THE INTERPRETATION OF INVERTED METAMORPHIC ISOGARDS USING SIMPLE PHYSICAL CALCULATIONS

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Abstract. Several zones of major thrust faulting exhibit a juxtaposition of rocks of higher metamorphic grade over rocks of lower grade. This configuration may indicate, but does not require, that temperature gradients were temporarily inverted near the fault. We examine the physical conditions under which such a temperature inversion could occur. The overthrusting of hotter rock on colder rock can cause a temporarily inverted gradient both above and below the fault only if the time taken to underthrust rock from the land surface to a given depth, \( z_f \), on the fault is less than \( \pi \) times the characteristic time, \( \tau^* \), for diffusion of heat through the block above the fault, where \( \kappa \) is thermal diffusivity. An inverted gradient cannot form without heat generation in the fault zone unless \( V \times z_f \times \sin \delta \gtrsim 100 \), where \( V \) is the rate of underthrusting (in millimeters per year), \( z_f \) is in kilometers, and \( \delta \) is the dip of the fault. This simple criterion is sufficient to demonstrate that several examples of inverted metamorphic gradients cannot be explained simply by the thrusting of hot on cold rock without heat sources in the fault zone. Dissipative heating accompanying deformation can cause an inverted temperature gradient within and beneath the thrust zone, but whereas the overthrusting of hot upon cold rocks cools the fault zone, dissipation heats it. Thus the overthrusting of hot on cold rock and dissipative heating affect the temperature gradient and the maximum temperatures differently. We show that the magnitudes of the inverted temperature gradient and of the maximum temperature above the inverted gradient yield independent estimates of the rate of dissipative heating. Discrepancy between these estimates implies that some additional process must have occurred, such as the post-thrusting disruption of the isograds. If there is such a discrepancy, the maximum temperature probably provides the more reliable estimate of the rate of heating at the fault. We illustrate this analysis by applying it to reported inverted metamorphic zonation in the Pelona Schist, the St. Anthony Complex, the Mt. Everest region of the Main Central Thrust, and the Olympos Thrust. Petrological inferences of maximum temperatures, depths of metamorphism, and magnitudes of apparent inverted gradients, imply that shear stresses of about 100 MPa accompanied thrust faulting in some of these regions, and that some zones of inverted metamorphism have been tectonically thinned after the metamorphism occurred.

1. INTRODUCTION

Inverted metamorphic isograds are commonly observed in orogenic belts and in other settings where rapid convergence and thrust faulting can be seen to have been important [e.g., Graham and England, 1976; Hubbard, 1989; Le Fort, 1975]. Whether the high grade rocks were still hot when they were thrust over the lower grade rock is, however, unresolved for many areas. In some regions, the emplacement of high grade on lower grade rock seems to postdate the setting of the metamorphic isograds, and therefore the temperature structure may never have been inverted [e.g., Brunel and Kienast, 1986]. For regions where the temperature gradient is presumed to have been inverted, some authors have attributed the inverted gradient simply to rapid emplacement of hot on cold material, whereas others have ascribed it to localized heating due to dissipation on the fault surface.

One of the reasons for the different points of view is surely that different conditions apply to different regions, but another is that many studies have not considered quantitatively the physics of heat transfer. As a result, some deductions are inconsistent with the reported data. For instance, it is a common misconception that the emplacement of a hot upper block over a cold lower block can lead to an inverted gradient within which the temperature at the top of the lower block approaches the original temperature at the base of the upper block. A homely analogy is sometimes made with the movement of an iron over an ironing board [e.g., Spray, 1988]. We would not even mention this, obviously flawed, analogy were it not widely employed. Unlike the hanging wall of a thrust sheet, an iron has a far greater heat capacity than the thin layer of fiber over which it is drawn, and furthermore heat is generated within the iron at a rate designed to keep its temperature constant. The geological analogues of an electric cord and socket are not readily apparent.

While ignoring the quantitative aspects of the physics of heat transfer has led to some misunderstanding, the relationship of inverted isograds to the physical processes responsible for them has been almost totally obscured by the supposition that numerical solutions of the heat transfer equation are necessary for the interpretation of inverted isograds [e.g., Shi and Wang, 1987]. In some cases the ease with which numerical solutions can be made complicated has led to the incorporation of specific geometries that obscure the simple but fundamental physical processes and which make it virtually impossible for others to apply such solutions to their own problems.

We do not claim that inverted isograds always form by simple processes, but rather that an understanding of the simple processes that could form them is a prerequisite for the interpretation of inverted isograds. We focus this analysis on the aspects of inverted isograds that can be measured using metamorphic geothermometry and geobarometry and geologic mapping. These include not only an apparent inverted temperature gradient, but also the values of inferred temperatures at the top of a zone of inverted isograds, the thickness of the zone of inverted isograds, and
its depth. In the absence of dissipative heating, the base of a thrust sheet will cool as heat is conducted into the lower block. The value of the temperature at the top of the inverted sequence is therefore vital for evaluating the role of dissipative heating during thrusting. As we discuss further below, the width of the zone of inverted isograds can place an important constraint on the duration of the processes that could have generated an inverted gradient.

Our main objective here is to derive simple formulae, based on the physics of heat transfer, that describe the relationships between the parameters of thrust faulting (such as the rates of slip and dissipative heating, and the geometry of the fault) and the quantities that can be measured geologically. This approach allows us to set up a formalism within which one can test specific hypotheses about the origin of inverted metamorphic sequences. For example, one may evaluate (1) whether or not the temperature gradient is likely to have been inverted in a given set of tectonic circumstances, (2) how much it might have been inverted, (3) whether dissipative heating would be necessary for such an inverted gradient, and (4) what such an inverted gradient implies for the duration and amount of slip on the fault. We then apply these formulae in an illustrative fashion to selected published examples of inverted isograds.

2. BASIC CONSIDERATIONS

As is well known, yet often overlooked, a sequence of metamorphic isograds, or of pressure-temperature points determined from petrologic geobarometers and geothermometers, does not necessarily record a temperature gradient that ever existed. Metamorphism is likely to have been diachronous, so that conditions of pressure and temperature recorded at different levels and different times in a developing thermal regime will not, in general, reflect the temperature profile at any instant during that development [e.g., England and Richardson, 1977]. Bearing this in mind, our approach is first to examine the plausible range of thermal gradients that can be produced during thrust faulting. Then we examine inverted isograds from different settings in order to ask what physical processes could have caused their inversion.

There are three obvious sources of heat in a crustal environment: heating from below the lithosphere, radiogenic heating within the crust and the mantle lithosphere, and dissipative heating due to irreversible deformation. In general, heat is conducted outward from the interior of the Earth to its surface. The existence of an inverted temperature gradient implies the conduction of heat downward in the Earth, and there are two simple mechanisms for causing heat to flow in this direction. Sufficiently rapid emplacement of hot material onto colder material can cause heat to flow from the upper hotter block into the lower block. Heat can also flow downward from transient heat sources localized within a narrow depth range. Dissipative heating on an active fault is an obvious localized heat source that can cause an inverted gradient.

Radioactivity can contribute 50% of the surface heat flux in continental regions and clearly must be considered in the total heat budget. In principle, radiogenic isotopes in the crust could provide a localized heat source, if thrust over a lower block with less radiogenic heat production. Because radiogenic heating in the crust is distributed over many kilometers in depth, its contribution to the shape of the geotherm is merely to introduce a slight curvature. We will, however, be concerned mainly with the transient, markedly curved shape of the geotherm during thrust faulting. In these circumstances, the inclusion of modest curvature due to radiogenic heat production serves only to introduce tedious algebra that obscures the relevant physics. We ignore it except where we need to calculate steady state temperatures in the crust.

