then becomes  $\text{Var}(u_i) = \sigma_b^2 X_i^2 + \sigma_w^2 (1 - X_i)^2 + 2\sigma_{bw} X_i (1 - X_i)$  (again, with exogenous  $X_i$ ), which is heteroskedastic as long as  $X_i$  varies across precincts. How much this heteroskedasticity matters in any given situation is largely an empirical matter—it depends on the heterogeneity among the  $X_i$ , which can be tested by means of standard regression diagnostics (e.g., using a Breusch-Pagan Lagrange Multiplier statistic).

Note that in contrast to much of the theory and practice in the random coefficient literature, the error covariance,  $\sigma_{bw}$ , is taken to be non-zero since the Goodman identity implies a linear relation between the two parameters.<sup>4</sup> King exploits this relation to visualize the constraints on the acceptable parameter pairs through a tomography plot.

<sup>1</sup>For example, King's reference to "estimators" for the individual  $\beta_i^b$  and  $\beta_i^w$  parameters is clearly non-classical. In a classical treatment, those would be referred to as predicted coefficients. Also, King's EI produces a full posterior (simulated) density for these predictors, whereas a classical approach would be limited to point estimates. Other differences are highlighted in Anselin (2000).

<sup>2</sup>The main role of the bounds is to yield a truncated normal density as the basis for the likelihood and to provide additional information for use in the derivation of the posterior density for each individual coefficient. The other features of the EI approach are standard to random coefficient estimation.

<sup>3</sup>It is not totally clear from King's description of the procedure whether the common means must actually exist, or function purely as a device to incorporate some "common" patterns in the data. If there is no common mean, for example when the distribution is a mixture of two distributions with different means, then the resulting estimator will not have any desirable properties.

<sup>4</sup>The constraint is  $\beta_i^b = \frac{T_i}{X_i} - \beta_i^w \frac{(1-X_i)}{X_i}$ . Unless  $X_i = 1, \forall i$ , this implies a non-zero covariance between the two coefficients. In contrast, see for example Griffiths et al. (1979), where the covariance between the random components is set to zero.

As long as there is a common underlying mean, or as long as the precinct-specific bounds do not logically preclude the existence of a common mean, it can be estimated consistently without any further distributional assumptions. Specifically, a consistent estimator such as feasible GLS does not require an assumption of normality. However, this does not hold when the truncation is explicitly considered as well. For example, King's EI approach imposes normality in order to estimate the common means and covariances from a likelihood that does incorporate the parameter constraints in the form of a truncated bivariate normal.

Once the overall parameters are obtained, they can be used to construct optimal predictors for the individual coefficients. As shown in Griffiths (1972), in the standard random coefficient model such an optimal predictor takes the form

$$\hat{\beta}_i = \hat{\beta} + \hat{\Sigma} x_i (x_i' \hat{\Sigma} x_i)^{-1} (y_i - x_i' \hat{\beta}),$$

where  $\hat{\beta}$  is a vector of common means,  $\hat{\Sigma}$  is a matrix with the estimates of the random error variances and covariances,  $x_i$  is a vector of observations on the explanatory variables, and  $(y_i - x_i' \hat{\beta})$  is the residual for observation  $i$ . In other words, the best linear unbiased predictor is obtained by allocating the residual to each of the individual  $\beta_i$ , using weights that are a function of the value of the  $x_i$  and the covariance of the random errors. As a result, the model yields a perfect fit for each observation.<sup>5</sup> Consequently, diagnostics based on the usual notion of fit or lack of fit are meaningless. The only notion of fit that may be used to construct diagnostics would be the one based on the overall mean  $\hat{\beta}$ , but there is no observable counterpart to assess the properties of the individual  $\hat{\beta}_i$ . King's EI uses Bayesian constructs to derive the posterior distribution of the  $\beta_i$ , conditional upon the common parameters  $\hat{\beta}$  and  $\hat{\Sigma}$  (following the principles outlined in Griffiths et al. 1979), while incorporating information on the precinct-specific bounds through  $T_i$  (using a truncated bivariate normal as the underlying distribution in the likelihood function).

