CORNER FREQUENCIES OF $P$ AND $S$ WAVES AND MODELS OF EARTHQUAKE SOURCES

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ABSTRACT

$P$- and $S$-wave spectra of 144 aftershocks $\frac{1}{2} \leq M \leq 4\frac{1}{2}$ of the February 9, 1971 San Fernando earthquake corroborate previous work showing that the corner frequencies for $P$ waves in general are greater than those for $S$ waves. This observation is consistent not only with models that treat earthquakes as volume sources, but also with physically reasonable dislocation models for which (1) the source is approximately equidimensional, (2) both the duration of slip at each point on the fault and the time for the ruptured area to develop are not long compared with the time for seismic waves to cross the ruptured area, and (3) much of the source radiates essentially simultaneously. There may be other physically reasonable dislocation models compatible with the observations. Savage's calculations indicate that models that involve propagating dislocations on long thin faults are not adequate for describing most moderate and small earthquakes studied.

INTRODUCTION

Most earthquake source theories predict a far-field displacement spectrum that is constant at low frequencies and inversely proportional to some power of frequency at high frequencies (Aki, 1967; Brune, 1970; Haskell, 1964; Savage, 1966; et al.). A spectrum's corner frequency is defined as that frequency where the high- and low-frequency trends intersect. Different theories relate the corner frequency to different properties of the earthquake source and predict different relationships between the corner frequencies of $P$ and $S$ waves. Hence, observations of corner frequencies for both $P$ and $S$ waves from the same earthquake provide an important constraint on the various theories.

A point source at which stress is relieved instantaneously would radiate $P$- and $S$-wave displacement pulses that are Dirac delta functions (e.g., White, 1965, Chapter 5). Their spectra would, therefore, be flat. When the stress is relieved over a finite time, the radiated pulses are broadened proportionally and a corner in both the $P$-wave and the $S$-wave spectrum occurs at a frequency proportional to the reciprocal of the time for the stress to be relieved at the source.

Similarly, if stress is relieved instantaneously but the source size is made finite, the $P$ and $S$ pulses will be broadened, and their spectra will have corner frequencies. The spectra remain flat at low frequencies because the source is effectively a point source for long wavelengths. At frequencies higher than the corner frequency, the spectra decrease with frequency at least as fast as $\omega^{-3/2}$ because the energy is bounded. Because the $P$-wave velocity is higher than the $S$-wave velocity, the corner frequency due to interference effects is higher for $P$ waves than for $S$ waves. (At a given frequency the wavelength of $P$ waves is longer than that of $S$ waves.) This relation between $P$ and $S$ corner frequencies is common to most models that treat earthquakes as volume sources (Archerbou, 1968; Bullen, 1963 (pp. 75–77); Honda, 1962; Kasahara, 1957; Randall, 1966) and has been assumed for fault models (e.g., Hanks and Wyss, 1972; Trifunac, 1972; Wyss et al., 1971). (We use the term "volume source" for models that assume boundary conditions in or on the surface of a finite volume instead of on a plane surface.)
Another phenomenon that affects the spectrum is the manner in which the rupture area develops (Savage, 1966a, 1972). In general, it is not possible to isolate this phenomenon from those above. For example, in one dislocation model involving unilateral rupture (Haskell, 1964), the corner frequencies of both the $P$ and the $S$ waves depend on azimuth but, on the average, are approximately equal. Moreover, Savage (1972) showed that for bilateral rupture, on the average, the $S$-wave corner frequency is slightly larger than $P$-wave corner frequency. These relationships are characteristic of models of earthquakes representing long thin faults such as those of Haskell (1966) and Aki (1967).

