Trademark Infringement and

Endogenous Innovation

Michael W Nicholson
Department of Economics
University of Colorado, Boulder
nicholso@colorado.edu

Abstract:
This paper introduces trademark infringement into a dynamic, general equilibrium setting. I elaborate the conception of intellectual property rights beyond an incremental rise in imitation costs. An increase in the strength of intellectual property protection increases the rate at which firms shift production to the South. It also increases the innovation rate, regardless of whether technology is transferred by FDI or through imitation. Trademark enforcement may enhance welfare by broadening the gap between the amount some consumers are willing to pay for a good and the actual price charged.

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1. Introduction

Intellectual Property Rights (IPRs) and their protection have been the subject of intense debate in both academic circles and the arena of public policy. The Millennium Round of the WTO featured IPRs as a central focus of discussion. The technically advanced countries of the world have a large vested interest in the protection of IPRs, due to the fact that a large majority of the world’s intellectual property is created within their boundaries. The U.S. International Trade Commission (1988) supports these claims in a survey of major firms in the United States. Respondent firms estimated losses of $23.8 billion due to IPR infringement, or 2.7% of sales affected by intellectual property, citing lost revenues from fees and licensing or reduced profit margins as the primary components.

Many authors, notably Helpman (1993), provide the counterargument that IPRs strengthen the existing monopoly power of advanced countries to the detriment of the developing world. While IPRs may yield dynamic benefits through increased research and development (R&D) efforts extended to technological innovations (although this point, too, is debated), they produce static losses when potential growth in developing countries is constrained by limited access to existing knowledge. Feinberg and Rousslang (1990), using the U.S. ITC survey data, show that the static gains to consumers from infringement might exceed the profit losses of legitimate suppliers.¹ An optimal level of IPR protection balances these effects.

One of the elements at the heart of the debate on IPRs is trademark protection. Trademarks identify a firm’s reputation for quality. In some cases, the trademark itself
captures a large portion of the firm’s intellectual property. Besen and Raskind (1991) discuss trademarks as a method of product identification within orderly markets. Trademarks protect the consumer from fraudulent signaling. According to Besen and Raskind, the trademark depends on both the quality of the product and public familiarity. In this context, the firm has much to lose as the power of its symbol erodes.

Landes and Posner (1987) describe the various costs associated with marketing intellectual property, including the difficulties inherent in the non-excludability of knowledge goods. They argue that the trademark derives much of its value by lowering search costs of consumers, and that it also encourages firms to invest in maintaining the quality of trademarked goods. In their model, consumers and firms both gain from the reputation of a trademark, even if it differentiates goods of identical quality.

Maskus (2000) claims that trademark infringement is fairly common in developing countries, and affects both international brands and local enterprises. He describes a “Trademark Complex” of products that are susceptible to infringement, such as status goods, but warns that trademark infringement spreads across many sectors, including brand names but also machinery, transport equipment, and medicines. In the U.S. ITC survey, trademarks reportedly played great or very great importance in 83% of sales affected by intellectual property.

A preponderance of academic papers, both theoretical and empirical, has investigated the above claims, but none has focused on the role of trademarks in the global economy. In this paper, I present a dynamic, general equilibrium model of the

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1 As pointed out by Helpman (1993), the strong assumptions of Feinberg and Rousslang may dramatically affect this result.
2 Landes and Posner (1987) focus on search costs, rather than the risk of infringement in an international context.
international effects of IPRs that incorporates trademarks and introduces new techniques for modeling IPR protection. I elaborate the conception of IPRs beyond an incremental rise in imitation costs, integrating such aspects the scope of trademark protection.

I show that an increase in the strength of IPR protection that diminishes the threat of trademark infringement tends to increase the innovation rate in the economy. This result holds regardless of whether technology is transferred by FDI or through imitation. I also show that IPRs may tend to increase consumer welfare. These results extend current theoretical arguments on the role of IPRs to include trademark protection.

Krugman (1979) develops a standard model for analysis of international trade and growth. He presents a world with two regions, the North and the South. The North represents the industrialized countries concentrated in Europe and North America, while the South signifies the developing countries in Latin American and Southeast Asia that contain a production infrastructure and a market for sophisticated goods, but face technological lags behind the North. By assumption, the North enjoys a Ricardian advantage in innovation, to the extent that all innovation takes place there. New products are produced in the North and exported to the South. The South has the technical capability to imitate new products, which are then exported back to the North in a version of a product cycle.