A well-known practical problem in the estimation of random coefficients models is the lack of a positive definite covariance matrix  $\hat{\Sigma}$ . There are a number of ad hoc solutions that have been proposed to deal with this problem. King avoids these issues all together by enforcing positive definiteness (as well as an adherence to the parameter bounds) in the constrained maximum likelihood. Also, in contrast to the classical treatment of prediction of the individual coefficients, King generates a full posterior distribution that incorporates, in addition to the common parameters,  $\hat{\beta}$  and  $\hat{\Sigma}$ , the observation-specific bounds that are evident from the tomography plot as well.

In sum, the EI solution deals with a particular aspect of heterogeneity, conceptualized as random variation around a common mean. While this is a general method for dealing with hetero-

---

<sup>5</sup> $y_i - x_i' \hat{\beta}_i = y_i - x_i' (\hat{\beta} + \hat{\Sigma} x_i (x_i' \hat{\Sigma} x_i)^{-1} (y_i - x_i' \hat{\beta})) = 0.$

generality, the method does not incorporate any information about the particular structure of the heterogeneity. The heterogeneity may be “spatial,” for instance. There are, however, no attempts to exploit the underlying nature of the heterogeneity. In the next three sections, we review some spatial models that can be categorized as different conceptualizations of a substantive ecological inference problem, but with a focus on the spatial/geographic aspects of the issue. The first is based on spatially random coefficients, which follows the same general rationale as King’s EI estimator, except that spatial autocorrelation is introduced explicitly. The other two specifications deal with spatial heterogeneity in the form of respectively discrete changes in the underlying parameters (spatial regimes) or continuous variation over space (spatially varying coefficients).

#### SPATIALLY RANDOM COEFFICIENTS

It is not straightforward to incorporate full two-dimensional and multidirectional spatial dependence in a random coefficient specification such as EI. In previous studies of this issue (King 1997; Cho 1998), a simple form of dependence was considered, which is essentially one-dimensional as well as uni-directional. As a consequence, some of the standard features associated with spatial autoregressive processes are not present, such as simultaneity and induced heteroskedasticity (Anselin 1988). King’s specification (King 1997, 167), described in the context of Monte Carlo simulation experiments, is as follows:

$$\beta_i = \delta \beta_{i-1} + (1 - \delta) u_i,$$

where  $u_i$  is a (draw from) truncated normal variate and  $\beta_1 = u_1$ . In this expression,  $\delta$  is referred to as the “degree of autocorrelation”. However, upon closer examination, it is obvious that this is not an autoregressive process, but rather a weighted average of two truncated random variables. Moreover, there is nothing “spatial” about the process. This is not without consequences, since true spatial dependence precludes a recursive formulation for the process and instead causes all variates simultaneously to be determined. In other words, rather than taking  $\beta_1$  as the starting point and building up the other  $\beta_i$  recursively, in a true spatial process, all the  $\beta_i$  are determined jointly and simultaneously.

In a random coefficient model, the spatial dependence pertains to the deviations around the common mean, in other words, to the observation-specific error terms. A spatial autoregressive process for these error terms implies that large or small deviations from the common mean will tend to occur in spatial clusters, rather than spatially random as is assumed in the standard model.

Formally,

$$(\beta_i - \beta) = \rho \sum_{j \neq i} w_{ij} (\beta_j - \beta) + \xi_i,$$

or

$$\varepsilon_i = \rho \sum_{j \neq i} w_{ij} \varepsilon_j + \xi_i$$

where  $\varepsilon_i$  are the error terms,  $\xi_i$ , are i.i.d. innovation terms, and the index  $j$  pertains to the “neighbors” of  $i$ , as defined by the non-zero elements,  $w_{ij}$ , of a spatial weights matrix  $W$ .<sup>6</sup> In matrix notation, with  $\varepsilon$  as the  $p \times 1$  vector of random deviations from the common mean  $\beta$ ,  $W$  as the  $p \times p$  matrix of spatial weights and  $\xi$  as a  $p \times 1$  vector of i.i.d. innovations, the usual spatial autoregressive process is given as

