Slip-line field theory and large-scale continental tectonics

Paul Tapponnier
Laboratoire de Géologie Structurale, U.S.T.L., Montpellier, France

Peter Molnar
Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

A simple analogy is made between the tectonics of Asia and deformation in a rigidly indented rigid-plastic solid. India is analogous to the indenter and the great strike-slip faults correspond to slip lines. For various indentation geometries, the sense and linearity (or curvature) of strike-slip faults, convergence at the Burma arc and the existence of the Himalayan Burman Syntax, the conjugate strike-slip faults in Mongolia and the extension at the Baikal and Shansi graben can be predicted. Given the horizontal force necessary to support Tibet, an average shear stress of a few to several hundred bars along faults in Asia is predicted, corresponding to the yield stress of rigid-plastic material.

Rigid–plastic materials and plane strain slip-line fields

A plastic material is often assumed to be homogeneous, isotropic and perfectly plastic. There is no volume change and no strain hardening. A large class of problems, and those of interest here, assume a state of plane strain for which the two common criteria for the onset of plastic flow, or yielding (Tresca’s and Von Mises’s), are equivalent. Yielding occurs when the maximum shear stress, \( \tau = (\sigma_1 - \sigma_3)/2 \), reaches a limiting value, the yield stress, \( \tau_Y \), where \( \sigma_1 \) and \( \sigma_3 \) are the maximum and minimum compressive stresses, respectively. Because of the mathematical difficulties in matching boundary conditions across the boundary between regions of elastic and plastic deformation and because the deformation in the plastic region is much greater than that in the elastic region, the elastic region is often treated as a rigid solid. Hence the material is said to be rigid–plastic: plastic where the yield stress is reached and rigid elsewhere. Figure 3a shows the stress–strain curve for such a solid.

All analyses are made at the yield point, so that \( \tau = \tau_Y \) throughout the region which deforms plastically. In this plastic region the slip lines correspond to the maximum shear stress trajectories along which \( \tau = \tau_Y \). The mean (hydrostatic) stress, \( \sigma_n = (\sigma_1 + \sigma_3)/2 \), changes along the slip lines when they are curved, in a simple manner that can be determined from the equilibrium (Henceky) equations. Thus, from these equations, the geometry of the slip lines and the stress boundary conditions, the complete stress field in the plastic medium can be obtained easily.

Pure shear occurs on and parallel to the slip lines. They mark lines across which the tangential component of displacement can be discontinuous, as at faults in the Earth. The two types, \( \alpha \) and \( \beta \) lines, differ only in sense, and in geologic terminology are right- and left-lateral respectively. As slip lines are lines of maximum shear stress, they must intersect each other at right angles and all traction-free boundaries at angles of 45°.

Solutions to rigid–plastic problems can be divided into those of steady and of unsteady flow. For steady flow, the boundaries do not change shape or relative position, and the pattern of flow is independent of time. The drawing of wire is an example. For most tectonic problems, however, it appears that solutions only for unsteady flow are of interest. Again two types of solution can occur. In some cases, the deformation occurs in such a way that, although the region of deformation increases with time and, hence, the boundary conditions change continuously, the geometry of deformation changes only in scale. An example is the indentation of a rigid–plastic medium by an infinite rigid wedge (Fig. 2b). The pattern of deformation changes only by increasing in extent with time, proportionally to the rate of indentation. For most problems, however, the actual shape of the boundaries, and, hence, the boundary conditions, change as deformation progresses. The plane indentation solution shown in Fig. 2a is an example; the slip lines are appropriate only for the onset of yielding.

In principle, it is possible to determine the finite deformation of a rigid–plastic medium by successively and incrementally calculating the deformation for the given boundary conditions and re-evaluating the boundary conditions on the deformed boundaries. For the analogy with Asian tectonics, however, we do not know with confidence the detailed history of deformation, and therefore sophisticated computation of complicated deformation histories probably will not be very useful at present. On the other hand, we think we do know, qualitatively, how deformation occurred during the past few million years, and we are more concerned with understanding it than speculating on the previous history of deformation. Fortunately, because the solutions for plasticity problems are incremental, in so far as strain hardening can be neglected, knowledge of the previous history of deformation is not necessary for calculation of the slip lines at a given stage of the deformation.
Assumption of plasticity and plane strain in Asian tectonics

Although in detail there are obvious violations of the basic assumptions made for the solutions for slip lines of rigidly indented rigid-plastic media when this formalism is applied to Asia, the similarity of the patterns alone is enough to pursue the analogy further. Although strain may accumulate elastically during intervals between earthquakes, the total strain in Asia since the collision of India is a few tens of percent. Elastic strain at any instant and the total change of volume in the region will be negligible portions of the total deformation.