Published data, however, indicate that, in general, the $P$-wave corner frequency is higher (Figures 1 and 2). These data include spectra determined from recordings made at teleseismic distances (Hanks and Wyss, 1972; Molnar and Wyss, 1972; Wyss and Hanks, 1972; Wyss and Molnar, 1972), and spectra of large aftershocks of the San Fernando earthquake recorded at short distances (Trifunac, 1972). These observed ratios of $P$-to $S$-wave corner frequencies range from about 1 to 2. This range is consistent with ratios of predominant periods of $S$ waves to those of $P$ waves determined by Russian scientists (Antonova et al., 1969, pp. 54–58). The most striking evidence for relatively higher frequencies for $P$ waves than $S$ waves comes from simple inspection of strong-motion records of earthquakes. For example, the aftershocks of the Imperial Valley earthquake of 1940 discussed by Trifunac and Brune (1969) show an obvious difference in predominant frequencies of more than a factor of 2. Such data are especially convincing because the hypocentral distances are small, and, thus, there is relatively little influence.
of propagation path on the waves. Such results are common to most strong-motion recordings of earthquakes, including the San Fernando earthquake (Trifunac, 1972).

**Data**

As further evidence that the $P$-wave corner frequency is higher than the $S$-wave corner frequency, we offer spectra from aftershocks of the February 9, 1971, San Fernando earthquake. The Richter (local) magnitude of these aftershocks ranged from $\frac{1}{2}$ to $4\frac{1}{2}$. The spectra obtained are perhaps more reliable than those used in previous body-wave spectral studies because (1) the corrections for propagation effects were small since the average hypocentral distance was short (12 km), and the propagation path was through granite, and (2) the recordings were made on well-calibrated, high-sample-rate (150/sec), high-dynamic-range (90 db) digital equipment. The recording equipment, characteristics of the $S$-wave spectra, and sources of error are discussed in Tucker and Brune (1973).

The geophones measured horizontal components of ground motion in order to enhance the capability for recording $S$ waves. As a result, clearly defined $P$ waves were recorded for fewer events than were used for the study of $S$ waves. Nevertheless, a spectrum was determined for as many $P$ waves as possible. These spectra were corrected for instrument response and for propagation-path attenuation by multiplying the observed spectra by $\exp(\pi f D/Q c)$, where $f$ is frequency, $D$ is hypocentral distance, and $c$ is seismic-wave velocity. For $S$ waves, we used $c = 3.5$ km/sec and $Q = 250$, and for $P$ waves $c = 6.0$ km/sec and $Q = 500$. The hypocentral distance was estimated from the relative arrival

![Diagram](image)

**Fig. 2.** $P$-wave corner frequencies versus $S$-wave corner frequencies for large aftershocks of the 1971 San Fernando earthquake by the strong-motion instrument at Pacoima Dam.
times of the $P$ and $S$ waves. The vast majority of the aftershocks studied here had hypocentral distances between 8 and 18 km. The majority of the depths of the aftershocks located by Wesson et al. (1971) and Whitcomb et al. (1973) ranged from 4 to 12 km. On

![Image of seismograms and spectra]

**Fig. 3A**

Fig. 3. Examples of seismograms and spectra for some of the events studied. Arrows beneath seismograms indicate portion of $S$-wave Fourier analyzed. Preceding signal used for $P$ waves. Top example of each pair is for $S$ wave. $P$-wave spectrum is shifted down by one unit. Dots show our choices for the corners in the spectra. Spectra are corrected for attenuation and instrument response. Examples in (A) are typical of those used. Examples in (B) include examples for which $f_s < f_c$ or $f_s > f_c$.

the average, then, for the aftershocks studied by us, the epicentral distance was about equal to the hypocentral depth. (See note added in proof.)

Since the correction for propagation-path attenuation consists of multiplying by an
exponential function of hypocentral distance and of frequency, it affects the spectra most strongly at high frequencies and when the hypocentral distance is large. Therefore, if the choice of \( Q \) were grossly in error, there would be a correlation between corner frequency and hypocentral distance and between the slope of the high-frequency part of the spectrum and hypocentral distance. To test this possibility, we plotted the corner frequencies and high-frequency slopes of S-wave spectra as a function of hypocentral distance. The spectra were considered both collectively and in groups with comparable magnitudes; no correlation was found.

The ratio of the assumed values of \( Q \) for \( P \) and for \( S \) of \( \frac{1}{4} \) is consistent with theoretical calculations of Savage (1966b) and Walsh (1966) and with experimental data cited by
them. The individual values are consistent with estimates of $Q$ for attenuation of seismic waves propagating in the crust (e.g., Press, 1964; Sutton et al., 1967). Thus, for an S wave $Q$ of 250, a $Q$ of 500 is probably reasonable for P waves. Trifunac (1972) used values of 150 and 200 for the $Q$ of S and P waves, respectively, while the four studies summarized in Figure 1 used Julian and Anderson's (1968) $Q$ values.