$$\varepsilon = \rho W \varepsilon + \xi,$$

or

$$\varepsilon = (I - \rho W)^{-1} \xi.$$

The inverse matrix represents the so-called spatial multiplier which demonstrates the joint (simultaneous) nature of the spatial dependence. The complex nature of the resulting covariance matrix for the random coefficient model can be seen by substituting  $\sum_j a_{ij}^b \xi_j^b$  for  $\varepsilon_i^b$  and  $\sum_j a_{ij}^w \xi_j^w$  for  $\varepsilon_i^w$ , with  $a_{ij}$  as the row elements in the spatial multiplier inverse. This yields an error term of the form

$$u_i = \sum_j a_{ij}^b \xi_j^b X_i + \sum_j a_{ij}^w \xi_j^w (1 - X_i).$$

Since the  $\xi_j^b$  and  $\xi_j^w$  terms are uncorrelated across observations, the variance terms are still a function of the variance and covariance of  $\xi^b$  and  $\xi^w$ . However, the spatial multiplier terms,  $a_{ij}$ , induce extra heteroskedasticity. In addition, the autocorrelation yields non-zero covariance terms between the errors for different precincts. Ignoring this extra variance and covariance will yield inefficient estimates for the  $\beta$  and biased estimates for the variance terms.

It is important to note that the spatial autoregressive process of the “spatial lag” variety cannot be implemented in the EI model without violating the Goodman identity. For example, assume

---

<sup>6</sup>For a recent review of the issues involved with specifying spatial weights and incorporating them into regression models, see Anselin and Bera (1998).

such a process in the dependent variable,  $T$ . For ease of notation, represent  $T$  as a vector:

$$T = \rho WT + \beta^b * X + \beta^w * X^c$$

with the  $X$  and  $X^c$  as suitable vectors,  $\beta^b$  and  $\beta^w$  as vectors of precinct-specific coefficients and  $*$  as the direct (element by element) product. The corresponding reduced form is

$$T = (I - \rho W)^{-1}(\beta^b * X + \beta^w * X^c)$$

which is incompatible with the fundamental identity because turnout in a precinct would then be a function of the racial turnout characteristics in all other precincts in the systems, not just its own. In part, this is due to the assumption of an exogenous  $X_i$ . If one relaxes this assumption, one could conceive of the actual (spatially correlated) pattern of  $X_i$  to be the result of a spatial process yielding  $(I - \rho W)^{-1}X^l$ , where  $X^l$  is the latent spatially random original layout. However, this clearly falls outside the standard approach to the Goodman identity and so is not further pursued. In the simulations considered below, we will implement spatial autoregressive processes for the error terms.

#### SPATIAL REGIMES

Spatial regimes consist of geographic subsets in which the model parameters assume distinct values (Anselin 1988). For example, one can think of two subregions of precincts,  $S_g$  and  $S_h$ , that together exhaust the district. Rather than allowing the parameters,  $\beta_i^b$  and  $\beta_i^w$ , to be different for each precinct  $i$ , they may assume two distinct values, say  $\beta_g^b$  for  $i \in S_g$  and  $\beta_h^b$  for  $i \in S_h$  (Cho 2000). The constancy of parameters across subregions is a testable hypothesis, for example, by means of a spatial Chow test (Anselin 1990). A crucial assumption in the spatial regime approach is the delineation of subregions. This should be exogenous to the model, or aggregation bias will occur. Ideally, the delineation and regime estimation should be carried out jointly (e.g., regimes with endogenous switching). However, at this point this has not been attempted in a setting that also incorporates spatial dependence. In practice, exploratory spatial data analysis techniques such as local indicators of spatial association, or LISA (Anselin 1995) may be useful in suggesting local clusters or spatial outliers that may form the core of a subregion. In the simulations that follow, we will implement spatial regimes in the form of distinct spatial subsets for the coefficients and assess the extent to which EI estimates are affected by this particular form of heterogeneity. It should be noted that spatial regimes can (and often do) co-exist with spatial autoregressive error processes. However, we will not pursue this avenue here.