Without radiogenic heating, the initial temperature gradient through the crust and uppermost mantle, before thrusting begins, can be treated as linear:

\[ T = T_0 + G_0 z \]  

(1)

where \( T \) is temperature and \( z \) is depth. Even in very young oceanic lithosphere (~10 Ma), the curvature of the geotherm in the outer 30 km of the Earth is not so great that treating it as linear introduces an appreciable error. Again for simplicity, we ignore temperature dependence and spatial variability of thermal conductivity; England's [1978] numerical experiments show that differences between calculated temperatures obtained using models with variable conductivities and those using their average are well within the uncertainties of that average, and of the metamorphic inferences of temperatures. Accordionly we assume a constant value for the thermal conductivity, \( K \), of 2.25 W m K\(^{-1}\). The reader may adjust all temperatures given below for a different assumed value of conductivity, \( K' \), by multiplying them by \( K/K' \).

The overthrusting of hot on cold material and dissipative heating affect the temperature structure differently. The overthrusting of hot on cold material causes an inverted gradient both at the base of the upper block and at the top of lower block. In contrast, heat due to dissipation near the fault will flow from the fault so as to steepen the gradient in the upper block, while contributing to an inverted gradient only in the lower block. As we discuss below, when the temperature structure of the upper block approaches steady state during slip on the fault, the overthrusting of hot on cold material cannot produce an inverted gradient in either block, whereas dissipatively generated heat flowing downward from the fault can cause an inverted gradient in the lower block.

In principle, geologic mapping of the position of the fault with respect to the inverted isograds could distinguish between these two causes. In fact, many such thrust zones are broad shear zones (several kilometers wide), and it is difficult or impossible to determine how dissipative heat sources were distributed through such a zone when it was active. Moreover, because the rate of dissipative heating per unit volume is the product of shear stress and strain rate, the zone of maximum heating need not coincide with the zone of greatest strain.

Both potential contributors to an inverted gradient require diffusion of heat, and the characteristic distance from the fault affected by diffusion is virtually the same in each case: approximately \( \sqrt{Kt} \), where \( \kappa \) is thermal diffusivity and \( t \) is elapsed time. (Thermal diffusivity is related to the thermal conductivity, \( K \), by: \( \kappa = K/\rho c \), where \( \rho \) is density and \( c \) is specific heat.) Apart from the similarity in their characteristic lengthscales, the processes differ in how
temperature and the temperature gradients at the fault change with time, and an understanding of them is most easily obtained if they are treated separately. Because the heat flow equation is linear the two processes can be analyzed independently, and their contributions can then be added.

Two basic time constants characterize the thermal development of major thrust faults. The first is the time, \( t_1 \), when material originally at the toe of the thrust fault before the beginning of slip passes below the point of interest on the fault:

\[
t_1 = \frac{z_f}{V \sin \delta}
\]

where \( z_f \) is the depth of the fault at the point in question, \( V \) is the rate of slip, and \( \delta \) is the dip of the fault. The second characteristic time, after which temperature changes near the fault become sensitive to the fixed temperature condition near the surface of the earth, is the time constant for the diffusion of heat through a slab of thickness \( z_f \):

\[
t_2 = \frac{z_f^2}{\pi^2 \kappa}
\]

where \( \kappa \) is thermal diffusivity. Molnar and England [1990] showed that when \( t \) exceeds the sum of \( t_1 \) and \( t_2 \), steady state is approached, defined as the condition in which temperatures within the upper block do not change significantly with time.

3. ANALYSIS

In this section we derive simple formulae for the temperature at the fault, \( T_f \), and the thermal gradient in the upper block immediately above the fault, \( G_{ab} \), and in the lower block immediately below the fault, \( G_{lb} \) (see Figure 1). The fundamental condition that allows the thermal analysis of thrust faulting to be simple is that diffusion of heat parallel to the fault is negligible compared with the rate at which heat is advected by slip on the fault. There is, however, appreciable diffusion of heat perpendicular to the fault, because slip on the fault perturbs the gradient perpendicular to the fault much more than it does the gradient parallel to it. Numerical experiments addressing two-dimensional heat transfer in a thrust environment confirm that for slip rates of more than a few millimeters per year we may ignore diffusion of heat parallel to the fault [Molnar and England, 1990]. Therefore, they show that the conduction of heat into either the upper or the lower block from the fault can be treated using the one-dimensional heat conduction equation and appropriate boundary conditions.

We take as the geometry of thrust faulting that of a wedge of material, whose upper surface is held at a constant temperature of zero, sliding over a half-space whose temperature gradient tends to a constant value, \( G_0 \) at great depth. The interface between the wedge and the half-space represents the fault. Because diffusion parallel to the fault is negligible, the temperature distribution in any vertical slice through this geometry is the same as that within a slab of thickness \( z_f \) overlying the half-space, where \( z_f \) is the depth to the fault in the chosen slice.

3.1. Overthrusting of hot onto cold material without dissipation

For short times after the beginning of slip, the temperature at the fault, in the reference frame of the overlying block, decreases essentially linearly with time \( t \):

\[
T(z_f, t) = G_0 \left( z_f - \frac{1}{2} V t \sin \delta \right)
\]

(4)

(See Molnar and England [1990] for a derivation of (4) and

![Fig. 1. Sketch geotherms to illustrate calculations described in this paper. Curves show geotherms in a vertical section through a column containing a thrust fault at depth \( z_f \). Before underthrusting begins, the temperature is given by (1) and the geotherm is a straight line of slope \( G_0 \). In the absence of dissipative heating on the fault, temperatures drop below their initial values once underthrusting begins; inverted thermal gradients can form in the early stages of slip, but in steady state the temperature gradient below the fault cannot be negative (curve A, see section 3.1). Dissipative heating at the fault increases temperature both above and below the fault (curve B). In the presence of dissipative heating the geotherm can show a local maximum in temperature at the fault, the temperature gradient immediately above the fault (\( G_{ab} \)) being positive, and that immediately below the fault (\( G_{lb} \)) being negative (curve C, see section 3.2). The temperature at the fault (\( T_f \) and the depth of burial of the fault (\( z_f \)) may be inferred from metamorphic observations. A sequence of inverted isograds observed below a fault may imply that the metamorphism took place under conditions in which \( G_{lb} \) was negative. The paper develops simple formulae that use \( T_f, z_f \) and \( G_{lb} \) to obtain estimates of the rates of dissipative heating required to generate metamorphism observed in four different environments. One conclusion of this analysis is that postmetamorphic deformation in at least three of those environments has produced an apparent inverted thermal gradient that is far steeper than the thermal gradient, \( G_{lb} \), that existed during the metamorphism.)
for numerical experiments that verify it.) Physically, this equation merely states that the temperature on the fault at any time is an average of the temperatures in the upper and lower blocks an instant before the levels in each block are placed in contact at \( z = z_f \). This temperature regime holds until approximately the time \( t = t_1 \), when rocks that lay on the land surface before thrusting began first pass below the upper block at the point of interest.

We take (4) as the boundary condition for temperature at the upper surface of the semi-infinite half-space, representing the lower block, and at the lower surface of the plate of thickness \( z_f \), representing the upper block. In the following discussion, we consider only the temperature field of the lower block. Appendix A discusses that for the upper block and shows that, as would be expected, the two solutions yield gradients near the fault that are indistinguishable for the period of time, \( t_1 \), for which (4) applies, provided that \( t_1 \) is not much more than \( \pi \) times the thermal time constant, \( t_g \), of the upper block.