Of the 151 recordings for which both P- and S-wave spectra were determined, we used only 144. For the present study, we did not include events for which there was not a well-defined corner frequency for P or for S. Although important information about earthquake sources may be contained in such data, the small number of examples for which there is not a well-defined corner frequency permits us to ignore them here. Figure 3 shows examples of recordings of $P$ and $S$ waves for several events that were used. In general, the $P$ waves contain a higher predominant frequency than the $S$ waves. Figure 3 also shows the spectra and our choices of corner frequencies for these recordings. As discussed by Tucker and Brune (1973), there are errors in the estimated corner frequencies due to uncertainty in propagation-path attenuation, lack of azimuthal averaging because the signal was recorded usually at just one station, and ambiguity in the choice of corner frequency. They estimated that their precision in measuring the logarithm of the corner frequency was 0.17, or a factor of 1.5 in the corner frequency. Thus, if an estimate of corner frequency was 10 Hz, a second estimate of the same aftershock's corner frequency would probably lie between 15 and 7 Hz.

Figure 4 shows that, for these data, the $P$ corner frequencies are almost always greater than those for $S$ waves. Guidelines for which $f_p = f_s$ and for which $f_p = 1.5 f_s$ are also shown. These lines are not fits to the data. Although there is an uncertainty of approximately a factor of 1.5 in each estimate of $f_p$ and of $f_s$, the data do not indicate a simple functional relationship between them. The data are consistent with the idea that the ratio of $f_p$ to $f_s$ is not a constant for all earthquakes or in all directions of the radiation pattern.

**Implications for Earthquake Sources**

A popular model of an earthquake source is a long thin fault along which a dislocation propagates. This model is reasonable for very large earthquakes at subduction zones and for moderate earthquakes on shallow faults. In many such cases, aftershock distributions define long thin rupture zones, and usually the main shock is at one end of the rupture zone (e.g., Eaton et al., 1970; Kelleher et al., 1973). Moreover, in some cases, the model of a propagating dislocation has been successful in accounting for amplitudes of surface waves (e.g., Filson and McEvilly, 1967; Kanamori, 1970a, b). However, Savage (1972) showed that for such models, when averaged over azimuth, $f_s$ and $f_p$ are approximately equal. Thus, such models are not adequate for most of the smaller earthquakes, for which $P$-wave and $S$-wave corner frequencies have been determined. Probably most smaller earthquakes do not have long thin rupture zones. In fact, aftershock zones for earthquakes in the Aleutians with magnitudes as large as 7 are often equidimensional (Sykes, 1971) suggesting that the rupture zones of the main shocks for these events are not long thin faults.

On the other hand, several models of earthquakes that treat the source as a volume which radiates $P$ and $S$ waves predict a higher value of $f_p$ than $f_s$ (Archambeau, 1968; Bullen, 1963, pp. 75–77; Honda, 1962; Kasahara, 1957; Randall, 1966). Linde and Sacks (1972) measured $P$- and $S$-wave spectra for several earthquakes and interpreted these spectra as having peaks in them with the peak in the $P$-wave spectrum at a systematically higher frequency than in the corresponding $S$-wave spectrum. They cite these data as
evidence for Archambeau's (1968) theory, which models an earthquake as a relaxation surrounding a volume in which the shear modulus vanishes.