## SPATIALLY VARYING COEFFICIENTS

Spatially varying coefficients are a hierarchical approach towards modeling the spatial variation of the model parameters across observations. In its simplest form, also referred to as spatial expansion (Casetti 1997), each precinct-specific parameter becomes a function of a (small) set of exogenous variables such as the terms in a spatial trend surface (a polynomial in the latitude and longitude of the locations). For example, the individual  $\beta_i^b = \beta^b + \gamma^1 z_i^1 + \gamma^2 z_i^2 + \varepsilon_i^b$ . The presence of the random error leads to the same heteroskedastic disturbance as in the random coefficient model, but the regression includes several additional terms as cross-products of the  $z_i^k$  and the  $X_i$  (for technical details, see Anselin 1992). As in the spatial regimes model, the constancy of the parameters is a testable assumption and in principle the determination of the variables to be included in the expansion should be the subject of a careful specification search. In the context of ecological inference, additional complications results for the bounds on the parameters, in the sense that the  $\beta_i^b$  that results from the expansion specification must be contained in the 0–1 interval. A recent variant of a spatially varying coefficient model is the geographically weighted regression (e.g., Fotheringham et. al 1998). Essentially, this is a form of spatial kernel estimation which may be useful as an exploratory technique or as a diagnostic to assess the presence of spatial heterogeneity, but it is not a model of that heterogeneity, so it will not be further pursued in the current context.

## SOME MONTE CARLO RESULTS

In order to assess the effect of various forms of spatial effects on the performance of EI, we carried out a small number of Monte Carlo experiments. These experiments use the results of the EI software for both the EI estimator itself, as well as for Goodman’s estimator (essentially OLS).<sup>7</sup> At this point, we have not assessed any estimators that explicitly account for the forms of spatial heterogeneity described above. The simulations are carried out for a square grid of 10 by 10 cells, or 100 observations. To keep matters simple, each cell is assumed to have the same population (this eliminates the inherent variance instability due to proportions computed on a different base). Lastly, the  $X_i$  are the same for all simulations and are generated as uniform random variates.<sup>8</sup>

Three different “true” situations are modeled and considered. The first is the standard case used by King (1997), where the  $\beta_i^b$  and  $\beta_i^w$  are drawn from a truncated bivariate normal distribution by rejection sampling from an untruncated bivariate normal with parameters,  $\beta^b = \beta^w = 0.5$ ,  $\sigma_b = \sigma_w = 0.2$ , and correlation coefficient 0.3. The second case is the spatially random coefficient model,

<sup>7</sup>All tuning parameters for the EI estimation software were kept at their default settings unless otherwise specified in the Table.

<sup>8</sup>This latter feature may be unrealistic in the sense that empirical patterns for  $X_i$  show more regularity than a uniform random variate would suggest. On the other hand, it ensures the absence of aggregation bias.

where the error around the common mean is subject to a spatial autoregressive process. Both queen (8 neighbors on average) and rook (4 neighbors on average) spatial weights are considered, with spatial autoregressive coefficients of 0.2, 0.5, and 0.8. The other parameters were the same as in the base case. The third model is one of spatial regimes in which the  $\beta_i^b$  and  $\beta_i^w$  are drawn from truncated bivariate normal distributions with different means in different “regions” (the variance and correlation are kept to the same value as in the base case). A number of different situations are considered in which the regional means are symmetric and asymmetric around 0.5, and with both two (north, south) and four (four quadrants) subregions.

Loosely speaking, these simulated data exhibit patterns that we would see in actual voting data. Consider, for example, the voting behavior in Chicago and its surrounding areas. The core of Chicago is purely Democratic while the collar of Chicago tilts Republican. Throughout the rest of the state, patches of both types of behavior reside side by side. Distinguishable patterns of behavior that are related to neighboring areas are not uncommon. In fact, they are expected, and are not anomalous in most areas. Many neighborhoods are well-defined and similar to nearby locales. Provincial elections in the Vancouver area provide another such example. The westside of Vancouver exhibits a strongly competitive area for liberals, socialists, and conservatives. As one moves further east, the stronghold of the New Democratic Party becomes evident. Further east still, one notices a weakening of the socialist and a strengthening of the conservative Reform party. These evident patterns of spatial autocorrelation, spatial regimes, and simple spatial effects are captured in our simulated data.