Let us consider the temperature distribution in the lower block of the thrust fault and perpendicular to the fault. Initially, the temperature gradient in the direction perpendicular to the fault is [from (1)]

\[
T(z^*, 0) = G_0 (z_f + z^* \cos \delta) \tag{5}
\]

where \( z^* \) is distance below the fault, measured perpendicular to the fault. The dependence upon time of temperature at the top of the foot wall is given by (4). Thus the perturbation of the temperature at the top of the lower block, away from its original value of \( G_0 z_f \), is

\[
\Delta T(0, t) = -\frac{1}{2} G_0 V t \sin \delta \tag{6}
\]

The change in temperature in a semi-infinite solid on whose surface the temperature varies as \( \Delta T(0, t) = at \) is given by Carslaw and Jaeger [1959, p. 63]:

\[
\Delta T(z^*, t) = 4at \text{erfc} \left( \frac{z^*}{2\sqrt{kt}} \right) \tag{7}
\]

where \( z^* \) is the distance into the half-space and \( \text{erfc}(z) \) is the second integrated error function [Carslaw and Jaeger, 1959, pp. 483-484]. Although the evaluation of (7) at different values of \( z^* \) and \( t \) might be useful for some problems, what is more important for us is the perturbation to the temperature gradient at the surface of the half-space, \( z^* = 0 \), as a function of time:

\[
\Delta G_{ib} = \frac{2at}{\sqrt{\pi kt}} \tag{8}
\]

Equation (8) shows that the gradient at the surface is simply the change in temperature at the surface, \( \Delta T(0, t) \), divided by a characteristic distance,

\[
\Delta z^* = \frac{1}{2} \sqrt{\pi kt} \tag{9}
\]

From (8), with \( a = -1/2G_0 V \sin \delta \), the perturbation, to the temperature gradient below and perpendicular to the fault is

\[
\Delta G_{ib} = -\frac{G_0 V t \sin \delta}{\sqrt{\pi kt}} \tag{10}
\]

Hence the full temperature gradient perpendicular to the fault in the lower block as a function of time is

\[
G_{ib} = G_0 \left( \cos \delta - \frac{V t \sin \delta}{\sqrt{\pi kt}} \right) \tag{11}
\]

For most major, active thrust faults, \( \delta \) is small (<20°), and little is lost by letting \( \cos \delta = 1 \). Moreover, in many geologic settings, it is impossible to determine accurately the dip of the fault when it was active. Thus in most of what follows, we rewrite (11) assuming that \( \cos \delta = 1 \):

\[
G_{ib} = G_0 \left( 1 - \frac{V t \sin \delta}{\sqrt{\pi kt}} \right) \tag{12}
\]

The decrease of the temperature gradient given by (12) cannot continue indefinitely. Because there are no heat sources in the upper block, the steady state temperature in it must eventually become approximately linear, and with no heat source at the fault, the gradient across the fault must be the same in both blocks at all times. Hence an inverted gradient, if it forms at all without dissipative heating, must vanish as steady state is approached.

The steady decrease in the temperature on the fault given by (4) must slow down after the time \( t \) exceeds \( t_1 \) (given by (2)). Therefore (4) and equations based on it should not be applicable when \( t \) exceeds \( t_1 \). In fact, for rapid rates of underthrusting (\( V \approx 30 \text{ mm/yr} \) and sufficiently great depths (~25 km), the temperature on the fault can decrease essentially linearly with time for perhaps 10% longer than \( t_1 \), but for lower rates (<10 millimeters per year) and especially from shallow depths, (4) ceases to be valid before \( t \) reaches \( t_1 \) (see Figure 4 of Molnar and England [1990]). For typical depths, dips, and slip rates in continental crustal settings, \( t_1 \) is only a few million years.

If heat could diffuse sufficiently rapidly through the upper block, then even if the temperature on the fault decreased rapidly, the gradient throughout the entire upper block might decrease so rapidly that near the fault it would not become inverted. The condition for this to occur may be found by using \( t_1 \) and \( t_2 \), defined by (2) and (3), and rewriting (12) as

\[
G_{ib} = G_0 \left( 1 - \frac{\sqrt{\pi t_2}}{t_1} \right) \tag{13}
\]

Because this equation applies only for \( t < t_1 \), it can be used to place a limit on how large \( t_2 \), or \( z_f \), must be for an inverted gradient to form. Substituting \( t = t_1 \) in (13) yields the following condition for an inverted gradient to form:

\[
t_2 \geq t_1 / \pi \tag{14}
\]

Alternatively, an inverted gradient is possible only if

\[
\frac{V \sin \delta z_f}{\pi k} \geq 1 \tag{15}
\]

These inequalities are only approximate, because they are based on the assumption that the temperature on the fault will cease decreasing rapidly when \( t > t_1 \). In any case, only a negligible inverted gradient could form if (14) or (15) did not apply. In more familiar units, the condition for an inverted gradient, (15), is that

\[
V \text{ (millimeters per year)} \times z_f \text{ (in km)} \times \sin \delta \geq 100 \tag{16}
\]
Hence for $x_f = 20$ or $40$ km, and $\delta = 15^\circ$, for the gradient to become only slightly inverted $V$ must be greater than about 20 or 10 millimeters per year. Clearly inverted temperature gradients produced by thrusting hot or cold rock are likely only where rapid underthrusting occurs, or, for underthrusting at $\approx 10$ millimeters per year, at depths greater than $\approx 40$ km.

Consequently, although the overthrusting of hot on cold rock clearly can affect the temperature gradient near the fault, a restrictive range of conditions must be met if this is to be the sole cause of inverted isograds. Once underthrusting ceases, the inverted gradient decays and temperatures in the thickened crust rise above their syn-thrusting values on a time scale that is comparable with $t_2$. If $x_f = 40$ km, $t_2 \approx 5$ Myr and if $x_f = 20$ km, $t_2 \approx 1$ Myr. In order that such inverted gradients should be preserved in metamorphic assemblages and not be overprinted by minerals grown during subsequent temperature rises, substantial exhumation must occur within these few million years.

The discussion above has considered only the lower block. Appendix A presents a solution for the temperature gradient in the upper block, again assuming no lateral conduction of heat. It yields a condition on $t_2$ that differs from (14) by about 1%. Thus for all practical purposes, (12) and (13) apply to the upper block also.

3.2. Dissipative heating as a cause of inverted temperatures gradients.

In this section we combine expressions for the temperatures and temperature gradients near a thrust fault due to thrusting hot material on cold (section 3.1) with expressions for dissipative heating at the fault. Our goal is to obtain algebraic formulae that permit the calculation of shear stress and rate of slip on an exhumed fault from conditions of temperature, temperature gradient, and pressure inferred from observations made on rocks near the fault.

Dissipation in a fault zone provides a localized, essentially planar heat source whose magnitude is given by the product of shear stress, $\tau$, and the rate of slip, $V$. Of course in general dissipation occurs over a finite width but, provided that this width is small compared with $\sqrt{\tau t}$, the heat source can be treated as being planar. Because heat must diffuse away from such a planar source, dissipation in an infinitely thin zone can produce metamorphism of large volumes of rock. It is a mistake therefore to contend (as do, for example, Mohan et al. [1989, p. 107]) that because shear heating is a localized source of heat it must produce only local temperature rises.

