Mantle convection

(Section 6.19 of Turcotte and Schubert addresses convection with more rigor and less clarity)

Conduction of heat

How is heat moved in the Earth? We are all familiar with conduction of heat. Conductive heat flow q is related to the temperature gradient: $q = -k\frac{dT}{dz}$, where T is the temperature with depth and k is the coefficient of thermal conductivity (the minus sign reflects heat going from hot to cold and often is omitted in the simple 1-D case but becomes important in three dimensions). For typical crustal rocks, k is 2-3 W m^{-1} °C^{-1}, though this increases somewhat to 3-5 W m^{-1} °C^{-1} for dunite and peridotite. Most heat is conducted out of the Earth through conduction (some is by fluid flow, which is a separate problem). The average heat flow in ocean basins is 101 mW/m^2, continents 65 mW/m^2, and the global average is 87 mW/m^2 for a global total heat loss of about 4.43 x 10^{13} W. In steady state, without a change in temperature boundary conditions, temperatures should vary linearly.

A simple application of this is the cooling of a warm body. If the body has material with a specific heat of C (the specific heat per unit mass, in units of J kg^{-1} °C^{-1}) and a density ρ, then a decrease in temperature of ΔT represents the loss of $(\Delta T)\rho C$ J/m^3 of heat. The loss (or gain) of heat is the difference between the heat flow in and that out, which occurs at a rate of $\frac{dq}{dz}$, so we may write

$$\frac{dq}{dz} = -\frac{d}{dt}(\rho C T) \tag{1}$$

where the minus reflects the fact that a positive change in heat flow results in a negative change in temperature. By using the relationship between heat flow and conduction, (1) can become

$$\begin{aligned} k\frac{d^2T}{dz^2} &= \rho C\frac{dT}{dt} \\ \frac{dT}{dt} &= \kappa\frac{d^2T}{dz^2} \end{aligned} \tag{2}$$

where $\kappa = k/\rho C$ and is termed the thermal diffusivity. This equation is the basis for several thermal phenomena we will consider later in the course, but for now simply consider the thermal diffusivity. Its dimensions are $length^2/time$, so it suggests a characteristic time for a thermal perturbation to travel a certain distance; that time is l^2/κ. Thus, to first order, a body of radius l will reach ambient temperature after a time about equal to l^2/κ. Note that this does not depend on the magnitude of the initial temperature difference. For mantle rocks, κ is about 1 mm^2/s, so a millimeter sized object will cool in a second; a kilometer scale object will cool over about 10^{12} seconds, or about 30,000 years.

Heat and the Age of the Earth
An interesting sidelight is that nearly a hundred and fifty years ago, Lord Kelvin (still William Thompson at the time) challenged the idea that the Earth's age was limitless (recall the popular geologic quote of Hutton's "*…that we find no vestige of a beginning,—no prospect of an end.*") This was anathema to Kelvin, who thought it deserved a test. Although he originally proposed several tests, the one that stuck the longest was that based upon heat flow measured in England. Basically, Kelvin assumed that if you started with a molten sphere turned solid, as it cooled the surface temperature gradient decreases over time (fairly rapidly, in fact). The thermal gradient he used is a bit higher than typical of continents (35°C/km), but even adjusting for that, if you assume (as Kelvin did) that heat was lost through conduction, you find that even adding in radioactivity will not let you make the Earth older than a few tens of millions of years (Richter, 1986).

Most histories hold that once radioactivity was discovered, it allowed for an extra heat source and thus Kelvin's hold on the age of the Earth was broken. In a sense, this is historically accurate (it removed the last shackles from geologists' hands and also provided a means of dating igneous rocks), but scientifically it is wrong. Heat produced today (or in the past 100 million years) cannot account for the extra heat loss of the Earth (Richter, 1986): modern estimates of total heat production from radioactive U, Th, and K are only about half the present day heat loss (and as most of these are near the surface, they help even less with the thermal gradient problem). Ironically for the standard story, the solution was proposed the year before radioactivity was recognized; in 1895, some 30 years after Kelvin had launched his campaign to show that the Earth was of limited age, John Perry showed very simply that a variation of thermal conductivity with depth would greatly change the results. In particular, he noted that convection is likely in the core and that this would greatly alter Kelvin's results (Burchfield, 1975). Because Kelvin disputed this and held to his original assumptions, it didn't sway most of the geological or physical communities. However, this is the key: convection in the interior allows for more efficient transport of heat, which causes the thermal gradients at the Earth's surface to remain relatively high over longer periods of time.