If the stress on and near active faults alternately accumulates and then drops during earthquakes, the regional stress-strain curve may resemble that shown in Fig. 3b. Although the stress may vary with time, when averaged over a long time (many earthquakes), the regional stress-strain curve would be similar to that shown in Fig. 3a, for a perfectly rigid-plastic solid. In the Earth, at slow strain rates, strain hardening may be a minor effect, particularly along fault zones where gouge alteration and strain heating are likely to occur. Moreover, at depths ≥ 20 km fault creep is likely to play an increasingly important role in the deformation of the crust. At even greater depths, the temperature is appropriate for plastic deformation to be dominant in most rocks at grain scale. Thus if most of the lithosphere at depth behaves plastically, in spite of the pressure dependent strength of the rocks in the top part of the crust, the assumptions of rigid plasticity and isovolumetric deformation are likely to be reasonable over a large scale and long time (10^5 yr) for the behaviour of the continental lithosphere as a whole.

Also, a plane horizontal strain analysis is justified for most of the deformation in Asia in the past 40 x 10^4 yr since the largest displacements (of the order of several hundred km (refs 5, 6)) seem to be horizontal movements along great vertical strike-slip faults.

Plane horizontal strain, however, does not take place everywhere in Asia. Important crustal thickening occurs at the Himalayas and Pamirs, and to a lesser degree in the Tien Shan and Nan Shan (Fig. 1). Further to the north-east (Fig. 1), crustal thickening occurs in the Baikal and Shansi graben belts. The deformation in these narrow mountain and graben belts, in fact, is close to plane vertical strain^7^, but is likely to be a fraction of that due to large scale strike-slip faulting, for two reasons: first, whereas strike-slip faulting is compatible with steady state deformation for long periods of time, in any given area crustal thinning is limited by the thickness of the crust itself and crustal thickening by the fact that, with increasing elevation...
of the mountains, an increasing amount of work must be done against gravity. Second, the horizontal maximum principal
stress necessary to cause plane strain ($\sigma_1 = \sigma_3 + 2\tau_\alpha$) is less for
plane horizontal strain through strike-slip faulting where
$\sigma_3 < \sigma_2 < \sigma_1$ than that for plane vertical strain through
thrusting and crustal thickening where $\sigma_3 = \sigma_1$. Crustal thickening
will occur when $\sigma_1$ is especially high as is the case in front of
the indenter or where there are pre-existing weak fault zones
of the right orientation, as might be the case in old orogenic belts
such as the Tien Shan or the Nan Shan (Fig. 1). In fact, that
the crust and upper mantle structure of Asia is not homogeneous
and may not be isotropic even on a scale of a few tens of km
may be one of the causes of the differences between the fault
pattern in Asia and the slip-line fields.

**Indentation geometry and Asian tectonics**

The deformation of a rigid–plastic body caused by indentation
depends strongly on the shape of the indenter. If the indenter is
flat, deformation extends into the semi-infinite rigid–plastic
medium to a distance approximately equal to the width of the
indenter (Fig. 2a). If the indenter is a wedge, at any given time
the deformation may or may not extend further into the medium
than the wedge itself depending upon the angle of the wedge
(Fig. 2b).

Assuming the boundary of (rigid) India to be along the
Himalayan Front and through the Kirthar and Sulaiman Ranges
in Pakistan, parallel to the Chaman Fault, it seems that India
has penetrated far into (plastic) Asia (Fig. 1), and therefore
the analogy in Fig. 2a can be improved upon. In the western
Himalayas and in Pakistan, the boundary of India is similar to
that of a wedge. Note that the Herat and Altyg Tagh faults are
approximately parallel to the slip lines that would be created
by wedge indentation (Fig. 2b). As the direction of motion
between India and Eurasia is approximately north–south, the
western edge of the Indian wedge is steeper than the eastern
dge. Correspondingly, the Altyg Tagh fault strikes more north-
easterly on the Herat fault. The right lateral North Anatolian
fault in Turkey and the left lateral Great Kabavi fault in Iran
have an analogous relationship to Arabia, which appears to
act as a wedge into Eurasia further west.

