AVERAGE REGIONAL STRAIN DUE TO SLIP ON NUMEROUS FAULTS OF DIFFERENT ORIENTATIONS

Peter Molnar
Department of Earth and Planetary Sciences
Massachusetts Institute of Technology

Abstract. If a region is cut by faults with different orientations and if the faults intersect the boundaries of the region of interest, then the average finite (rotational) strain, \( \varepsilon_{ij} = \partial u_i / \partial x_j \), is given by \( \varepsilon_{ij} = \Sigma M_{ij} / \mu V \) where \( \mu \) is the shear modulus, \( V \) is the volume of the region, and \( \Sigma M_{ij} \) represents the sum of asymmetric moment tensors, \( M_{ij} \). For each fault, \( M_{ij} = M_{ij} \hat{u} \hat{n} \), where \( M_{ij} \) is the scalar seismic moment and \( \hat{u} \) and \( \hat{n} \) are the unit vectors parallel to the slip vector and perpendicular to the fault plane. This result is applicable when the average regional strain occurs primarily by displacements of essentially rigid blocks that together compose the region of interest and hence when both elastic strains and inelastic strains due to folding and bending are small.

Introduction

Unlike simple plate boundaries, where slip seems to occur on one fault or on a narrow fault zone, in continental areas such as Asia or Western United States, deformation between lithospheric plates is diffusely distributed over broad areas. Deformation is not strictly continuous for it occurs (apparently mainly) by slip on faults. Hence, if we are to estimate the relative velocity or displacement of the plates bounding the broad zone of deformation, we must sum the effects of slip on faults of different orientations and with different rates or amounts of slip. Much of this slip occurs during earthquakes, for which our historic record is quite short. Nevertheless, the earthquake history can be used to make an estimate of the rate of deformation or rate of slip.

After Aki [1966] recognized the significance of the seismic moment as the fundamental parameter that quantitatively describes the size of an earthquake, Brune [1968] and Kostrov [1974] developed methods for summing the seismic moments of many earthquakes to estimate the cumulative slip on a fault or the strain in a region where earthquakes have occurred. Brune considered the simple case of a single, planar fault. Kostrov later used the moment tensor [Gilbert, 1971] to estimate the strain in a body due to slip on faults with a variety of orientations. Although neither formulation is wrong, Brune's is not a specific case of Kostrov's more general formulation for Kostrov's leads to different results from Brune's for the particular case considered by Brune. The purpose of this paper is to generalize Brune's formulation by modifying Kostrov's. This modification should allow an estimate of the average strain in a region due to slip on faults of different orientations when the boundaries of the region are not far from the faults that are active.

The essential difference between Kostrov's and Brune's formulations is that Kostrov assumes that there is no rotation of the region under consideration so that the average strain is pure shear. Whereas Brune, in fact, calculates the rotation due to repeated events on a single fault. When slip occurs on a fault, at the fault there appears to be a rotation about an axis lying in the fault plane and perpendicular to the slip vector. Geologists sometimes use the words clockwise and counterclockwise to define the sense of slip. Yet conservation of angular momentum requires that the moment causing the slip be balanced by an equal and opposite moment, which manifests itself by deformation concentrated near the ends of the fault. This deformation is (thought to be) elastic. Hence it is not necessarily permanent, and the strain energy stored at the ends of the fault can be released by subsequent earthquakes. Thus if one considers a region through which an active fault passes, slip on that fault will eventually cause a net rotation of material within that region. The surrounding volume, however, must undergo a rotation in the opposite sense to maintain equilibrium.

Kostrov [1974] implicitly considered a volume in which the margins are in the far field from each earthquake source. Since the far field cannot undergo a rotation due to the spontaneous release of strain energy within the volume, the average deformation caused by one or many events must be pure shear, with no net rotation. He developed a method for estimating the irrotational strain,

\[
\varepsilon_{ij} = \frac{1}{2} (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})
\]

Brune [1968], however, was not concerned with a volume but with a plane of finite dimensions. He therefore ignored the elastic deformation near the ends of the faults and considered only the slip that rotates one side of the fault past the other. The complementary moment associated with each event causes deformation outside of the region considered by him.

Thus Brune and Kostrov started from very different assumptions. Neither formulation is incorrect, and both are applicable to specific cases. The circumstances under which one is valid, however, violate the assumptions of the other. Moreover, neither is applicable to the situation in which many faults of different orientations are active and intersect the boundaries of the region (or volume) of interest.

Figure 1 shows a simple example where the faults intersect the edge of the region under consideration and Kostrov's formulation breaks down. Slip occurs on parallel faults so that the region under consideration undergoes simple
Consider a rectangular parallelepiped with dimensions and orientations shown in Figure 2 and with a vertical fault cutting it from one side to the other. Let slip $\Delta u$ occur on the fault in a purely strike slip sense. Let dextral shear on planes perpendicular to the x axis and sinistral shear on planes perpendicular to the y axis be positive, and let extension be positive.