Derivation of Conditions Allowing Convection
Convection is simply the movement of heat along with material. Consider a viscous fluid with a temperature gradient ΔT across a distance d from top to bottom . If it is conducting heat only and there is no heat production, then the temperature profile is linear. For convection to work, material has to move fast enough that the heat in the moving material isn't lost by conduction faster than the material is moving. So let us have a body, let us approximate this as a sphere of radius l, moving upward with speed v. Our body needs to move faster than heat travels conductively, so $d/v > l^2/\kappa$ (from above), where we've let the body traverse the full width of our conducting layer d.

The body will only continue to move upward if it is buoyant. The buoyancy forces are due to it being somewhat less dense than its surroundings, for in rising quickly the body has not lost (much) heat and so is warmer than its surroundings. This buoyancy force/unit mass (i.e., acceleration from buoyancy) is $g\delta\rho/\rho$, where $\delta\rho$ is the change in density from rising up the distance d. If we use the coefficient of thermal expansion, α, we get the force/unit mass is $g\alpha\delta T$.

The temperature difference δT is proportional to the rate of decline of surrounding temperatures times the diffusion time, which is

$$\frac{\Delta T\, v}{d}\frac{l^2}{\kappa} \tag{3}$$

so the overall force pushing the body up is

$$g\alpha\frac{\Delta T\, l^2 v}{d\kappa} \tag{4}$$

Balanced against this is the viscous drag keeping the material from rising up. Our body has velocity *v* and dimensions on order of *l*, so we might expect a velocity gradient around its sides on the order of *v/l* (which has units of strain rate, $time^{-1}$).Recall that the shear stress in a Newtonian fluid is the dynamic viscosity η times the velocity gradient, here *v/l*. Our body has a surface with size on the order of l^2, so the total force is $\eta v l$. The force per unit mass is the force divided by the product of the density and the volume, and the volume is proportional to l^3, so the resistive force is

$$\frac{\eta v}{\rho l^2} \tag{5}$$

Obviously the greater the upward forces in (4) are than the downward forces in (5), the more apt the region is to have bodies convecting. So if we take the ratio of (4) to (5) we get

$$g\alpha\frac{\Delta T\, l^2 v}{d\kappa}\frac{\rho l^2}{\eta v} = \frac{\rho g\alpha l^4 \Delta T}{d\kappa\eta} \tag{6}$$

Obviously the ratio is greatest for the largest possible value of *l*, so we replace *l* with *d* (which certainly seems the largest dimension a convecting body could have) and we get

$$\mathrm{Ra} = \frac{\rho g\alpha d^3 \Delta T}{\kappa\eta} \tag{7}$$

where Ra denotes the Rayleigh number; being the ratio of two forces, it is dimensionless. When the Rayleigh number is above a critical value Ra_{crit} (on the order of 1000, it turns out), convection can occur. There are a number of other derivations in the textbooks that can give different insight into this parameter. In the Earth, the Rayleigh number of the mantle as a whole is on the order of 10^6, which is easily enough to drive convection. The style of convection described here is Rayleigh or Rayleigh-Bénard convection to most earth scientists, though it is sometimes called Bénard or rarely Bénard-Boussinesq convection (thankfully, the Rayleigh number's name is unchanged through this).

(On occasion, you will see the Rayleigh number defined with no density term; this is because the kinematic viscosity is used, which is = η/ρ).

There is a separate derivation for the case of a fluid that is heated internally and not by greater temperatures at the base; this is usually the solution applied when considering convection in the Earth's mantle. An interesting difference from the classic Rayleigh convection is that while downwellings are distinct, upwelling material is quite broad and distinct upwelling sheets do not exist. This version of the Rayleigh number is

$$\mathrm{Ra_H} = \frac{\rho^2 g \alpha d^5 A}{k \kappa \eta} \quad (8)$$

where A is the heat production per unit mass. The mantle has a value on the order of 10^9 for this version of the Rayleigh number, which similarly has a threshold near 1000 for convection.