The thermal development of a major thrust fault falls into two distinctive parts: in the time interval $t < t_1$, where $t_1$ is given by (2), temperatures on the fault change with time, whereas for $t > t_1 + t_2$, with $t_2$ given by (3), temperatures in a reference frame fixed to the hanging wall change very little [Molnar and England, 1990]. The temperature at the upper surface of the footwall, however, changes with time because of its movement down the fault. Thus the distribution of temperature on the fault in the interval $t < t_1$ differs from that after $t > t_1 + t_2$, and consequently the thermal gradient in the footwall is different in the two cases even if all other quantities remain the same.

3.2.1. Temperature gradients during slip for time $t < t_1$. In the time interval $t < t_1$ the temperature on the fault in the absence of dissipation is given by (4) and the gradient below the fault is given by (12). If, during this interval, heat is generated at a rate of $\tau V$ on the fault, its contributions to temperature at the fault, $\Delta T_{diss}$, and to the inverted gradient below the fault, $\Delta G_{diss}$, are given by Carslaw and Jaeger, 1959, p. 263.

\[
\Delta T_{diss} = \frac{\tau V}{K} \sqrt{\frac{\tau t}{\pi}}
\]

and

\[
\Delta G_{diss} = \frac{-\tau V}{2K}
\]

We can combine (4) with (17) and (12) with (18) to give expressions for the temperature at, and temperature gradient below, the fault at time $t < t_1$:

\[
T(x_f, t) = G_0 \left( x_f - \frac{1}{2} V t \sin \delta \right) + \frac{\tau V}{K} \sqrt{\frac{\tau t}{\pi}}
\]

\[
G_b(x_f, t) = G_0 \left( 1 - V \sin \delta \sqrt{\frac{t}{\tau \kappa}} \right) - \frac{\tau V}{2K}
\]

where we have ignored a factor of $\cos \delta$ in the first term on the right-hand side and therefore neglected the difference in directions between the normal to the fault and the vertical.

In principle, equations (19) and (20) could prove useful for the investigation of inverted metamorphic gradients formed early in the slip of major thrust faults and not subsequently expunged by later metamorphism. In practice, their quantitative utility depends on measurement of the time interval, $t$, of slip on an uncertainty much less than $t_1$. Because, for most major thrust faults, $t_1$, is a few million years or less and because the likelihood of obtaining precise dates for both the onset and the end of slip seems remote, we do not devote extensive analysis to equations (19) and (20). One specific case is worth noting, however: $t = t_1$, for which the inverted gradient (20) should be at its maximum value.

\[
T(x_f, t) = \frac{G_0 x_f}{2} + \frac{\tau V}{K} \sqrt{\frac{\kappa x_f}{\pi V \sin \delta}}
\]

and

\[
G_b(x_f, t) = G_0 \left( 1 - \sqrt{\frac{x_f V \sin \delta}{\pi \kappa}} \right) - \frac{\tau V}{2K}
\]

3.2.2 Steady state temperature gradients. When $t > t_1 + t_2$, the temperature distribution in the upper block does not change significantly with time [Molnar and England, 1990]. The temperature in the lower block, however, changes continuously with time as it slides beneath the upper block. The upper and lower blocks undergo thermal evolutions for different durations. The upper block is influenced by thrust faulting throughout the duration of slip. But when the top of the lower block reaches a depth $x_f$ on the fault, it has undergone a thermal perturbation only for a time $x_f/(V \sin \delta) = t_1$. During the preceding interval, $t - t_1$, it lay at the Earth's surface. The temperatures of rocks at the top of the lower block during their movement down the fault can be calculated from the ex-
pressions below, which give the temperature at the fault as a function of depth.

At steady state the temperature in the upper block is simply the steady state temperature resulting from the existing heat sources divided by a divisor $S$:

$$ S = 1 + b \frac{\sqrt{V \sin \delta_z J}}{\kappa} $$

(23)

See Molnar and England [1990] for a derivation of (23) and for numerical examples showing that $b \sim 1$. Equation (23) can also be written as $S = 1 + \pi \sqrt{\frac{J}{3}} / K$. Typical values of $S$ are between 1 and 10. Without dissipative heating,

$$ T(z_f) = \frac{G_0 z_f}{S} $$

(24)

(Radioactivity can be included in such an expression by using equations (19) and (24) of Molnar and England [1990] (see Appendix B), but we proceed without that adjustment here.) In the absence of dissipative heat sources, the gradient at the fault,

$$ G(z_f) = \frac{G_0}{S} $$

(25)

applies to both the entire upper block and to the upper part of the lower block. Clearly, this gradient cannot be inverted (negative).

With dissipative heating, the steady state temperature at the fault is

$$ T(z_f) = \frac{z_f}{S} \left( G_0 + \frac{\tau V}{K} \right) $$

(26)

and the gradient in the upper block is

$$ G_{ub} = \frac{1}{S} \left( G_0 + \frac{\tau V}{K} \right) $$

(27)

The fraction of the dissipative heating rate, $\tau V$, conducted through the upper block is $\tau V / S$. Hence the remainder, $(1 - 1/S)\tau V$, is conducted into the lower block, and when steady state applies to the upper block, the temperature gradient in the lower block is

$$ G_l = \frac{1}{S} \left( G_0 + \left( 1 - S \right) \frac{\tau V}{K} \right) $$

(28)

Clearly, this can be negative for sufficiently large values of $S$, $\tau$, and $V$. Equations (26) and (28) may be rearranged to give two alternative expressions for the rate of dissipative heating on the fault as a function of the initial gradient, $G_0$, the depth of the fault, $z_f$, the steady state temperature gradient below the fault, $G_{lb}$, and $S$. From (26):

$$ \tau V = K \left[ \frac{ST(z_f)}{z_f} - G_0 \right] $$

(29)

or, from (28):

$$ \tau V = \frac{K [SG_{lb} - G_0]}{1 - S} $$

(30)

Note that temperature gradients are positive if temperature increases with depth. Appendix B shows that a simple adjustment may be made to (29) to account for radioactive heat production in the upper block. In practice, as is illustrated in section 4.3 below, this adjustment leads to small changes in the estimated stress.

The two expressions (29) and (30) are related to quantities that can be inferred or measured. At first sight, it might seem natural to combine (29) and (30) to solve for both $\tau$ and either $S$ or $G_0$, but to do so involves making the assumption that $G_{lb}$ records a temperature gradient that did exist. This assumption is better tested than accepted. Accordingly, we adopt an alternative strategy that allows this test to be made.

Using (29), estimates of preslip thermal gradient and of $S$ may be combined with metamorphic estimates of $T_f$ and $z_f$ to give values of $\tau V$. Using (30), estimates of $G_0$ and $S$ may be combined with a metamorphic estimate of the synslip inverted gradient, $G_{lb}$, to give $\tau V$. In either case the preslip thermal gradient, $G_0$ and the magnitude of $S$ must be estimated independently of any quantity that can be measured in the rocks themselves. For a given combination of $G_0$ and $S$, (29) and (30) provide independent estimates of the rate of heating because they depend on different quantities inferred from metamorphic observations. If these independent estimates differ, as they do in some of the cases discussed below, there is a strong indication that a link is broken in the chain of logic joining temperatures and temperature gradients inferred from metamorphic observations to the actual temperature regime that obtained during slip on the fault. A plausible cause of such a break is synmetamorphic to postmetamorphic deformation that thins or thickens an inverted metamorphic sequence, hence giving a misleading impression of the magnitude of the inverted gradient, $G_{lb}$. In the analysis that follows, we attempt to reconcile the different estimates of shear stress by calculating the degree of thickening or thinning required to reconcile the shear stress calculated from the inverted gradient with that calculated from the temperature and pressure at the fault. By choosing this means of reconciliation we are placing relatively greater weight on the observations that give temperature and pressure conditions at the fault. Geologically speaking, this amounts to assuming there has not been appreciable metamorphic overprinting of the mineral assemblages after the main thermal event, but that the relative positions of rocks near the fault have been changed, thus biasing estimates of gradients made from their metamorphic assemblages. (Of course, metamorphic overprinting is a common occurrence, but overprinting that still leaves an inverted metamorphic gradient is a process that we do not consider.)