We evaluate the performance of EI and OLS through our simulate data. Note that the evaluation of the performance of the estimators merits some special attention. Unlike “traditional” simulation exercises, the proper standard of reference is not unambiguous. EI reports a district-wide estimate, as well as precinct-specific point estimates (the mean of the posterior distribution) and the full distribution of the simulated posterior density. By contrast, the Goodman estimator provides only the district-wide parameter.<sup>9</sup> We consider a number of different criteria. First, to provide a sense for the effect of truncation and the sampling error of the simulations, we report the district-wide coefficients computed from the simulated “truths” for the individual  $\beta_i^b$  and  $\beta_i^w$ . In most of our designs, the truncated and untruncated means should be very close, since there is no truncation effect on the mean with  $\beta^b = \beta^w = 0.5$  for a symmetric distribution such as the normal distribution. The “bias” of the estimators is not computed with respect to this district-wide average however, but with respect to each individual  $\beta_i$ . In other words, what is reported as bias is in fact the average error (over all precincts) between the “predicted”  $\beta_i^b$  and its simulated value. For EI, the mean of the

<sup>9</sup>Technically, the Goodman model does provide precinct-level estimates as well. The estimate for each precinct is simply the same as the district-wide estimate. This is the constancy assumption.

posterior distribution is taken as the predicted value (for Goodman's method, the predicted value is of course the same in each precinct). An estimate of precision is provided by the mean absolute percentage error (MAPE), where again the deviation is measured between the predicted coefficient and its simulated value for each precinct. Finally, we also consider a more complete measure of the posterior density generated by EI by assessing the proportion of cases where the simulated truth fell outside the 80% coverage of the density (more precisely, where the simulated parameter was either in the first or last decile of the posterior density).

The results are reported in Table 1 for the EI estimates and in Table 2 for the OLS estimates of Goodman's regression. In general, each Monte Carlo run involved 250 repetitions. The Esims parameter that sets the number of simulations within each Monte Carlo run has a default value of 100. In general, we found that increasing this value to 1000 does not add much precision, if any at all. Note that the two asymmetric regime runs differed by this parameter only. The results from these two runs are indistinguishable.

#### PRELIMINARY CONCLUSIONS

Although our set of simulations could be more extensive, a few patterns in the performance of the two estimators under conditions of spatial effects are striking even in this small set. First, in contrast to earlier evidence regarding the effect of spatial autocorrelation, we find considerable increase in both MAPE and lack of coverage by the 80% range for high spatial autocorrelation ( $\rho = 0.8$ ). This effect is even more pronounced when the underlying pattern for the  $\beta_i^b$  and  $\beta_i^w$  is more polarized, which we refer to as "Asymmetric Means" in the tables. For both high and low levels of autocorrelation ( $\rho = 0.2$  and  $\rho = 0.8$ ), the EI estimates are biased, have an MAPE that is an order of magnitude larger (54.5% for  $\beta_i^b$  with  $\rho = 0.8$ ) and have more than 50% of the true values falling outside the 80% range.

The evidence of bias is interesting, since it does not seem to be present (or exists only slightly for the higher levels of autocorrelation) for the spatial autoregressive case with symmetric means. Moreover, for the Goodman or OLS estimates, while the effects on MAPE are similar, strikingly, there seems to be no evidence of bias for either the cases with symmetric or asymmetric means. In other words, our evidence refutes King's claim that spatial autocorrelation does not pose difficulties for EI (King 1997, 94). In other words, we have identified yet another strong assumption (no spatial autocorrelation) that must be met before EI can be considered a reasonable estimator. The EI model is not robust to violations in this assumption, and that the resulting effects when it is violated are significant and interfere with proper inference. The presence of a spatial autoregressive process has a biasing influence on the estimates of the standard errors (and possibly also on the estimates of the

Table 1. Monte Carlo Simulation Results for EI Model.