Since earthquakes are usually assumed to result from slip on faults, an important problem is to find a model of slip on a fault for which $f_p > f_s$. From dislocation theory (e.g. Aki, 1967; Burridge and Knopoff, 1964; Haskell, 1964) the far-field displacement signal, $U(t)$, for $P$ or for $S$ waves is given by

$$U(t) = \frac{\mu R_{\phi}^p}{4\pi \rho c^3} \int \int \frac{\hat{D}(\rho', \phi', t - r/c) dS'}{r}$$

where $\mu$ is the shear modulus, $R_{\phi}^p$ is the point source radiation pattern coefficient for $P$ or $S$ (e.g., Keilis-Borok, 1957), $\rho$ is the density, $c$ is the $P$- or $S$-wave velocity, $\hat{D}$ is the discontinuity in particle velocity at each point $(\rho', \phi')$ on the fault, $r$ is the distance from each point $(\rho', \phi')$ on the fault to the receiver and integration is over the ruptured area, $S'$. The spectrum of the displacement is given by

$$U(\omega) = \int_{-\infty}^{\infty} u(t) e^{i\omega t} dt.$$
\[
eq \frac{\mu R_{\theta \phi}^S}{4\pi \rho c^3} \int \int \hat{D}(\rho', \phi', \omega) \frac{e^{i\omega t/c}}{r} dS',
\]

where

\[
\hat{D}(\rho', \phi', \omega) = \int_{-\infty}^{\infty} \hat{D}(\rho', \phi', t) e^{i\omega t} dt.
\]

Following Savage (1966a), in the far-field \( r \approx r_0 - \rho' \sin \theta \cos(\phi - \phi') \) where \( r_0, \theta, \) and \( \phi \) are the polar coordinates of the receiver in a frame centered at a point on the fault and with the line, \( \theta = 0, \) parallel to the normal to the fault (Figure 5). Then, with \( r_0 \ll \rho' \)

\[
U(\omega) = \frac{\mu R_{\theta \phi}^S}{4\pi \rho c^3 r_0} \exp \left[ \frac{i\omega r_0}{c} \right] \int \int \hat{D}(\rho', \phi', \omega) \exp \left[ -i\omega \rho' \sin \theta \cos(\phi - \phi') \right] d\rho' d\phi'.
\]  

(2)

\( \hat{D}(\rho', \phi', \omega) \) is the spectrum of the discontinuity in particle velocity across the fault at the point \((\rho', \phi')\). In general, it is complex, but if the particle velocity does not change sign,

![Fig. 5. Geometry of a quadrant of fault and receiver used in equation (2).](image)

at low frequency \( \hat{D} \) is a real constant, equal to the total slip during the earthquake at \((\rho', \phi')\). If the displacement does not occur instantaneously, \(|\hat{D}(\rho', \phi', \omega)|\) has a corner frequency above which it decreases (Aki, 1972). The modulus, \(|\hat{D}(\rho', \phi', \omega)|\), is independent of the time history of growth of the rupture area, but the phase of \( \hat{D} \) does depend upon the time at which slip begins, and, thus, on the development of the rupture area. Note that \( \hat{D}(\rho', \phi', \omega) \) does not depend on the wave velocity.

The factor \( \exp [i\omega \rho' \sin \theta \cos(\phi - \phi')/c] \) is a phase factor that describes the constructive and destructive interference due to different path lengths, \( \rho' \sin \theta \cos(\phi - \phi') \), of radiation from different parts of the fault. At low frequencies, the factor is unity, and (except for \( \theta = 0 \)) at high frequencies it is rapidly oscillating. Clearly, this factor depends upon the orientation of the fault with respect to the observer, \( \theta \). In the direction normal to the fault, the factor is unity; the observer sees essentially a point source. The importance of this factor for the present study is that, in all other directions, it depends upon the wave velocity, \( c \). To illustrate this dependence, consider a model in which the dis-
placement occurs instantaneously, with the displacement equal to that for a uniform stress drop over a circular rupture area (Keilis-Borok, 1959). From the solution given by Randall (1973) and determined in a different way in the appendix, with $\theta = 0$, in the far-field the $P$- and $S$-wave displacements are of the form

$$ U \sim 1 - \left( \frac{ct - r_0}{\rho_0 \sin \theta} \right)^2, \quad -1 \leq \frac{ct - r_0}{\rho_0 \sin \theta} \leq 1 $$

$$ = 0 \text{ otherwise}, $$

where $\rho_0$ is the radius of the rupture area and $t$ is the travel time. The pulse is a section of a parabola, and its duration depends upon $\rho_0$ and $\theta$, and the wave velocity $c$. For any direction $\theta$, the ratio of the pulse widths for $P$ and $S$ waves is equal to the inverse ratio of the wave velocities. Therefore, the ratio of the corner frequencies is equal to the ratio of the wave velocities. Both the $P$- and $S$-wave spectra decrease as $\omega^{-2}$ at high frequencies (Randall, 1973; and appendix).