Clearly, slip of an amount $\Delta u$ causes an average strain

$$\varepsilon_{yy} = \frac{\Delta u}{w} = \frac{\Delta u \cos \theta}{w}$$

(1)

Note that the equivalent seismic moment is

$$M_o = \mu L h \Delta u$$

(2)

where $\mu$ is the shear modulus, L the length of the fault, and h the thickness of the rectangular parallelepiped and hence the width of the fault. Note also that

$$l = L \sin \theta$$

(3)

With (2) and (3) we can rewrite (1) as

$$\varepsilon_{yy} = \frac{M_o}{\mu V} \sin \theta \cos \theta$$

(4)

where $V = L \cdot w \cdot h$ = volume of the region. Similarly there is an average shear strain

$$\varepsilon_{xy} = \frac{\Delta u \sin \theta}{w}$$

which with (2) and (3) becomes

$$\varepsilon_{xy} = \frac{M_o}{\mu V} \sin \theta$$

(5)

Expressions for the other two components of strain are slightly less obvious. Note that in Figure 2, the mean displacement of the left side of the body is $(\Delta u \sin \theta) w_1 / w$, and the mean displacement of the right side is $(\Delta u \sin \theta) w_3 / w$. Hence, the average change in length is

$$\Delta x = \frac{\Delta u \sin \theta (w_3 - w_1)}{w}.$$  

Since $w_1 - w_3 = L \cos \theta$,

$$\varepsilon_{xx} = \frac{\Delta u \sin \theta}{w} \left( \frac{w_3 - w_1}{w} \right) = -\frac{M_o}{\mu V} \sin \theta \cos \theta$$

(6)

Similarly, the difference in the mean displacements in the y direction of the ends of the region leads to

$$\varepsilon_{yy} = \frac{\Delta u \cos \theta}{w} \left( \frac{w_3 - w_1}{w} \right) = -\frac{M_o}{\mu V} \sin \theta$$

(7)

Equations (4)-(7) define an asymmetric strain tensor that is proportional to a seismic moment $M_o$. Note that this relationship will be the same for all earthquakes, so that $M_o$ can alternatively represent the sum of seismic moments of many earthquakes on the same plane.

An Asymmetrical Seismic Moment Tensor

The relationships expressed by (4)-(7) are reminiscent of those derived by Kostrov [1974].

![Fig. 2. Rectangular parallelepiped cut by vertical fault with strike $\theta$. Dimensions of body are $l$, $w$, and thickness $h$. Displacement $\Delta u$ occurs on a fault of length $L = l / \sin \theta$. $\Delta u$ is small compared with $l$, $w$, and $L$. The width $w$ is cut into segments of length $w_1$ and $w_2$ on the left and $w_3$ and $w_4$ on the right. Hence $w_1 + w_2 = w$, $w_3 + w_4 = w$, $w_1 + \cos \theta = w_1$, and $w_2 + \cos \theta = w_4$.](image)
He showed that the increment of average strain $\Delta \varepsilon_{ij}$ can be expressed as

$$\Delta \varepsilon_{ij} = \frac{M_{ij}}{2 \mu \nu}$$

(8)

where

$$M_{ij} = M_0 ( \hat{u} \hat{n} + \hat{n} \hat{u} )$$

(9)

and $\hat{u}$ and $\hat{n}$ are the unit vectors parallel to the slip vector and perpendicular to the fault plane, respectively. The strain due to different events, with different values of $M_0$, $\hat{u}$, and $\hat{n}$, can be summed linearly by adding the elements of the moment tensor.

Note that for the case in Figure 2,

$$\hat{u} = \sin \theta \hat{x} + \cos \theta \hat{y},$$

$$\hat{n} = -\cos \theta \hat{x} + \sin \theta \hat{y}$$

Hence, if we let $M_{ij}$ define an asymmetric moment tensor

$$M_{ij}^* = M_0 \hat{u} \hat{n}$$

(10)

Then from (4)-(7),

$$M_{ij} = \frac{\varepsilon_{ij}}{\mu \nu}$$

(11)

Note the similarities of (10) and (11) to (9) and (8), derived by Kostrov [1974].

Clearly, (10) and (11) are general, and the resulting average strain due to slip on many faults of different orientations can be obtained by summing the appropriate asymmetric moment tensors. Moreover, these expressions are applicable to the case where many faults are active with different rates of slip, either by slip during earthquakes or by fault creep. Using the amount of slip on the particular fault, one first calculates the appropriate values of $M_{ij}^*$ for each fault for a specific length of time. Then one adds the elements of $M_{ij}^*$ to estimate the amount of average strain that developed during that period of time. Finally, note that if faults are randomly distributed in orientation so that for each fault there is a conjugate fault with comparable displacement, then (10) and (11) yield the same average strain as (9) and (8). Note also, these expressions are not limited to earthquakes and large areas but also can be used to study the average deformation at the scale of outcrop and even hand specimens if amounts of slip on planes of different orientations can be determined.

**Discussion**

Equations (10) and (11) apply to the case where the average strain of a body is the result of movement of essentially rigid blocks that together compose the region of interest. These expressions neglect both elastic strain, which accumulates as faults slip and is stored in the region, and inelastic strain that cannot be described by displacement of rigid blocks along faults. Elastic strains probably never reach \%\% except perhaps locally, but the average strain of large regions, such as Asia or western United States, clearly is greater than 10\%. Thus neglecting elastic strains seems justified. The amount of inelastic strain due to bending or folding and the depths at which these processes occur are still uncertain. Consequently, by neglecting such inelastic deformation, (10) and (11) yield minimum estimates of the average strains.

In principle, (10) and (11) should be exact if we know the amounts of slip and orientations of all of the faults in a region. For most regions, however, our historic record of earthquakes is shorter than the recurrence interval of major earthquakes at each locality on the fault, and our knowledge of the faulting is incomplete. Thus an application of (10) and (11) is subject to errors that are difficult to evaluate. Nevertheless, insofar as the seismicity in a short time is representative of the long-term earthquake history of the region and the earthquakes occur on faults that cross the boundaries of the region of interest, then (10) and (11) should give a reasonable estimate of the average strain resulting from faulting.

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**References**


P. Molnar, Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139.

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