| Type             | Rep. | Esim | $\beta_i^b$ | $\beta_i^w$ | $\rho$ | District-wide |             | $\beta_i^b$ | Bias    | MAPE   | 80% out | $\beta_i^w$ | Bias    | MAPE   | 80% out |  |
|------------------|------|------|-------------|-------------|--------|---------------|-------------|-------------|---------|--------|---------|-------------|---------|--------|---------|--|
|                  |      |      |             |             |        | $\beta_i^b$   | $\beta_i^w$ |             |         |        |         |             |         |        |         |  |
| base             | 250  | 1000 | 0.5         | 0.5         | 0.5    | 0.5004        | 0.4998      | 0.5003      | 0.0001  | 5.988  | 16.46   | 0.4998      | -0.0001 | 4.700  | 16.48   |  |
| Symmetric Means  |      |      |             |             |        |               |             |             |         |        |         |             |         |        |         |  |
| queen            | 250  | 100  | 0.5         | 0.5         | 0.2    | 0.5001        | 0.5003      | 0.5005      | -0.0004 | 6.059  | 18.04   | 0.4993      | 0.0009  | 5.067  | 18.05   |  |
| queen            | 250  | 100  | 0.5         | 0.5         | 0.5    | 0.5034        | 0.4985      | 0.5013      | 0.0021  | 7.814  | 18.44   | 0.4985      | 0.0000  | 7.237  | 18.28   |  |
| queen            | 250  | 100  | 0.5         | 0.5         | 0.8    | 0.5002        | 0.5018      | 0.4892      | 0.0110  | 15.436 | 24.98   | 0.5095      | -0.0077 | 13.649 | 24.86   |  |
| rook             | 250  | 100  | 0.5         | 0.5         | 0.2    | 0.5015        | 0.5003      | 0.5028      | -0.0013 | 6.356  | 18.11   | 0.5005      | -0.0001 | 5.315  | 18.12   |  |
| rook             | 250  | 100  | 0.5         | 0.5         | 0.5    | 0.5018        | 0.4987      | 0.4989      | 0.0029  | 7.774  | 16.80   | 0.5008      | -0.0021 | 6.981  | 16.87   |  |
| rook             | 250  | 100  | 0.5         | 0.5         | 0.8    | 0.5064        | 0.4964      | 0.4943      | 0.0121  | 16.681 | 32.04   | 0.5076      | -0.0112 | 15.053 | 31.80   |  |
| Asymmetric Means |      |      |             |             |        |               |             |             |         |        |         |             |         |        |         |  |
| rook             | 250  | 100  | 0.2         | 0.6         | 0.2    | 0.2006        | 0.6017      | 0.2336      | -0.0330 | 31.084 | 37.02   | 0.5732      | 0.0285  | 8.568  | 36.28   |  |
| rook             | 250  | 100  | 0.2         | 0.6         | 0.8    | 0.1990        | 0.6029      | 0.2931      | -0.0940 | 54.553 | 51.70   | 0.5258      | 0.0770  | 18.25  | 51.464  |  |
| Regimes          |      |      |             |             |        |               |             |             |         |        |         |             |         |        |         |  |
| regimes          | 250  | 100  | 0.2         | 0.2         | 0.2    | 0.5017        | 0.5000      | 0.5192      | -0.0175 | 4.804  | 14.44   | 0.4859      | 0.0141  | 4.185  | 14.48   |  |
| regimes          | 250  | 100  | 0.4         | 0.7         | 0.8    | 0.5731        | 0.5348      | 0.5631      | 0.0010  | 4.913  | 24.72   | 0.5547      | -0.0199 | 5.099  | 24.60   |  |
| regimes          | 250  | 1000 | 0.4         | 0.7         | 0.4    | 0.5739        | 0.5349      | 0.5643      | 0.0096  | 4.813  | 25.18   | 0.5532      | -0.0184 | 5.212  | 24.98   |  |
| regimes          | 250  | 100  | 0.2         | 0.2         | 0.4    | 0.4986        | 0.4995      | 0.5112      | -0.0126 | 4.927  | 11.80   | 0.4877      | 0.0118  | 4.481  | 12.01   |  |
|                  |      |      | 0.4         | 0.4         |        |               |             |             |         |        |         |             |         |        |         |  |
|                  |      |      | 0.6         | 0.6         |        |               |             |             |         |        |         |             |         |        |         |  |
|                  |      |      | 0.8         | 0.8         |        |               |             |             |         |        |         |             |         |        |         |  |

Table 2. Monte Carlo Simulation Results for OLS Model.