Thus, the radiated spectrum depends in a complicated way on the time history of slip at each point on the fault. Yet, certain generalizations are possible: (1) If the duration of slip and the time for growth of the ruptured area are long compared with the time for $P$ and $S$ waves to traverse the dimensions of the fault, the effect of interference due to radiation from different parts of the fault will be important only at frequencies much larger than those at which $\textbf{D}$ is no longer constant. Because $\textbf{D}(\rho', \phi', \omega)$ does not depend on the wave velocity, the measured $P$- and $S$-wave corner frequencies would probably be the same. At the opposite extreme, (2) if the entire fault radiates nearly simultaneously for a relatively short amount of time, interference of radiation from different parts of the fault will be important in determining the corner frequency and, as a result, $f_p$ will be greater than $f_s$.

These two extreme cases do not consider the growth or propagation of the rupture front, which is usually parameterized by a “rupture velocity”. Case (1), above, includes the case of a very slow rupture velocity. Although Kovach and Archambeau (1972) consider this to have occurred during the Terminal Island earthquakes, the data from the ratios of corner frequencies suggest that it is, in general, not common.

Case (2) corresponds to an infinite rupture velocity, which defies causality and, therefore, is unreasonable. However, it may not be unreasonable to expect that most of the fault will radiate essentially simultaneously. A rupture could nucleate at a point and grow outward. Because of variations of stress or of strength on the fault, most of the displacement might occur later. Stress waves transmitted up and down the fault by slip at other parts of the fault might cause further slip, and most of the energy would be radiated nearly simultaneously, or at least not by a uniformly propagating source. Perhaps much of the energy is radiated by the edges of the rupture area when it stops growing. The radiation from stopping phases (Savage, 1965) at the edge of the fault might be more important than that from slip near the center of the ruptured area, especially at high frequencies. In either of these cases, the essentially simultaneous radiation from a large portion of the rupture area would cause $f_p$ to be greater than $f_s$.

Although the situations described above include the extremes of zero and infinite “rupture velocity”, the spectra radiated by them do not span the range of possible radiated spectra. A finite rate of growth of the rupture area can affect the spectrum markedly. To explore this possibility, we investigated a simple model of displacement in time $[\dot{D}(\rho', \phi', t)]$ and integrated equation (2) numerically for both $P$ and $S$ waves at many different values of $\theta$. The spectra were plotted on a log-log scale. The corner frequencies were determined in the same manner as with real data, by drawing straight lines
through the low- and high-frequency portions of the spectra and by measuring the frequency where they intersect.

We let

\[ D(\rho', \phi', t) = D_0 \left\{ H\left( t - \frac{\rho'}{v} + \frac{\rho_0}{v} \right) - H\left( t + \frac{\rho'}{v} - \frac{\rho_0}{v} \right) \right\} H(\rho_0 - \rho') \]

where \( \rho_0 \) is the radius of a circular ruptured area, \( v \) is the "rupture velocity", and \( H(\cdot) \) is a step function. The rupture nucleates at the center, grows radially in all directions at a constant velocity, \( v \), to the radius \( \rho_0 \), and then contracts back to the center at the same velocity. The particle velocity, \( D = D_0 \), is a constant over the fault, but because the center of the fault radiates for a longer time than the edges, there is a greater displacement there. With this model, we tried to approximate a relaxation source, in which slip does not occur at a point on the fault until the rupture front reaches the point and continues to slip until information from the edges of the fault is radiated back to the point. This model crudely approximates the phenomena that occur in the two-dimensional models of Burridge (1969) and Burridge and Halliday (1971), and the conditions used in deriving the Brune (1970) model.

We experimented with different "rupture velocities". In all cases, from equation (2) the spectra for \( P \) and \( S \) are alike in the direction \( \theta = 0^\circ \). When \( v = \beta/2 \), in all directions the \( P \)- and \( S \)-wave spectra were quite similar; the corner frequencies were nearly the same and varied insignificantly with \( \theta \). Except for \( \theta = 0^\circ \), the spectra decayed as \( \omega^{-3} \) at high frequency as predicted by Savage (1966a, 1972).