The basic story has been elaborated to describe many aspects of the global economy. Two major contrasting results have developed in regard to IPRs. Helpman (1993) argues that an increase in IPR protection may tend to hurt both the North and the South through its influence on the market power of Northern firms and the composition of manufacturing. Since IPRs slow down the product cycle through the reduced rate of
imitation, production of good remains in the North using resources that may otherwise be used in R&D towards the innovation of new goods. A combination of higher prices (due to market power) and slower innovation leads to diminished welfare.

Lai (1998) demonstrates that Helpman’s results are sensitive to his assumptions, primarily on the stationary location of production. Lai allows for Northern firms to engage in FDI, while continuing control of their innovation. Northern firms maintain their incentive for innovation in monopoly rents while opening resources in the North for R&D. With these assumptions, an increase in IPRs that leads to a higher rate of FDI also leads to lower prices and a faster rate of innovation; thus, IPR protection improves welfare.  

These contrasting results identify an important complexity in discussions of IPRs. Lai’s model captures the virtues of comparative advantage – the North specializes its resources in innovation, while the South, with lower wages, specializes in production. Lai’s representation of IPRs, however, is overly simple, to the point that Lai says that they can be interpreted as any positive incentive for FDI. Helpman, on the other hand, emphasizes the market power of IPRs. As the monopoly for any given innovation is extended, firms invest fewer scarce resources into innovation.

This paper joins an array of others that analyze the subject using quality-ladders. Krugman, Helpman and Lai, with utility functions based on Dixit-Stiglitz preferences, all use horizontal differentiation of goods. In these models, innovation means the introduction of new products. Grossman and Helpman (1991) formalize vertical  

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3 Helpman (1993) includes a section on FDI, but with the assumption of exogenous innovation, so that the extra resources do not lead to a welfare gain. In addition, he assumes factor-price equalization, so the welfare gains from lower prices are also not realized.
differentiation of goods, where the number of products remains constant, and innovation on each increases its individual quality level.

The fundamental premise holds that consumers are willing to pay a premium for quality (call it $q$) for the new version of the good. Grossman and Helpman show how this assumption leads to a product cycle where production shifts from the North to the South, and then made obsolete when the new generation of quality is introduced.

Glass and Saggi (1995) incorporate costly, endogenous FDI into the quality-ladders framework. Firms pay an “adaptation” cost to take advantage of lower wages by shifting production overseas. Yang and Maskus (2000a) show how firms can directly license technology to Southern producers under similar assumptions.

The quality-ladders framework is a sensible way to model trademark infringement. I assume that goods encompass two forms of intellectual property. One is the knowledge of production, which represents the intrinsic value of any innovation. This production knowledge is the result of innovative R&D efforts, and is proprietary to the firm.

When firms innovate a new quality level, they must signal its value to potential consumers. To do so, they receive a trademark that differentiates it vertically from previous innovations. This trademark indicates the other form of intellectual property embodied by a good, its reputation for quality. The premium $q$ consumers are willing to pay covers both the value of the good and the reputation for quality.

2. Basic Model

2.1 Consumption
The basic model builds on Grossman and Helpman’s (1991) quality-ladders model of vertical differentiation. Consumption is determined by the following utility function and budget constraint,

\[ U = \int_{0}^{\infty} e^{-\rho t} \log u(t) dt, \]

where \( \rho \) is the discount rate. A continuum of goods exists, indexed by \( j \in [0,1] \). Each good \( j \) can be innovated on to yield a new quality premium \( q^m \). Thus, \( \log u(t) \) is determined by

\[ \log u(t) = \int_{0}^{1} \log \left( \sum_{m} q_{m}(j)x_{mt}(j) \right) dj, \]

where \( x_{mt}(j) \) indicates consumption of quality \( m \) of good \( j \) at time \( t \). Quality innovations enter multiplicatively – the \( m^{th} \) version is valued \( q \) more than the \( (m-1)^{th} \) version. Consumers purchase the good with the highest quality per price; as shown in the pricing strategy below, this means they purchase only the latest innovation.

Aggregate spending by consumers is given by \( E(t) = \int_{0}^{1} \left[ \sum_{m} p_{mt}(j)x_{mt}(j) \right] dj \).

They optimize their lifetime spending at any time \( t \) for the instantaneous expenditure \( E \). Since all goods enter the utility function equally, instantaneous expenditures are split across all goods evenly. That is, consumers spend \( E(t)/J \) for each good, and since \( J \to 1 \) on the continuum, consumers spend \( E \) on every good \( j \) at each point in time.