4. INTERPRETATIONS OF EXAMPLES OF INVERTED ISOGRADS

The analysis given above considered two sources of heat for inverted temperature gradients and two periods in the evolution of thrust faulting when inverted gradients could exist. Heat conducted from a hot overthrust block into a cold underthrust block and dissipative heat generated at the fault and conducted into the lower block can each contribute to inverted gradients. Transient inverted gradients can form early in the history of thrust faulting, either without dissipative heating, or enhanced by it. When the temperature field in the upper block reaches steady state, an inverted gradient can exist in the lower block only if dissipative heating is sufficiently large. Here we analyze selected zones of inverted isograd to address spe-
specific questions: could the inverted sequence represent an undeformed, in situ, inverted temperature gradient generated within the rock during thrust faulting? Could such a gradient have formed without dissipative heating? If not, what value of shear stress on the fault is necessary?

The purpose of this section is to illustrate the use of the formalism developed in section 3. In order to answer unequivocally the questions just posed, one would need to enter into a detailed analysis of the metamorphic and field observations relating to the area of interest. The metamorphic observations used to infer inverted thermal gradients are generally of the mineral parageneses or mineral compositions of the rock. Inferences of temperature and pressure in the tectonic settings discussed here are commonly based on the assumption that the observations relate to a near-equilibrium condition attained not far from the maximum temperature attained by the rock. We do not consider here the uncertainties, nor indeed the errors, involved in this chain of inference, and in the discussion below we refer to maximum temperatures, to pressures and to temperature gradients as though they could be determined without error. The rigorous discussion of uncertainty and error in metamorphic petrology is an invidious and time-consuming business which lies beyond our scope. Equally, we do not address the uncertainty in the structural or stratigraphic widths of the zones of inverted metamorphism.

Equation (12) provides an uncomplicated expression for estimating the slip rate required to generate a given inverted thermal gradient without dissipative heating. If dissipative heating occurs, however, the thermal evolution of the slip zone at short times (t < t₁) is likely to be influenced by factors not included in our simple analysis. Foremost among these may be the dependence of shear stress upon temperature [e.g., Hacker, 1990; Yuen et al., 1978]. For this reason, and for those set out in section 3.2.1 above, when we analyze temperatures during the interval t < t₁, we do not consider dissipative heating.

Because the expressions (29) and (30) are obtained from consideration of the steady state temperature on slip zones, they are independent of any complexity in thermal evolution that may result from temperature-dependent rheology in the slip zone. Thus provided that steady state conditions obtained in the slip zone of interest, (29) and (30) are appropriate expressions to use to evaluate the shear stress. Steady state is reached very rapidly once rocks that were at the Earth’s surface before slip began pass beneath the hanging wall at the point of interest (section 2 and see Molnar and England [1990]). Because the inverted metamorphism we discuss in this section developed in just such rocks, we use these steady state expressions. An alternative analysis for such rocks, using equations (21) and (22), would yield lower estimates of shear stress.

We analyze four examples of inverted isograds in order to consider a variety of tectonic environments and different aspects of the analysis (Table 1). The St. Anthony Complex in Newfoundland illustrates steeply inverted isograds at the sole of an ophiolite complex [Jamieson, 1986] which probably developed during rapid emplacement of an initially very hot upper block (the ophiolite). The Pelona Schist, California, shows metamorphism that may be characteristic of the upper part of a subduction zone [e.g., Graham and Powell, 1984]. The Main Central Thrust Zone near Mount Everest in Nepal [Hubbard, 1989] contains inverted metamorphism associated with a moderate rate of slip (10–25 millimeters per year) deep within continental crust. The Olympus Thrust in Greece offers an example of a shallow thrust fault above inverted isograds [Barton and England, 1979]. Our objective is merely to illustrate how the analysis presented above can be used. Where our conclusions are disputable, or where we cannot satisfactorily distinguish between hypotheses, further refinement of measurement and reapplication of this analysis may provide a resolution.

### 4.1. St. Anthony Complex

Jamieson [1980, 1986] reported a thin zone, ~1 km, of inverted isograds beneath a complete ophiolite suite of metamorphosed peridotite, gabbro, pillow basalt, and sedimentary cover. The peridotite at the base of the ophiolite suite was metamorphosed at temperatures of 850°-1050°C. An underlying “amphibolite” unit was metamorphosed at temperatures of about 650°-900°C, and the lowest, metavolcanic and metasedimentary unit in the sequence were metamorphosed at temperatures between 300°C and about 550°C. The thickness of the ophiolite suite at the time of development of the metamorphism is difficult to determine. Metamorphic pressures vary from about 0.3 GPa to over 0.7 GPa (Jamieson, 1986, Table 2).

<table>
<thead>
<tr>
<th>T, °C</th>
<th>V, mm/yr</th>
<th>S, °C/km</th>
<th>δ, deg</th>
<th>r, MPa from (29)</th>
<th>from (30)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pelona Schist</td>
<td>620</td>
<td>100</td>
<td>6.4</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Everest Region</td>
<td>750</td>
<td>25</td>
<td>3.0</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>Olympus Thrust</td>
<td>400</td>
<td>25</td>
<td>2.7</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>from (29)</td>
<td>from (30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Shear stresses are calculated from equations (29) and (30), using a value of 2.25 W m⁻¹ K⁻¹ for thermal conductivity, K, and a value of 10⁻⁶ m² s⁻¹ for thermal diffusivity, k.
We take a range of thickness of 10 to 20 km.

Jamieson [1986] emphasized that the peridotite and amphibolite units were cooling down during their metamorphism, and that only the lowermost, "greenschist," unit records prograde metamorphism. In view of these relations, it is natural to ask whether the metamorphism could be the result solely of the thrusting of hot rock on cold, as analyzed in section 3.1. If this were the case, the peridotite and the amphibolite unit would represent the cooling base of the upper block (Appendix A), while the greenschist would represent the warming top of the lower block (section 3.1). Because the greenschist units contain pillow lavas, it must have lain close to the earth's surface before thrusting began, and the condition for this unit to attain metamorphic temperatures of 300°-550°C without dissipative heating on the thrust fault is that the prethrusting temperature at the base of the overlying block should have been above 600°-1100°C. The peridotites within the St. Anthony Complex record a phase of deformation at temperatures of 850°-1050°C, which appears to be related to the earliest phase of metamorphism associated with the displacement of the ophiolite [Jamieson, 1986, Table 2]. Thus the ophiolite appears to have been hot enough to have produced the observed prograde metamorphism by contact.

That these rocks, at a depth of 10-20 km within the oceanic lithosphere, should have been at temperatures of 850°-1050°C, requires that the age of the ocean floor should have been between 1 Ma (1050°C at 10 km) and 10 Ma (850°C at 20 km) at the beginning of thrusting. This requirement is in agreement with the common presumption that ophiolites represent young oceanic lithosphere thrust rapidly onto nearby continental crust.