This simple analogy would predict strike-slip motion along
the boundaries of the wedge and the rigid–plastic medium (Fig.
2b). Strike-slip motion of the right sense is observed for both
the Chaman fault and the Dead Sea fault. There is also evidence for
strike-slip faulting along the Zagros suture zone and the
Karakanom fault. In addition, however, thrust faulting and
crustal thickening occur in both the Zagros and the Himalayas.
The direction of thrust faulting is approximately parallel to the orientation of the maximum compressive stress
in the plastic material (Fig. 2b).

An obvious feature of Asian tectonics is that deformation occurs over a vast area north-east of the Himalayas. If the wedge
indentation analogy held for all of Asia, we might expect
deforation north-east of the Himalayas to be bounded approximately by the Altyg Tagh fault, just as deformation
does not extend far north-west of the Herat fault. A possible
explanation for the widespread deformation north-east of the
Himalayas lies in the fact that the trend of the Himalayas to the
east curves and becomes approximately perpendicular to the
direction of relative motion of the plates. Thus the analogy
with the indentation solutions in Fig. 2a would be more applicable
to this region.

We think that the pattern of faulting north, east and even
south-east of the eastern Himalayas supports the analogy with
plane indentation by a flat indenter. The Kunlun and Kangtung
faults are approximately parallel to $\beta$ lines. The marked curvature,
particularly of the Kangtung fault, compared with the
straight Altyg Tagh and Herat faults would be expected from
their proximities to the edge of a flat indenter (Fig. 2a) and to a
wedge (Fig. 2b) respectively.

For the region at the eastern end of the Himalayas an even

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**Fig. 3** a. Deviant stress–strain curve for rigid–plastic and elastic–
plastic body. Once the yield stress, $\tau_\alpha$, is reached, plastic strain
proceeds at constant stress (no strain hardening). $b$. Qualitative
regional stress–strain behaviour of rocks in top part of crust.
Slow accumulation of elastic strain is followed by sudden stress
drops during earthquakes. With increasing regional strain,
average deviant stress may oscillate about roughly constant
value.
more appropriate analogy than plane indentation of a flat-sided rigid-plastic medium is with the indentation of an already indented or a hollowed-out medium (Fig. 2c). For this case, the slip lines continue to curve around 180° in order to intersect the ‘north–south’ boundaries at 45°. In our interpretation of the Landsat photos, the Kangting fault curves around so that east of the Himalayas it trends north–south before becoming lost in the complications of the tectonics further to the south. In addition, the right lateral Red River fault is approximately parallel to an a line. Although this fault is longer than might be expected from the analogy in Fig. 2c, this analogy is strongly supported because it predicts not only the curvature and sense of motion on the Kangting fault but also the sense of motion and trend of the Red River fault.

For us a particularly puzzling feature was the Burman arc, in front of which are a series of folds and east-directed thrusts and beneath which lies a belt of intermediate-depth earthquakes. Both observations imply a recent eastwards underthrusting beneath the arc. Thus, underthrusting of India beneath the Himalayas in a northerly direction and beneath the Burman arc to the east or south-east occurred simultaneously. In the indentation problem in Fig. 2c, material flows around the edge of the indenter and back in towards it. By analogy, we would expect India to squeeze part of Asia out of the way, so that this part of Asia extrudes around the edge of India and then encroaches upon its eastern side. As the Indian subcontinent is very narrow in eastern India between the Bay of Bengal and the Himalayas, oceanic lithosphere would have underthrust the Burman arc. Therefore resistance to the encroachment of extruded Asia would be less than if it encountered continental lithosphere. The Himalayan–Burman syntaxes and the surrounding tectonics are analogous to a corner in an indenter and the

![Diagram](image)

Fig. 5 Distribution of tectonic styles in Asia. Bold lines represent major faults (Fig. 1) (open thrust fault symbol for Pacific subduction zones). Dark shaded area—region of major crustal thickening, dotted area regions—where strike-slip faulting occurs, hatched area—normal faulting and crustal thinning. Corresponding stress states are indicated.
surrounding deformation when the indenter penetrates deeply into a rigid-plastic medium.

Boundary conditions at the limits of the indented block
The pattern of deformation in plane plastic strain also depends upon the shape of the boundaries, of the rigid-plastic medium, as is already clear from the differences in slip-line fields for the situations shown in Fig. 2a and c. For plane indentation as shown in Fig. 2a, it is assumed that the rigid-plastic medium extends to very large distances compared with the width of the indenter. When a plastic block of finite width is indented, the slip-line field is of the type shown in Fig. 4a. Plastic yielding occurs in the region between the indenter and the symmetric pair of slip lines that leave the corners of the indenter and intersect each other at the opposite side of the block. Thus the plastic region becomes wider as the ratio of the width of the block (b) to the width of the indenter (a) increases (Fig. 4a).