The intrinsic value of the good as it enters the utility function is the value \( q^m \) for the \( m^{th} \) innovation. Consumers thus value \( q^{+} \) goods at a quality premium \( q \) over the next best goods. Since, by assumption, all previous goods are sold at marginal cost \( w_s \), consumers would be willing to pay \( qw_s \) for the \( q^{+} \) good. With full trademark protection, this \( q \) captures both the value of the good and the reputation of the trademark, thus
consumers can be confident they are buying the latest innovation. Most quality-ladder models implicitly assume this signal. If trademarks can be infringed, however, the signal is imperfect.

Note this concerns only the Southern market. Full trademark protection is offered in the North, so the signal is perfect. An example of the markets under consideration is a quality-sewn name-brand shirt, designated by the trademarked pocket emblem. In the North, every shirt with the emblem is assuredly the quality-sewn variety. In the South, infringing firms sell knock-off shirts of inferior quality, but with the same pocket emblem. The trademark is an imperfect signal.\(^4\)

2.2 Price decision

Trademark infringement affects both the pricing decision and the market share of innovating firms. Firms engage in price competition to maximize profits. In general, this means pricing to capture the full market. All consumers are always willing to pay \(q\) for any quality innovation when the nearest alternative is the last generation of the same good. With this \(q\), they are purchasing the intrinsic value of the good and the reputation of the trademark. Innovating firms, with a perfect signal, can charge \(q\) times the production cost of their nearest competitor. I assume all previous innovations are disseminated to the point that production and consumption takes place at perfectly competitive prices. Since the most nearest competitor faces a marginal cost equal to the Southern wage \(w_S\), innovating firms charge \(p^* = qw_S - \varepsilon\) to capture the entire market. As \(\varepsilon \to 0\), \(p^* = qw_S\). For simplicity, normalize \(w_S = 1\), so that \(p^* = q\).

\(^4\) Note this is not a case of multiple quality levels being sold in equilibrium, such as in Glass and Saggi (1998) or Yang and Maskus (2000b), since consumers always prefer to pay the premium for the highest quality good. It is a case of uncertainty. Someone buys a Coca-cola in Caracas and ends up with, say, New
In the presence of trademark infringement, however, firms are unable to perfectly signal the quality premium. The innovating firm has a monopoly on q+, which they market and sell under the trademark. With infringement, competitors are able to produce and market q- under the same trademark. I fix the rate they can do this at τ, which depends on the level of IPP. Thus, τ of all products sold under the trademark are infringed goods.

Infringing firms pay marginal cost \( w_s \), so they will make a positive profit whenever \( p^* > w_s \). If they charge \( p^* < w_n \), they will not sell anything, because no Northern firm would sell below *its* marginal cost and this low price would signal the low quality of the good. Consumers will also not pay \( qw_s \), the maximum price for the innovating firms, due to the possibility they would be purchasing an inferior product.

The expected value to the consumer is \( E\{u(q)\} = (1-\tau)u(q+) + \tau u(q-) \). Assuming risk neutrality, the expected utility is \( (1-\tau)q^m + \tau q^{m-1} \). The expected quality premium is \( (1-\tau)q \), since q- is otherwise sold at \( w_c=1 \). Thus, risk-neutral consumers are willing to pay \( (1-\tau)q \) for a good sold under the q+ trademark.\(^5\)

If both innovating and infringing firms charge \( p^* = (1-\tau)q \), both can steal the market by selling at an incremental discount. The resulting “Bertrand paradox” would drive prices to \( w_n \), leaving no economic profits for the Northern firm. For this reason, I extend the assumption on infringement so that τ represents the *maximum* market share of infringing firms. Neither firm will then charge below \( p^* = (1-\tau)q \), as this would lower Coke. Nobody likes New Coke, so the consumer has overpaid, and will pay less for any drink labeled “Coca-cola” in the future.

\(^5\) Risk averse consumers, of course, would be willing to pay less, since by Jensen’s inequality \( E\{u(c)\} \leq u(E\{q\}) \).
their expected profits. Since consumers will not pay more, this price holds as an equilibrium price.

2.3 Market structure

Following an innovation on a quality-level, a single firm in the North holds a monopoly on the production knowledge for q+ as well as its trademark. By investing resources, these firms can “adapt” production to Southern plants to take advantage of lower factor costs. This adaptation represents costly FDI, and firms successful at this adaptation are considered multinational enterprises (MNEs). MNEs face the same rate of trademark infringement as Northern firms who export to the South. I assume no transportation costs, so MNEs are able to service the Northern market (by “re-exporting”) at the Southern factor cost $w_s$.