The thickness of the zone suggests that the isograds have been disrupted, as Jamieson [1986] argues. A thickness of less than 1 km implies, via (9), that the rocks were underthrusted for less than 0.04 Myr. In that time, less than 4 km of slip would have occurred, even if the thrust fault that emplaced the ophiolite were slipping at 100 millimeters per year. If we assume that the ophiolite was emplaced along a thrust fault with a dip similar to that of the upper part of active subduction zones (say, about 15°), then for rocks initially at the surface to have been buried to a depth of 10-20 km would require slip of 40-80 km, at least. Such slip would require at least 0.4 to 0.8 Myr, for a slip rate of 100 millimeters per year. Again via (9), the thickness of the zone of inverted metamorphism would have been 3-5 km after this time. Thus considerable structural thinning of the metamorphic sequence must have occurred.

From this analysis, it seems, as has been suggested by Jamieson [1986], that the metamorphism of the St. Anthony Peridotite may be explained entirely by the thrusting of hot upon cold rock, without the need to invoke dissipative heating. However, in view of the differences between ophiolites and among workers, it is worthwhile emphasizing the dependence of this conclusion upon its key parameter. The conclusion relies on the admissibility of a high temperature for the base of the upper block immediately before thrusting (see section 3.1). If, for example, the ocean floor were only 30 Myr old at the beginning of thrusting, the temperature at a depth of 10-20 km within it would be 250°-500°C and peak metamorphic conditions in the footwall without dissipative heating would be only 125°-250°C. Equally, if the fault emplacing the ophiolite slipped for an interval longer than $t_1$ (equation (2)), then the base of the ophiolite would have cooled to a steady state value given by (24) before it came in contact with the rocks it now overlies, and again the observed metamorphism of the lower block could not have been explained by the thrusting of hot rock on cold.

Thus if observations from other ophiolites indicate that the ocean floor was older than about 10 million years when it was emplaced, or that the fault along which it was emplaced was active for a long time, an alternative analysis, involving dissipative heating, should be carried out. The following section provides a suitable example.

4.2. The Pelona Schist, California

The Pelona Schist in California comprises a complex that may have been metamorphosed in a subduction zone [e.g., Graham and Powell, 1984; Peacock, 1987]. Graham and Powell [1984] estimated a peak temperature of 620°-650°C at the Vincent thrust, decreasing to 480°C, 600-700 m below the fault, corresponding to an inverted gradient of about 200°-280°C/km; we shall use the smaller of these values. Graham and Powell's estimated pressures of 800-1000 MPa imply a depth, $z_f$, of 30-35 km.

Unlike the case of the St. Anthony Peridotite, the hanging wall to the Vincent thrust could never have been hot enough to explain the inverted metamorphism by the juxtaposition of hot and cold rock [Graham and England, 1976]. Because of the likelihood that the inverted sequence marks an old subduction zone, where subduction was rapid and a steady state regime surely was attained at some time in the history of subduction, we analyze the metamorphic observations using the steady state expressions given in section 3.2. For the purposes of illustration, let us use a dip of 15° which is typical for subduction zones shallower than 35 km, and a rate of slip of 100 millimeters per year. We use an initial gradient, $G_{10}$, of 15°C/km, as being representative of old ocean floor, or of average continental crust.

We make two estimates of the required stress, assuming that steady state was reached during slip. The first, using (29) and estimates of $T_f$ and $z_f$ (Table 1), yields an estimate of 220 mW m$^{-2}$ for $rV$, or 70 MPa at 100 mm/yr. The second, using (30) with $G_{10} = -200°C/km$, yields an estimate of 540 mW m$^{-2}$, or 170 MPa at 100 mm/yr. These estimates of shear stress differ only by a factor of 2, and neither of them exceeds greatly previous inferences made by Graham and England [1976].

The present thickness of the sequence cannot be much greater than 2 km [Graham and Powell, 1984, Figure 5]. This thickness, if it represented an undisturbed thermal profile, would require, from (9), that the rocks were underthrust for only 0.1 Myr, during which interval slip of only 10 km would have occurred. Even at 100 millimeters per year, underthrusting to a depth of 30 km along a fault dipping at 15° would take more than a million years. Thus as is the case for the St. Anthony Complex, the preserved width of the zone of inverted metamorphism implies thinning of the metamorphic sequence. We may estimate the degree of thinning by finding the inverted gradient, $G_{10}$ in (30) that gives the same shear stress on the fault that is calculated from (29).
Estimates of shear stress calculated using (29) and (30) coincide if $G_b = -80^\circ$C/km, or one-third the value inferred from the present juxtaposition of rocks. Thus tectonic thinning by a factor of 3 could account for the preserved inverted thermal gradient. An initial thickness of 6 km for this zone is consistent with a duration of about 1 Myr for the underthrusting of the rocks preserving the metamorphism. With the slip rate and dip of fault assumed above, these rocks would have reached a depth of 25 km in 1 Myr. (Note that the interval 1 Myr is the duration of underthrusting of rocks of the footwall, not the duration of slip on the fault, which is indeterminate.) Graham and Powell [1984] calculate metamorphic pressures of 830 to 900 MPa for the top of the inverted metamorphic sequence and of 1000 ± 100 MPa for its base. For a density of 2800 kg m$^{-3}$, this range of pressures is equivalent to a depth range of zero to 9 km. Clearly, tectonic thinning of the zone of inverted metamorphism, now no more than 2 km thick, cannot be ruled out from, and may be indicated by, the metamorphic inferences of pressure.

The thermal profile calculated assuming a shear stress of 70 MPa (from equation (29)) is consistent with Graham and Powell's metamorphic observations if it is assumed that the inverted sequence has been thinned by a factor of 3 (Figure 2). Note that without dissipative heating, the temperature on the fault would be about 100°C.

4.3. Main Central Thrust Near Mount Everest

The Main Central Thrust zone of the Himalaya is notable for its inverted isograds, and Hubbard's [1989] profile of pressures and temperatures across it near Mount Everest is the most complete quantitative analysis of it. She found maximum temperatures of about 750°C decreasing to about 500°C in a zone that is present about 3 km thick. Hence the preserved disposition implies an inverted gradient of about $-80^\circ$C/km. Her inferred pressure of about 550 MPa for the rocks recording the highest temperature implies a depth of about 20 km for the fault.

We assume a rate of slip of 10-25 millimeters per year on the Main Central Thrust [see Lyon-Caen and Molnar, 1985; Molnar, 1987] and assume a dip of 15°. For simplicity, we assume that the steady state expressions of section 3.2 apply. For a slip speed of 25 millimeters per year, the rate of heating calculated from (29) is $rV = 200$ mW m$^{-2}$, and that from (30), with $G_b = -80^\circ$C/km, is 300 mW m$^{-2}$. The corresponding shear stresses are 250 MPa and 375 MPa. The two estimates of shear stress can be reconciled if the inverted metamorphic sequence is assumed to have been thinned by a factor of 2. For a slip speed of 10 millimeters per year, (29) and (30) yield shear stresses of 430 MPa and 1150 MPa, respectively, and thinning of the metamorphic sequence by a factor of 3 is required to reconcile the two estimates.

Hubbard [1989] plotted her pressures and temperatures in terms of their present structural position. In Figure 3 we plot her pressure and temperature data from rocks of her Dudd Kosi section without reference to their present position. The data appear to define a right-way-up metamorphic sequence above 20 km (pressure 560 MPa) and an inverted sequence for 8-10 km below this depth, approximately 3 times the present thickness of the zone of inverted metamorphism. Thus the difference between the present width of the zone of inverted metamorphism and its width inferred from the petrological data (Figure 3) supports the inference of structural thinning, drawn from the discrepancies between shear stresses calculated using (29) and (30).