When this ratio reaches ~ 4.4, the deformation changes drastically to resemble that in Fig. 2a; material flows around the edges of the indenter instead of forcing the two parts of the block to separate rigidly away from a narrow zone of plastic flow between them. Because the lithospheric plates are bounded, the slip-line field in Fig. 4a might be more appropriate for some problems than that in Fig. 2a. In a perfectly plastic material, the slip-line field will presumably be that either in Fig. 2a or in Fig. 4a, but not both, and not a combination of them. In the Earth, it is possible that elements of both situations might find analogues simultaneously, both because the continents are not homogeneous and because the shape of the plate boundaries and the boundary conditions are more complicated than in the idealised situations of Figs 2 and 4.

Perhaps the most interesting characteristic of slip-line fields of the type shown in Fig. 4a is that a tensile state of stress (secondary tension) develops in the region adjacent to the boundary opposite the indenter. The cause of this tension is the decrease of $\sigma_n$ in the plastic medium away from the indenter. This decrease is expressed in Hencky's second theorem:

$$\sigma_n = \sigma_n(0) - 2\tau \gamma$$

where $\sigma_n(0)$ is the maximum hydrostatic pressure in front of the indenter and $\gamma$ the angle rotated clockwise along $\beta$ lines and counterclockwise along $\alpha$ lines (Fig. 4). Along with $\sigma_n$, both principal stresses $\sigma_2$ and $\sigma_3$ decrease proportionately to $\gamma$ (Fig. 4a). In the extreme case shown in Fig. 4a, where $h/a \approx 4.4$, $\sigma_n$ is reduced enough near the apex of the field that all stress components are tensile. $\sigma_3$ becomes negative already in the middle of the field. This basic pattern is observed for all slip-line fields of the class shown in Fig. 4.

If the analogy of plane horizontal strain was strictly applicable to the Earth, then at a given depth the lithostatic stress $\sigma_z$ would be equal to the intermediate stress $\sigma_i = \sigma_n$. In the Earth, $\sigma_z$ need not always be equal to $\sigma_n$ and when it is not, plane horizontal strain should not occur. There are two extreme situations in which this might occur: when $\gamma > \gamma_n = \pi/2$ or when $\gamma < \gamma_n = \gamma_i$. These are likely to happen far from the indenter or close to it, respectively. Far from the indenter, the stress state will be appropriate for crustal thinning and the formation of graben (Figs. 4b and 5). We think that the Baikal Rift and Shansi grabens are a consequence of the stress system in Fig. 5, which is analogous to that in the area of secondary tension in Fig. 4. The predominantly normal faulting, the direction of extension and the relative position of the area in which it occurs to the Himalayas are appropriate to the conditions in Fig. 4a.

Deformation near the indenter, and mountain building
Near the indenter, $\sigma_n$ is maximum and can be very large. When indentation starts it is likely that it will be large enough that $\sigma_n$ will be the least compressive stress. Consequently, thrust faulting and crustal thickening will take place in the Earth. This in turn will lead to an increase in elevation and therefore to an increase in $\sigma_n$ at a given depth below sea level. Plane horizontal strain will not take place until $\sigma_n$ is large enough to be approximately the intermediate principal stress. At this stage, deformation may proceed through strike-slip faulting. This pattern of a first major phase of deformation with shallow thrusts and folds and a late phase of predominantly strike-slip faulting is the basic pattern to be found in most ancient orogenic belts.

When plane horizontal strain is the dominant mode of deformation, then the mean elevation in any given area should reflect the local value of $\sigma_n$. The high altitude of Tibet and the general decrease in elevation away from Tibet in Asia may be a manifestation of the decrease in $\sigma_n$. Thus we think that both the distribution of tectonic styles in Asia (Fig. 5) and the decrease in mean elevation away from the Indian indenter may reflect a variation of $\sigma_n$ similar to that predicted by the analogy in Fig. 4a.

The yield stress of Asia
For simple indentation geometries, the pressure $\sigma_i$, applied by the indenter and the yield stress are simply related. For the case in Fig. 2a, $\sigma_i = 2\tau \gamma (1 + \pi/2)$; for that in Fig. 2c, $\sigma_i = 2\tau \gamma (1 + \pi)$; and for wedge indentation (Fig. 2b), $\sigma_i = 2\tau \gamma (1 + \psi)$, where $\psi$ is related to the wedge angle $\theta$ by (see ref. 1)

$$\cos(2\theta - \psi) = \frac{\cos \psi}{1 + \sin \psi}$$

For $\theta = \pi/4$, $\psi \approx 0.575$.