The inclusion of FDI allows the model to overcome the resource constraint problem that is present in Helpman (1993). It could be generalized to include any outsourcing activities, including licensing and joint ventures, that capture the interregional shift in production. In section 3.9 below, I facilitate comparison with Helpman by restricting the assumption on FDI.

In the central model, three types of firms exist. Northern firms and MNEs, who both have the monopoly on q+ but are differentiated by their location of production, and the infringing firms who sell q- under the q+ trademark. With these assumptions, the South only obtains new technology when a good is innovated again, and thus the model cannot be considered a depiction of a product cycle. In a section below, I do include imitation of goods, for comparison to those models where IPRs feature prominently on the imitation
rate, but the central model focuses on the intellectual property of the quality reputation captured by the trademark.

2.4 Profit equations

Firms selling products in the North do not have to consider the possibility of trademark infringement. Since the trademark works as a perfect signal, consumers with expenditure level $E^N$ pay the maximum price for the quality premium. Northern firms (indexed by N) charge price $p^N = q$, sell quantity $E^N/q$, and pay marginal cost $w_n$ (or $w$). Multinationals exporting to the North charge the same price for the same quantity, but pay the lower marginal cost $w_s = 1$. This leads to following profit equations for sales in the North:

(3) Northern firm profits: $\pi^{NN} = x_j(p_j - w_n) = \frac{E^N}{q}(q - w_n) = \frac{E^N}{q}(q - w)$

(4) MNE profits: $\pi^{MN} = \frac{E^N}{q}(q - 1)$.

I assume $q > w$ to ensure positive profits for the Northern firm.

In the South, firms must take into account the risk of trademark infringement when pricing their products. Moreover, they earn only $(1-\tau)$ portion of the market. The profit equations for sales in the South are:

(5) $\pi^{NS} = (1-\tau)\frac{E^S}{(1-\tau)q}(p^* - w_n) = \frac{E^S}{q}((1-\tau)q - w)$

(6) $\pi^{MS} = (1-\tau)\frac{E^S}{(1-\tau)q}(p^* - 1) = \frac{E^S}{q}((1-\tau)q - 1)$.
The full profits for Northern firms and MNEs are the sum of the two above, which simplify when substituting \( E = E^N + E^S \) and \( E^S = sE \), where \( s \) represents the share of world income going to Southern consumers.

\[
\begin{align*}
\pi^N &= E(1 - s\tau - \frac{w}{q}) \\
\pi^M &= E(1 - s\tau - \frac{1}{q})
\end{align*}
\]

I do not include depictions of the profits of infringing firms, since their R&D processes are given exogenously. They only affect the general equilibrium results through the resources used.

### 2.5 Research and Development

Northern firms invest resources at intensity \( \iota \) to innovate quality improvements, with the labor cost of innovation \( I \). The cost to innovation is \( wIdt \), with the expected gain \( \iota v^N dt \). If successful, they then invest resources in adaptation at intensity \( \alpha \) in order to shift production to the South to take advantage of lower wages, with labor cost \( A \). The expected gain to adaptation is \( \alpha v^M \) at labor cost \( wA \) and the opportunity cost \( \alpha v^N \). Table 1 summarizes the R&D sector of the economy.

**Table 1: R&D summary**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Cost</th>
<th>Gain</th>
<th>Labor Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innovation</td>
<td>WI</td>
<td>( \iota v^N )</td>
<td>( I )</td>
</tr>
<tr>
<td>Adaptation</td>
<td>( wA + \alpha v^N )</td>
<td>( \alpha v^M )</td>
<td>( \alpha A )</td>
</tr>
<tr>
<td>Infringement</td>
<td>costless, exogenous</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
I assume free entry in innovation, and profit-maximization in adaptation, so that for both R&D equations, the expected gain cannot be greater than the cost. This leads to the following expressions:

\[(9) \quad \nu^N \leq wI \quad \text{c.s.} \quad t > 0\]

\[(10) \quad \nu^M \leq wA + \nu^N \quad \text{c.s.} \quad \alpha > 0.\]