| Type             | Rep. | Esim | $\beta_i^b$ | $\beta_i^w$ | $\rho$ | District-wide |           |             | Goodman's Regression (OLS) |        |             |         |        |
|------------------|------|------|-------------|-------------|--------|---------------|-----------|-------------|----------------------------|--------|-------------|---------|--------|
|                  |      |      |             |             |        | $\beta^b$     | $\beta^w$ | $\beta_i^b$ | Bias                       | MAPE   | $\beta_i^w$ | Bias    | MAPE   |
| base             | 250  | 1000 | 0.5         | 0.5         |        | 0.5004        | 0.4998    | 0.5005      | -0.0001                    | 6.040  | 0.4995      | 0.0001  | 4.790  |
| Symmetric Means  |      |      |             |             |        |               |           |             |                            |        |             |         |        |
| queen            | 250  | 100  | 0.5         | 0.5         | 0.2    | 0.5001        | 0.500     | 0.5004      | -0.0003                    | 6.092  | 0.4994      | 0.0008  | 5.106  |
| queen            | 250  | 100  | 0.5         | 0.5         | 0.5    | 0.5034        | 0.4985    | 0.5010      | 0.0024                     | 7.883  | 0.4987      | -0.0002 | 7.450  |
| queen            | 250  | 100  | 0.5         | 0.5         | 0.8    | 0.5002        | 0.5018    | 0.4936      | 0.0066                     | 16.214 | 0.5059      | -0.0042 | 15.193 |
| rook             | 250  | 100  | 0.5         | 0.5         | 0.2    | 0.5015        | 0.5003    | 0.5028      | -0.0012                    | 6.171  | 0.5005      | -0.0002 | 5.223  |
| rook             | 250  | 100  | 0.5         | 0.5         | 0.5    | 0.5018        | 0.4987    | 0.4995      | 0.0023                     | 7.765  | 0.5003      | -0.0016 | 6.970  |
| rook             | 250  | 100  | 0.5         | 0.5         | 0.8    | 0.5064        | 0.4964    | 0.5099      | -0.0035                    | 17.531 | 0.4948      | 0.0016  | 17.132 |
| Asymmetric Means |      |      |             |             |        |               |           |             |                            |        |             |         |        |
| rook             | 250  | 100  | 0.2         | 0.6         | 0.2    | 0.2006        | 0.6017    | 0.2007      | -0.0001                    | 16.107 | 0.6003      | 0.0014  | 4.35   |
| rook             | 250  | 100  | 0.2         | 0.6         | 0.8    | 0.1990        | 0.6029    | 0.1986      | 0.0005                     | 50.622 | 0.6038      | -0.0010 | 13.75  |
| Regimes          |      |      |             |             |        |               |           |             |                            |        |             |         |        |
| regimes          | 250  | 100  | 0.2         | 0.2         |        | 0.5017        | 0.500     | 0.5313      | -0.0296                    | 7.095  | 0.4759      | 0.0242  | 5.981  |
| regimes          | 250  | 100  | 0.4         | 0.7         | 0.8    | 0.5731        | 0.5348    | 0.5631      | 0.0010                     | 4.848  | 0.5547      | -0.0198 | 4.959  |
| regimes          | 250  | 1000 | 0.4         | 0.7         | 0.4    | 0.5739        | 0.5349    | 0.5641      | 0.0098                     | 4.796  | 0.5534      | -0.0186 | 5.160  |
| regimes          | 250  | 100  | 0.2         | 0.2         |        | 0.4986        | 0.4995    | 0.5181      | -0.0195                    | 5.861  | 0.4820      | 0.0175  | 5.254  |
|                  |      |      | 0.4         | 0.4         |        |               |           |             |                            |        |             |         |        |
|                  |      |      | 0.6         | 0.6         |        |               |           |             |                            |        |             |         |        |
|                  |      |      | 0.8         | 0.8         |        |               |           |             |                            |        |             |         |        |

covariances) in the random coefficient model. The effect seems primarily to manifest itself in terms of the precision of the EI estimates (i.e. the MAPE and coverage indicators). However, in the case where the means are asymmetric, the spatial autoregressive process seems to lead to bias as well.

A second interesting finding arose from the Monte Carlo simulations that examine the effect of spatial regimes on the properties of the estimators. Here, in contrast to spatial autoregressive processes, spatial regimes do not violate any of the model's basic assumptions. However, we find strong evidence that the lack of coverage of the EI values at the 80% level (and to a much lesser extent, the MAPE as well) is similarly affected by the existence of spatial regimes. There does not, however, seem to be any meaningful bias. Here again, the role of asymmetry seems to be pivotal. The results for spatial regimes that are not "polarized" within regimes exhibit fewer undesirable properties than the cases where the polarization *between* regimes differs as well (see the 0.4–0.7, 0.8–0.4 case).