Hubbard's [1989] observations are compared in Figure 3 with a thermal profile calculated assuming a shear stress
Fig. 3. As Figure 2, except that calculations relate to the inverted metamorphism in the Mt. Everest region (Table 1). The calculations are for a time 4.4 Myr after the beginning of slip, when steady state has been approached in the hanging wall; fault depth is 20 km; the initial temperature gradient is 25°C/km and the shear stress on the fault is 250 MPa. Circles relate to Hubbard's [1989] calculation of temperature and pressure from geothermometry and geobarometry on samples from her Dugh Kosi section. Pressures are converted to depth assuming an average density of 2800 kg m⁻³.

of 250 MPa on the fault and a slip speed of 25 millimeters per year, with the other parameters as in Table 1. With the exception of the point with the greatest pressure, all the observations are fit by this curve to within their uncertainties.

Unlike the other cases considered in this paper, the thermal regime near the Main Central Thrust was certainly influenced by the large amount of radiogenic heat production in the thickened continental crust of the region. To assess the influence of radiogenic heat production on our estimates of stress, we therefore employ (B5), rather than (29), to calculate the shear stresses from \( T_f \) and \( z_f \). For the two cases above, with \( V = 10 \) and 25 millimeters per year, we find that the calculated stresses are 330 and 210 MPa, respectively. (We have assumed that \( A_0D = 25 \) mW m⁻² and \( D = 15 \) km [Rao et al., 1976; England et al., 1992].)

4.4. The Olympos Thrust, Greece

Barton and England [1979] described a zone of inverted temperatures, about 3 km thick, beneath the Olympos thrust. Metamorphic geothermometry of the limestone and dolomite beneath the fault yield temperatures of 400°C (±10°C) just beneath the fault, 321°C (±21°C) at 1.5 km beneath it, and 286°C (±17°C) at 3 km beneath it, corresponding to an average inverted gradient of roughly 40°C/km in the 3 km beneath this fault, somewhat shallower than the average of 53°C/km in the upper 1.5 km. The observed width of the inverted metamorphism is at least 3 km, though it may be as much as 5 km thick [e.g., Barton and England 1979, Fig. 4; Figure 4 this paper]. Schermer (1990, Table 2, and E. Schermer, personal communication, 1991) determined that the pressure at the fault during its slip was at least 0.4 GPa. Let us assume a depth of 15 km to the fault, a dip of the fault of 15°, and an initial gradient of 15°C/km, corresponding to a temperature of <250°C close to, but not at, the fault during its slip [Barton and England, 1979; Schermer, 1990].

Mindful of the possibility of postmetamorphic disruption, we investigate a range of slip speeds, \( V \). We first estimate stresses using \( V = 25 \) millimeters per year. The stress estimated from the maximum temperature on the fault using (29) is 166 MPa, and the stress estimated from the inverted gradient using (30) is 203 MPa. The combination of slip speed \( V = 25 \) millimeters per year and shear stress of about 200 MPa lies within the band of conditions that Barton and England [1979, Table 2] deduced could account for the temperature regime inferred for the Olympos Thrust. Such an estimate may be reduced only by assuming much higher slip speeds. For example, if we assume \( V = 100 \) millimeters per year, we obtain estimates of \( \tau = 75 \) MPa (29) or \( \tau = 40 \) MPa (30). Note that, in this case, the shear stress estimated from the inverted gradient is lower than that estimated from the tempera-

Olympos Thrust

Fig. 4. As Figure 2, except that calculations relate to the Olympos Thrust. The calculations are for a time 1 Myr after the beginning of slip, when steady state has been reached in the hanging wall; fault depth is 15 km; the initial temperature gradient is 15°C/km and the shear stress on the fault is 166 MPa. Circles relate to Barton and England's [1979, Table 1] estimates of apparent temperature in rocks of the footwall from calcite-dolomite geothermometry, with the depth of the thrust fault taken to be 15 km (Schermer, 1990).
ture on the fault, and thickening of the zone of inverted metamorphism after it formed would be required to reconcile the two estimates of shear stress. Schermer [1990, and E. Schermer, personal communication, 1991], in fact, reports a phase of compressional deformation postulating that associated with motion on the Olympos thrust.

5. SUMMARY

Inverted temperature gradients may form during slip on major thrust faults under two conditions (section 3). Temperature gradients above and below the fault may become inverted during the early stages of slip provided that slip is rapid. Dissipative heating associated with deformation may maintain an inverted temperature gradient within and below the fault zone for the duration of slip, provided that the product of shear stress and slip speed is great enough. Each of these conclusions has been reached before in studies that have considered subsets of the conditions investigated here, that is in one-dimensional calculations [Barton and England, 1979; Bickle et al., 1975; Graham and England, 1976; Oxburgh and Turcotte, 1974] or in two-dimensional calculations without dissipative heating [Peacock, 1987].

5.1. Inverted gradients without dissipative heating

Oxburgh and Turcotte [1974] noted that if a thrust sheet were emplaced rapidly an inverted gradient could form beneath it. They suggested that the requirement for this to occur is that (in the notation of this paper) \( V \dot{z} \gg \gamma \lambda_x \). We have shown (section 3.1, equation (12)) that the vertical component of the velocity, \( V \sin \delta \), should be used in this expression, rather than the total slip velocity, \( V \). The condition (15) for inverted temperature gradients to be formed during thrust faulting may be expressed as follows: The time interval required to underthrust material from the Earth’s surface to a given depth, \( z_f \), on the fault must be smaller than the time constant, given by (3), for thermal relaxation of the hanging wall. For hanging wall thicknesses of 10 km or 30 km this interval is 0.3 Ma or 3 Ma.

An important aspect of inverted temperature gradients formed in this fashion is that the temperatures usually lie well below the steady state temperatures that would be approached after thrusting ceased. Once thrusting ceases therefore such inverted gradients would be overprinted in a few million years or less unless they were rapidly exhumed (Barton and England, 1979; Bickle et al., 1975; Graham and England, 1976; Oxburgh and Turcotte, 1974). This conclusion is also illustrated for a specific case by Peacock (1987, Figure 9). Preservation of inverted gradients formed by thrust faults in the absence of dissipation usually requires therefore a combination of rapid underthrusting and rapid subsequent exhumation. An exception to this requirement is encountered if the block above the thrust fault is extremely hot immediately before thrusting begins, and if it is emplaced within a time short compared with \( t_2 \) (equation (3)). Such conditions may be met during the emplacement of ophiolites: see section 4.1.

Last, it must be emphasized that in the absence of dissipative heating on the fault, temperatures there can never exceed the temperature in the hanging wall before the onset of slip. In many reported cases of inverted metamorphism the rocks of the foot wall are thought to have been at, or close to, the land surface before slip began. For these cases, temperatures at the fault should have been close to the average of the initial temperatures of hanging and footwalls and thus approximately half the initial (centigrade) temperature of the hanging wall. The existence of inverted metamorphic gradients with peak temperatures in excess of 600°C (e.g., section 4) thus would imply contact with a hanging wall whose initial temperature exceeded 1000°C [e.g., Graham and England, 1976]. If the evidence for such high hanging wall temperatures is lacking, or if such temperatures can be ruled out, alternative mechanisms for generating the apparent inverted gradient must be sought.

