GEOL 5690 Homework 4

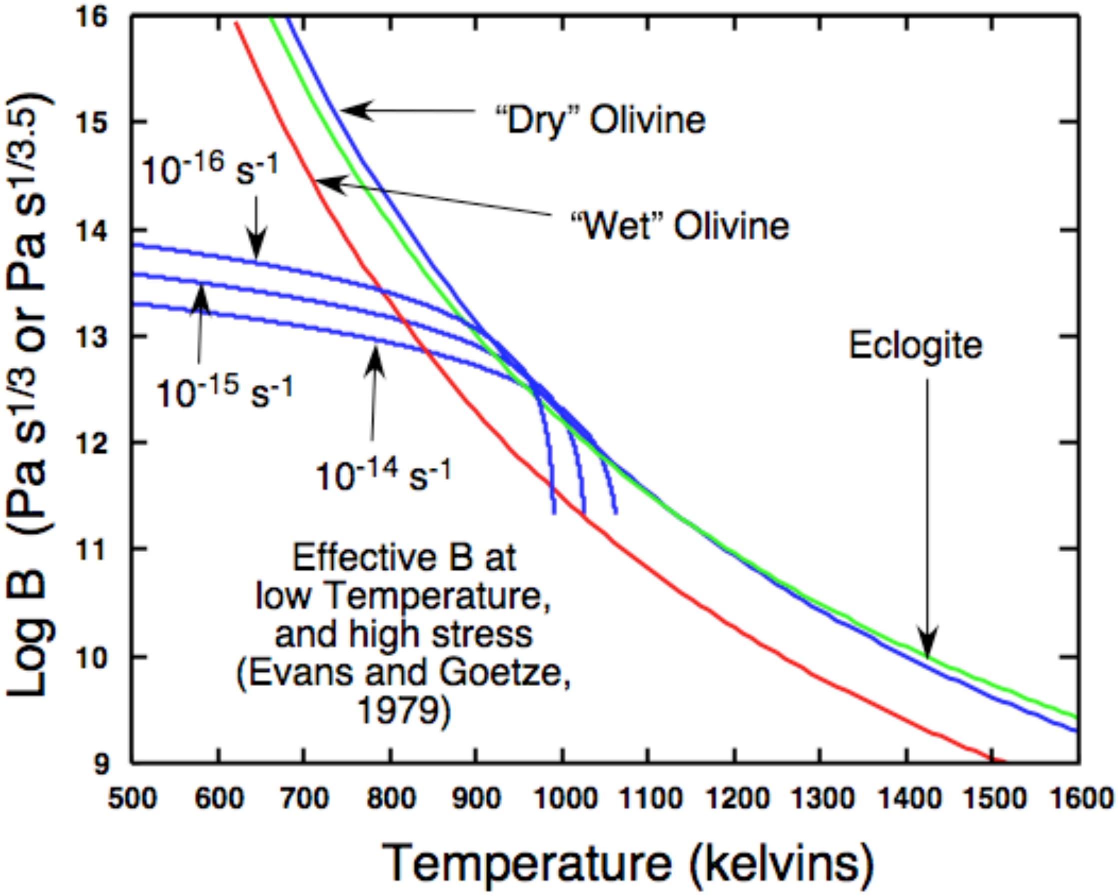
Lower lithospheric flow

Due: 5 March 2010

We have seen calculations based on a simple Newtonian rheology that allow for flow in the lower crust. The same equations can be used for Couette flow, which is shearing of a layer of viscous fluid (eqn. 6-13 in Turcotte and Schubert), by a simple change in the boundary conditions.

1) Convert eqn. 6-13 into a form that depends not on u_0 (the velocity of one side of the shearing fluid) but on τ (the shear stress acting on the fluid). Compute the average displacement of the fluid as a function of the shear stress, the viscosity, and the thickness of the layer.

2) We need a viscosity for the mantle lithosphere in order to use our simple Newtonian model. We will derive an effective viscosity from a more realistic laboratory-based power law rheology by noting that the power law equations can be rewritten as $\dot{\varepsilon} = (\sigma_1 - \sigma_3) B \mathrm{E}^{(1-n)/n}$, where E is the second invariant of the strain rate tensor: $\mathrm{E} = \sqrt{\tfrac{1}{2}\left[\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{zz}^2\right] + \dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{yz}^2 + \dot{\varepsilon}_{xz}^2}$. We will assume a dry olivine for this exercise. Plotted below (from Molnar and Jones, GJI 2004) are values of $B = 3^{-\frac{n+1}{2n}} \left(\frac{A}{2}\right)^{-\frac{1}{n}} e^{E_a/nRT} = 2\eta_{eff}\, \mathrm{E}^{(n-1)/n}$ as a function of temperature using recent values for dry olivine (Hirth and Kohlstedt, 1996, 2003, using E_a = 540 kJ/mole, $A = 2.4 \times 10^5$ MPa$^{-3.5}$ s^{-1} and n = 3.5. R, the ideal gas constant, is 8.314472 J K^{-1} mole^{-1}). Note then that $\eta_{eff} = \frac{1}{2} 3^{-\frac{n+1}{2n}} \left(\frac{A}{2}\right)^{-\frac{1}{n}} e^{E_a/nRT} \mathrm{E}^{(1-n)/n}$.



From this (ideally the equation, but check against the plot if necessary, especially being careful with the unit of MPa$^{-3.5}$), determine the effective viscosity for temperatures of 700K, 1000K, and 1300K.

3) If the length of flat slab subduction (parallel to shear) was 800 km, what would the necessary basal shear be to create an additional normal stress equal to about half of the strength of cold continental lithosphere in compression? (You should describe where your estimate of the strength of the lithosphere comes from).

4) Assume that the mantle lithosphere had an initial thickness of 80 km under the western United States and that flat-slab subduction lasted from 75 to 50 Ma (so 25 My). What is the average displacement of the mantle lithosphere assuming each of the three viscosities from (2) and requiring the shear stress calculated in (3)?

5) Suggest what your results mean for viability of the flat slab hypothesis. You might wish to consider the curves in the plot above, including that marked "Effective B at low Temperature and high stress": these are experiments indicating that the power-law

rheologies we’ve been using are limited at very low temperatures (a different deformation mechanism takes over, preventing extremely high stresses).

Extra credit: As the mantle lithosphere were to start to shear, what would be the impact of adding in a pressure term to the equations used in (1)? Will this tend to stabilize flow, accelerate it, or slow it? Defend your answer, ideally with some rigor.