When \( v = \beta \), except for \( \theta^\circ = 0 \) or \( \theta = 90^\circ \), the spectra decrease as \( \omega^{-3} \), but the corner frequency for \( P \) is always greater than that for \( S \) (Figure 6). As \( \theta \) approaches \( 90^\circ \), constructive interference of the \( S \) waves originating from different parts of the fault area causes the high-frequency portion of the \( S \)-wave spectrum to decay approximately as \( \omega^{-2} \), compared with \( \omega^{-3} \) for this portion of the \( P \)-wave spectrum. Nevertheless, if, as was done with the data, one measures the corner frequency at the intersection of the extrapolated low- and high-frequency portions, without regard to the rate of fall-off at high frequency, \( f_p \), is greater than \( f_s \) (Figure 6).

Similarly when \( v = \alpha \), the \( S \)-wave spectrum decays as \( \omega^{-2} \) for \( \theta \approx 35^\circ \) and the \( P \)-wave spectrum approximately as \( \omega^{-2} \) for \( \theta = 90^\circ \), because of constructive interference of radiation from a large part of the rupture area in these directions. In other directions, the spectra decrease as \( \omega^{-3} \). Nevertheless, again the measured corner frequencies of \( P \) waves were never less than those for \( S \) waves (Figure 6). For the cases with \( v = \beta \) and with \( v = \alpha \), the ratio, \( f_p/f_s \), is equal to one at \( \theta = 0^\circ \) and is a maximum at \( \theta = 90^\circ \).

Although these models are gross oversimplifications of the dynamic faulting processes, they do yield values of \( f_p > f_s \). The physical explanation for this result is that the source does not radiate for a long time compared with the time for seismic waves to traverse the source and that a large fraction of the source radiates simultaneously. However, the calculated ratio \( f_p/f_s \) was less than 1.3 in nearly all directions and was never as large as 1.7. Thus, for this particular form of the time function for displacement, the models do not yield the same range of this ratio as the observed data indicate. For other time histories and distributions of displacement on the fault, this ratio can be as large as 1.7 (= \( v_p/t_2 \)). The uncertainties in the data and the essential nonuniqueness of this problem, however, do not merit construction of more complex models.

Moreover, the ratios \( f_p/f_s \) in Figure 4 and the ratios of predominant periods of \( S \) and \( P \) waves observed on strong-motion instruments are often larger than 1.7 and can be as large as three or four. Because of the short hypocentral distances, these larger ratios are not likely to be effects of wave propagation, nor are they easily explained by simple disloca-
tion models or volume sources. Therefore, there appear to be other phenomena that affect the $P$- and $S$-wave spectra differently than we have not considered here.

A speculative explanation of this large difference in $P$- and $S$-wave corner frequencies is that the effective source for $P$ waves is smaller than that for $S$ waves. Although a large area may slip during an earthquake, most of the strain energy released by it may be concentrated in a localized area, as Hanks (1973) suggests for the San Fernando earthquake. Implicit in Brune's (1970) model was an assumption that during the earthquake the rupture area is opaque to shear waves, so that, in the far-field, radiation diffracts around the edges of the fault to interfere with the direct energy. If this were so, and if the rupture area did transmit $P$ waves, then it is possible that the $P$-wave corner frequency would be determined primarily by the dimensions of the localized area in which the stress is concentrated. The $S$-wave corner frequency would depend upon the larger, rupture area around which it must diffract. For such a phenomenon to occur, the rupture area would have to grow fast enough that the $S$ waves from the center of the fault must diffract around an area larger than the source of most of the $P$-wave energy. Such a model has not been considered explicitly, partly because the assumption that the rupture area is opaque to shear waves but not to $P$ waves is the same as assuming that there is a prescribed stress drop on the fault during the earthquake (and a peculiar one). Dislocation models have assumed a displacement and only continuity of stress on the fault. Such

\[\theta = 25^\circ\]

\[\theta = 55^\circ\]

\[\theta = 85^\circ\]

\[\omega_g/\alpha\]

\[\omega_g/\beta\]

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**Fig. 6.** Calculated spectra for a growing and contracting circular fault (radius $\rho_0$) for rupture velocity equal to $\alpha$, the $P$-wave velocity (*left*), and $\beta$, the $S$-wave velocity (*right*), at different directions ($\theta$).
a model is, of course, speculative at present and should be treated as such. Yet, if correct, it implies different boundary conditions on the fault than are commonly assumed.