5.2. Inverted gradients produced by dissipative heating

Dissipative heating localized on major faults has been invoked by several authors to explain inverted metamorphic isograds [e.g., Barton and England, 1979; Graham and England, 1976; Le Fort, 1975; Reitan, 1980] and convergent metamorphic grade [Scholz, 1980; Scholz et al., 1979]. This suggestion has sometimes been discounted on the basis that the required shear stress exceed those expected from laboratory studies of rheology. To sustain this objection would be to admit no independent test of laboratory rheologies and, of course, one of the reasons for investigating inverted metamorphism is to provide such an independent test. The magnitudes of shear stresses estimated from earlier one-dimensional studies, and from the present two-dimensional study, are in the range of a few tens to a few hundreds of megapascals.

The range of shear stresses calculated in section 4 is consistent with the range calculated by Molnar and England [1990] from surface heat flow measured above subducting slabs in the Japan and Peru trenches. Shear stresses of around 100 MPa were also inferred by England et al. [1992] to account for the generation of the Manaslu granite of central Nepal during slip in the Main Central Thrust of the Himalaya. Thus a number of independent lines of evidence suggest that dissipative heating may be an important component of the thermal history of major faults.

APPENDIX A: THE TEMPERATURE IN A LAYER WITH TEMPERATURE ON ITS BASE UNIFORMLY DECREASING WITH TIME

If there is negligible lateral conduction of heat within the block overlying a thrust fault, we may treat the block as a layer of finite thickness whose temperature, \( T \), is governed by the one-dimensional diffusion equation:

\[
\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t} \quad \text{(A1)}
\]

where \( z \) is the direction perpendicular to the fault, \( t \) is time and \( \kappa \) is thermal diffusivity. The temperature boundary
condition at the top of the block is \( T(0,t) = 0 \), and at the bottom:
\[
T(z_f, t) = G_0 z_f - \frac{1}{2} V G_0 t \sin \delta
\]  
(4.1)

The initial temperature within the block is
\[
T_i(z) = G_0 z
\]  
(4.2)

It is convenient to write
\[
T(z, t) = T_i(z) - \Delta T(z, t)
\]  
(4.3)

\( \Delta T(z, t) \) obeys \( \Delta T(z, 0) = \Delta T(0, t) = 0 \) and
\[
\Delta T(z_f, t) = -\frac{1}{2} V G_0 t \sin \delta
\]  
(4.4)

The solution for \( \Delta T \) can be evaluated using Carslaw and Jaeger’s [1959, pp. 102–104] more general solution:
\[
\Delta T(z, t) = \frac{k \pi V \sin \delta G_0}{z_f} \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{\alpha_n z}{z_f} \left[ t - \frac{1}{\beta_n} (1 - \exp(-\beta_n t)) \right]
\]  
(4.5)

where \( \alpha_n = 2 n \pi / z_f \) and \( \beta_n = n^2 \pi^2 \sin \delta / z_f^2 \). Using
\[
T(z, t) = \Delta T(z, t) + \frac{k \pi V \sin \delta G_0}{z_f} \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{\alpha_n z}{z_f} \left[ t - \frac{1}{\beta_n} (1 - \exp(-\beta_n t)) \right]
\]  
(4.6)

(4.7) can be combined with \( T_i(z) \) to yield
\[
T(z, t) = G_0 \left[ z - \frac{z V t \sin \delta}{2 z_f} - \frac{V t \sin \delta}{\pi} \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{\alpha_n z}{z_f} \left[ t - \frac{1}{\beta_n} (1 - \exp(-\beta_n t)) \right] \right]
\]  
(4.7)

We seek the temperature gradient, \( G_{ub} \), in the direction perpendicular to the fault and at the fault, depth \( z = z_f \).
Using \( t_1 = z_f / (V \sin \delta) \), \( t_2 = z_f^2 / (\pi^2 \lambda) \) and \( \sum_{n=1}^{\infty} 1/n^2 = \pi^2 / 6 \), we obtain
\[
G_{ub} = G_0 \left[ 1 - \frac{t}{2 t_1} - \frac{\pi^2 t_1}{6 t_2} + t_2 \sum_{n=1}^{\infty} \exp\left(\frac{n^2 \pi^2 t_1}{t_2} \right) \right]
\]  
(4.8)

(4.9) This solution is only applicable for \( t < t_1 \), after which the boundary condition (4.1) no longer applies. Numerical evaluation of (4.9) yields a dependence of the temperature gradient at the base of the upper block \( G_{ub} \) that differs by less than 1% from the expression obtained (in 11.1) for the gradient at the top of the lower block \( G_{ib} \), provided that \( t_2 < t_1 / \pi \).

**APPENDIX B: THE RELATIONSHIP BETWEEN MAXIMUM TEMPERATURE ON, DISSIPATIVE HEATING AT, AND THERMAL GRADIENT BENEATH A FAULT, WITH RADIOACTIVE HEATING IN THE CRUST**

Equations (26) and (28) give expressions for the temperature on the fault \( T_f \) and temperature gradient beneath the fault \( G_{ib} \) at steady state in the presence of dissipative heating. The temperature on the fault is reduced by a factor \( S \) below the steady state temperature that would obtain in the absence of advection (section 3, and Molnar and England [1990]). Consequently, to correct (26) for internal heat generation within the upper and lower blocks, we need simply to calculate the contribution, \( \Delta T_r \), to the one-dimensional, steady state geotherm from the internal heat generation, and divide this quantity by \( S \).

\[
\frac{d^2 \Delta T_r}{dz^2} = -\frac{A(z)}{K}
\]  
(4.10)

where \( A(z) \) is the rate of internal heat generation per unit volume. Consider a distribution of heating given by
\[
A(z) = A_0 \exp(-z / D)
\]  
(4.11)

If such a distribution continues to a depth, \( z_f \), then its contribution to the temperature distribution in the upper block is
\[
\Delta T_r = \frac{A_0 D^2}{K} \left( 1 - \exp(-z_f / D) \right)
\]  
(4.12)

A second block beneath the fault, also with internal heating given by (4.11), will contribute a further \( A_0 D / K \) to the temperature, giving a total from internal heating of
\[
\Delta T_r = \frac{A_0 D}{K} \left( D + z_f \right) \left( 1 - \exp(-z_f / D) \right)
\]  
(4.13)

Hence from (29),
\[
\frac{\tau V}{K} = \frac{1}{G_0} \left[ \frac{k ST(z_f)}{z_f} - G_0 \times \left( 1 + \frac{A_0 D}{K} \right) \left( 1 - \exp(-z_f / D) \right) \right]
\]  
(4.14)

This equation differs from equation (34) of Molnar and England [1990] in retaining the term in \( \exp(-z_f / D) \). Note that equivalent expressions may readily be obtained, in the same fashion, for any other assumed distribution of heat generation.

The expression (28) for the steady state gradient below the fault is derived from the result that the flux of heat into the base of the upper block is the steady state heat flux in the absence of advection, divided by \( S \) (above). If a total amount of heat \( A_0 D \) (equation (28)) is generated in the lower block, the heat flux into the upper block is
\[
K G_{ub} = \frac{1}{S} \left( K G_0 + A_0 D + \tau V \right)
\]  
(4.15)

The difference between this flux and the flux of heat into the lower block is the rate of dissipation of heat per unit area on the fault, \( \tau V \). Hence
\[
G_{ib} = \frac{1}{S} \left( G_0 + \frac{A_0 D}{K} + \frac{(1 - S) \tau V}{K} \right)
\]  
(4.16)

and
\[
\tau V = \frac{K \left[ S G_{ib} - G_0 - A_0 D \right]}{1 - S}
\]  
(4.17)

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