What the Rayleigh number tells us is just what makes convection likely. The cube of the height of the convecting region suggests that single layer convection is actually more likely than multiple layers. Clearly an increasing viscosity retards convection; as we saw before, the effective viscosity of Earth materials decreases with temperature. Thus as mean temperatures cool, we expect that convection would become less vigorous.

Not too surprisingly, there is a wavelength dependence of the critical Rayleigh number. If you want to make tall, narrow convection cells you have to have more energy in the system than if you have a more equant (square-shaped) convection cell. The cell geometry with the lowest critical number is one with a cell $\sqrt{2}$ times as wide as it is tall for convection rolls for bottom heated convection; for internally heated convection, it is closer to 2 times as wide as it is tall.

Adiabatic Temperatures

With vigorous convection, the temperature structure of the convecting fluid becomes nearly isothermal. The geotherm in a convecting medium is termed the adiabat and averages only about 0.4°C/km in the mantle. The adiabat is a limit: it is the temperature structure that would exist if material rose and fell without losing or gaining heat through conduction. It is derived from the fact that Earth materials heat up as they are compressed as the pressure does work by decreasing the volume of the rock. This is usually addressed thermodynamically, as the entropy of a body of rock moving up or down is assumed to stay constant. In this case, the entropy per unit mass *ds* is given by

$$ds = \frac{C}{T} dT - \frac{\alpha}{\rho} dP = 0 \quad (9)$$

We can manipulate this to be in terms of depth, obtaining

$$\frac{dT}{dz} = \frac{T\alpha}{C\rho} \frac{dP}{dz} = \frac{T\alpha g}{C} \quad (10)$$

Using some typical values from Turcotte and Schubert of T=1600K, $\alpha = 3\mathrm{x}10^{-5}\ \mathrm{K}^{-1}$, $C = 1\ \mathrm{kJ\ kg}^{-1}\ \mathrm{K}^{-1}$ and $g = 10\ \mathrm{m\ s}^{-2}$, the thermal gradient in the shallow mantle works out to be 0.5°/km.

Both the convection calculations and the adiabat assume there are no major phase changes. In fact, as we shall see, there are profound changes in mantle phases in the shallower mantle, that part termed the transition zone. Convection can be enhanced or damped by such features, depending on how densities and temperatures change across these boundaries. For the moment, we shall note that the discontinuity at 660 km appears to be one that can damp convective motions. Just how much it damps them has been the subject of fierce debate.

Thermal Boundary Layer

The last element of convective systems we consider are the boundaries. As the convecting fluid approached an adiabatic gradient, one might ask how this can work when the Earth's surface is a little above 0°C and the core mantle boundary is somewhere in the neighborhood of about 4000°C, as suggested from the observations of a molten outer core. The 0.4°/km mean adiabatic gradient would only produce no more than 1200° difference across the mantle.

The solution is to recognize that at the top and bottom, material is constrained to flow parallel to the edges and be exposed to a heat sink or a heat source. The material facing the heat sink (at the Earth's surface) is being cooled rapidly and develops a conductive geotherm until it cycles back down into the mantle. This is true to some degree under the continents as well, which are resistant to convection because they are gravitationally stable (cold continental crust is not denser than warm asthenosphere). The region at the top and bottom of a convecting region is the thermal boundary layer, and heat is conducted within it (with the exceptions of fluids and melts). This is one possible definition of lithosphere, one sometimes used by geodynamicists and very closely related to those frequently used by geochemists and petrologists.

This structure then brings to closure the problem Kelvin examined. With a purely conductive geotherm, the interior of the Earth had to be very hot and thus the total amount of heat within the Earth quite high, which would mean it had not lost much of its original heat. With a conductive geotherm at the top and then a long adiabatic geotherm, the temperatures in the Earth's interior are much lower, and thus the total heat in the Earth is much lower than Kelvin envisioned. Therefore the Earth could have lost a lot more heat and thus be a lot older than Kelvin had envisioned.

Non-text References

Burchfield, J. D., Lord Kelvin and the Age of the Earth, Univ. Chicago Press, Chicago, Illinois, 267 pp. 1975 (reprinted 1990).

Richter, F. M., Kelvin and the age of the Earth, J. Geol., *94*, pp. 395-401, 1986.