CONCLUSIONS

The principal result of this paper is the observation that, from reliably determined spectra of earthquakes in the magnitude range 0.5 to 4.5, the corner frequency for $P$ waves is systematically larger than that for $S$ waves. These data corroborate results presented in other studies that considered different sized earthquakes and that show obviously higher predominant frequencies of $P$ waves observed on strong-motion records of very close earthquakes and aftershocks.

Models that treat an earthquake as a volume source predict higher frequencies for $P$ waves than for $S$ waves (Archambeau, 1968; Bullen, 1963; Honda, 1962; Kasahara, 1957; Randall, 1966). Dislocation models for which both the duration of slip on the fault and the time for the rupture area to develop are not large compared to the time for seismic waves to cross the source area and for which most of the radiation is generated by essentially simultaneous slip on most of the rupture area also exhibit an $f_p > f_s$. These data, therefore, do not require a volume source to model the observed radiation from earthquakes.

Savage's (1972) calculations of corner frequencies for Haskell's model, which considers the propagation of a dislocation along a long thin fault, yield a value of the ratio $f_p/f_s \approx 1$. Models that employ long thin faults seem to be applicable to earthquakes large enough that the surface of the Earth and the bottom of the lithosphere constrain one dimension of the fault to be small compared with the other, but the data presented in this paper indicate that such models are not applicable to these smaller earthquakes. Most small earthquakes probably are caused by slip on nearly equidimensional faults.

APPENDIX

From Keilis-Borok (1959), the distribution of displacement as a function of distance $\rho'$ from the center on a circular rupture area of radius $\rho_0$ resulting from a uniform stress drop is proportional to $(\rho_0^2 - \rho'{}^2)^{1/2}$. Let this be applied instantaneously

$$D(\rho', \phi', t) = D_0 \frac{(\rho_0^2 - \rho'{}^2)^{1/2}}{\rho_0} \delta(t).$$

We may integrate equation (1) by letting

$$r = r_0 - \rho' \sin \theta \cos(\phi - \phi')$$

and by changing to rectangular coordinates

$$x = \rho' \cos(\phi - \phi'), \quad y = \rho' \sin(\phi - \phi').$$

Then for $r_0 \gg \rho'$

$$= \frac{\mu R_o^2}{4\pi \rho c^3 r_0} \int_{-\rho_0}^{\rho_0} dx \int_{-(\rho_0^2 - x^2)^{1/2}}^{(\rho_0^2 - x^2)^{1/2}} \frac{D_0}{\rho_0} (\rho_0^2 - y^2 - x^2)^{1/2} \delta \left( t - \frac{r_0}{c} + \frac{x \sin \theta}{c} \right) dy$$

$$= \frac{\mu R_o^2}{8 \rho c^2 r_0} \frac{D_0 \rho_0}{\sin \theta} \left[ 1 - \left( \frac{ct - r_0}{\rho_0 \sin \theta} \right)^2 \right]$$
the spectrum is given by

$$U(\omega) = \frac{\mu R_e^3}{2\rho \omega \sin \theta} \frac{D_0}{\rho \omega^3 \sin \theta} \left[ \sin \left( \frac{\omega \rho_0 \sin \theta}{c} \right) - \frac{\omega \rho_0 \sin \theta}{c} \cos \left( \frac{\omega \rho_0 \sin \theta}{c} \right) \right].$$

At high frequency it decays as $\omega^{-2}$, and the corner frequency is proportional to $c/(\rho_0 \sin \theta)$.

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REFERENCES


**Note added in proof:**

We have computed the P- and S-wave spectra from recordings of vertical ground motion for seven events and have obtained ratios of P and S corner frequencies similar to those reported here.