Assuming Tibet to be in isostatic equilibrium, we can estimate $\sigma_i$ for different depths of compensation. Because Tibet
is 5 km higher than the surrounding stable continental areas, for all depths shallower than the depth of compensation, the lithostatic pressure will be greater than that beneath the surrounding stable areas (Fig. 6). If we integrate this pressure difference from the surface to the depth of compensation, we obtain an estimate of the force per unit length, or the average pressure times depth of compensation, acting along the boundaries of Tibet. The value of this quantity depends upon how compensation occurs, but for all models in which compensation occurs primarily by crustal thickening beneath Tibet, the value is $\sim 5 \times 10^6$ Pa km$^{-1}$ (1 bar $\sim 10^6$ Pa).

By dividing this number by the thickness of the zone, $H$, assumed to behave analogously to a rigid-plastic material, we obtain an average estimate of $\sigma_1$. For a 100-km thick zone, $\sigma_1 = 5 \times 10^7$ Pa. From the relationships above, the yield stress, $\tau_y$, would be between $\sim 6 \times 10^6$ and $1.6 \times 10^7$ Pa. For a 50-km thick zone, these values should be doubled.

These estimates of $\sigma_1$ are only approximate and depend upon the assumption that the elevation of Tibet and the compensating mass deficiency are not maintained by vertical shear stress. Moreover, they represent average pressures, averaged over an assumed thickness ($H$), in which mechanical properties vary markedly with depth. If the yield stress estimated above was controlled essentially by the strength of the faults in the upper crust, where strain is relieved by earthquakes, the estimated strength of this brittle region would be larger than the values of $\tau_y$ given above.

For a range of thickness for brittle deformation of 20–40 km (ref. 16), the average stresses on faults would be 2.5–5 times larger than those estimated above, that is as small as $1.5 \times 10^7$ Pa or as large as $8 \times 10^7$ Pa. The number of assumptions make it impossible to be more precise. Moreover, stress could concentrate in localized regions on faults (asperities) to much higher levels, and the average stress may not reach $1.5 \times 10^7$ Pa for all earthquakes. In any case, the estimated range of average shear stresses on faults is higher than nearly all calculated stress drops of earthquakes ($10^8$–$10^9$ Pa), but is $< 10^8$ Pa.

**Summary**

When the pattern of faulting in Asia is compared with slip-line fields calculated for various plastic plane strain indentation problems several seemingly unrelated phenomena in Asia appear to have a common cause. India behaves like a rigid die that indents the rest of Eurasia, causing deformation over a large area that resembles deformation in rigid-plastic media. The linear Herat and Altyrn Tagh faults are similar to $a$ and $\beta$ lines, respectively, caused by wedge indentation by northwestern India. The more curved Kunlun and Kangting faults are similar in sense and trend to slip lines near the edge of a more planar indenter, in eastern India. The right lateral Red River fault and the convergence at the Burman arc are parallel to $a$ lines and the direction of plastic flow, respectively, for deformation near the corner of an indenter that has already penetrated deeply into a rigid-plastic solid. The simultaneous convergence at the Himalayas and flow around its eastern end causing convergence at the Burman arc is responsible for the Himalaya–Burman syntaxis. Tension is predicted at a large distance from a flat indenter, and the Baikal Rift Zone and Saniis Graben may be consequences of an analogous state of stress in Asia. From an estimate of the indentation pressure needed to maintain the high elevation of Tibet, we estimate that the average yield stress in a 100-km thick zone in Asia is $\sim 10^7$ Pa. If all of the yielding occurs by stress accumulation on faults and subsequent stress drops during earthquakes to a depth of 20–40 km, the average stress on the faults is a few hundred bars and less than a kbar.

We think that the analogy of plastic plane strain with the tectonic processes in Asia supports the contention that most of the tectonics of Asia are caused by the collision of India with Eurasia and provides a unifying explanation for the phenomena occurring there and in continental collisions in general. This type of approach may lead to an even more general understanding of the geographical distribution of different tectonic styles within continents at given epochs and the succession in time of different tectonic phases in a given continental area.

C. Goetz, M. Mattauer and F. McClintock clarified several points in this paper, and G. Garcia drafted the figures. This research was supported by the NSF, by the Laboratoire de Geologie Structurale and by an Alfred P. Sloan Fellowship.

Received June 28; accepted October 7, 1976.