Finally, we should emphasize that our results are in line with earlier findings (Cho 1998) that have demonstrated the virtually identical performance of EI and the "naive" Goodman OLS estimator. There is only a slight edge (if any) for EI in terms of MAPE, and an edge for OLS in cases of asymmetry. The estimators have otherwise performed similarly in the face of spatial effects.

- Achen, Christopher H. and W. Phillips Shively. 1995. *Cross-Level Inference*. University of Chicago Press.
- Anselin, Luc. 1988. *Spatial Econometrics: Methods and Models*. Boston: Kluwer Academic.
- Anselin, Luc. 1990. "Spatial Dependence and Spatial Structural Instability in Applied Regression Analysis." *Journal of Regional Science* 30: 185–207.
- Anselin, Luc. 1992. "Spatial Dependence and Spatial Heterogeneity: Model Specification Issues in the Spatial Expansion Paradigm." In J.P. Jones and E. Casetti (eds.). *Applications of the Expansion Method*. London: Routledge, 334–354.
- Anselin, Luc. 1995. "Local Indicators of Spatial Association—LISA." *Geographical Analysis* 27: 93–115.
- Anselin, Luc. 2000. "The Alchemy of Statistics, or Creating Data where No Data Exist." *Annals, Association of American Geographers* 90. Forthcoming.
- Anselin, Luc and A. Bera. 1998. "Spatial Dependence in Linear Regression Models with an Introduction to Spatial Econometrics." In A. Ullah and D. Giles (eds.). *Handbook of Applied Economic Statistics*. New York: Marcel Dekker, 237–289.
- Ansolabehere, Stephen and Douglas Rivers. 1997. "Bias in Ecological Regression Estimates." Paper in progress.
- Casetti, Emilio. 1997. "The Expansion Method, Mathematical Modeling, and Spatial Econometrics." *International Regional Science Review* 20: 9–33.
- Cho, Wendy K. Tam. 1998. "If the Assumption Fits...: A Comment on the King Ecological Inference Solution." *Political Analysis* 7:143–163.
- Cho, Wendy K. Tam. 2000. "Latent Groups and Cross-Level Inferences." *Electoral Studies*. Forthcoming.
- Fotheringham, A. Stewart, Chris Brundson and Martin Charlton. 1998. "Geographically Weighted Regression: A Natural Evolution of the Expansion Method for Spatial Data Analysis." *Environment and Planning A* 30: 1905–1927.
- Goodman, Leo A. 1953. "Ecological Regressions and Behavior of Individuals." *American Sociological Review* 18:663–64.
- Goodman, Leo A. 1959. "Some Alternatives to Ecological Correlation." *American Journal of Sociology* 64:610–625.
- Griffiths, William E. 1972. "Estimation of Actual Response Coefficients in the Hildreth-Houck Random Coefficient Model." *Journal of the American Statistical Association* 68: 329–335.
- Griffiths, William E., Ross G. Drynan and Surekha Prakash. 1979. "Bayesian Estimation of a Random Coefficient Model." *Journal of Econometrics* 10: 201–220.
- King, Gary, 1997. *A Solution to the Ecological Inference Problem: Reconstructing Individual Behavior from Aggregate Data*. Princeton: Princeton University Press.
- Lancaster, Tony. 2000. "The Incidental Parameter Problem since 1948." *Journal of Econometrics* 95: 391–413.
- Neyman, Jerzy and Elizabeth L. Scott. 1948. "Consistent Estimation from Partially Consistent Observations." *Econometrica* 16: 1–32.

- Openshaw, Stan and Peter Taylor. 1979. "A Million or so Correlation Coefficients: Three Experiments on the Modifiable Areal Unit Problem." In N. Wrigley (ed.). *Statistical Applications in the Spatial Sciences*. London: Pion, 127-144.
- Robinson, W.S. 1950. "Ecological Correlations and the Behavior of Individuals." *American Sociological Review* 15:351-57.