### 2.6 No-arbitrage

I assume the same rational-expectations stock market valuation as Grossman and Helpman (1991). Individuals invest in firms until they reach the same expected value as a riskless bond earning interest \(r\) times the value of the firm. Northern firms earn the profits \(\pi^N dt\), with capital gain \(\nu^N dt\). Northern firms adapt to FDI at intensity \(\alpha\). When successful, they earn the return \(\nu^M\), so the expected return is \(\alpha \nu^M dt\). The cost is \(\alpha A\) units of labor plus the opportunity cost \(\nu^N\), giving a total return \(\alpha (\nu^M - \nu^N - A) dt\). They also face the risk of capital loss \(\nu^N dt\), in which other firms innovate over their quality. MNEs earn profits \(\pi^M dt\), with capital gain \(\nu^M dt\), and face the risk of capital loss \(\nu^M dt\). When dividing through by \(\nu^N\) and \(\nu^M\) respectively, this yields the following no-arbitrage conditions:

\[(11) \quad \frac{\pi^N}{\nu^N} + \frac{\nu^N}{\nu^N} + \alpha \frac{\nu^M - \nu^N - A}{\nu^N} - \frac{t}{\nu^N} = r\]

\[(12) \quad \frac{\pi^M}{\nu^M} + \frac{\nu^M}{\nu^M} - \frac{t}{\nu^M} = r\]

Along the path that \(\nu / \nu = 0\), and \(\rho = r\), these can be rearranged to yield the following:
(13)  \( v^N = \frac{\pi^N}{\rho + \iota} \)

(14)  \( v^M = \frac{\pi^M}{\rho + \iota} \).

2.7 Resource constraints

Production and R&D efforts are constrained by the scarce resources available to both regions in the model. Northern labor is used for production and adaptation by Northern firms, and in innovation by firms engaging in R&D. The Northern firms with recent innovations, a measure \( n_N \), produce \( E^N/q \) goods to sell in the North and

\[
(1 - \tau) \frac{E^S}{(1 - \tau)q}
\]

goods to sell in the South, for a total labor use of \( n_N E/q \). This same measure of firms expends \( \alpha A \) units of labor adapting production to the South.\(^6\) Firms that are engaging in research to achieve new innovations expend \( \iota I \) units of labor on the full continuum of goods. These lead to the following expression of the Northern resource constraint:

(15)  \( L_N = \iota I + n_N \frac{E}{q} + \alpha An_N \)

The Southern labor is only used in the production of goods. Infringing firms sell quantity \( \frac{E^S}{(1 - \tau)q} \) to proportion \( \tau \) of the market. MNEs produce \( E^N/q \) for sales in the North and \( (1 - \tau) \frac{E^S}{(1 - \tau)q} \) for sales in the South, for a total labor use of \( n_E E/q \). This yields the following resource constraint:

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\(^6\) Many authors, including Glass and Saggi (1995), assume that adaptation uses Southern resources. Appendix A.2 shows how the results are unaffected by the change.
2.8 Constant measures

In the steady-state, the measures of every firm type must remain constant. That is, the number of firms that become MNEs must be equal to the number of firms who stop being MNEs. These values are summarized in Table 2. Since innovation occurs on all types of firms, at any given time the number of firms becoming Northern firms is \( t(n_N + n_F) = t \). Firms are no longer Northern firms after adaptation or innovation, thus the measure of firms leaving \( n_N \) is \( (1 + \alpha)n_N \). Firms become MNEs through adaptation by Northern firms and leave through innovation on the measure \( n_F \).

Table 2: Constant Measures

<table>
<thead>
<tr>
<th>Firm Type</th>
<th>In</th>
<th>Out</th>
</tr>
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<tbody>
<tr>
<td>N:</td>
<td>1</td>
<td>((1+\alpha)n_N)</td>
</tr>
<tr>
<td>F:</td>
<td>(\alpha n_N)</td>
<td>(t n_F)</td>
</tr>
<tr>
<td></td>
<td>(n_N + n_F = 1)</td>
<td></td>
</tr>
</tbody>
</table>

These calculations lead to the following relationships for firm measures:

\[
(17) \quad n_M = \frac{\alpha}{t + \alpha}
\]

\[
(18) \quad n_N = 1 - n_M = \frac{1}{t + \alpha}
\]

\[
(19) \quad \alpha = t \frac{n_M}{1 - n_M}
\]
Notice that (19) was solved by plugging (17) into (18), so that these three equations actually only capture two relationships among the four variables. Knowing the constant measure for $n_N$ and the summation of firm measures to 1 makes the constant measure for $F$ redundant.

2.9 Reduced form equations

The above equations can be combined to provide insight into the economy. Combine the profit equations (7) and (8), the R&D equations (9) and (10), and the value equations (13) and (14) to get:

\[(20) \quad E(1 - \frac{w}{q} - \sigma) = wI(\rho + \tau)\]

\[(21) \quad E(1 - \frac{1}{q} - \sigma) = w(A + I)(\rho + \tau).\]

Equation (20) shows the values that lead to zero economic profits for innovating firms. The left-hand side is the profits for successful innovations, and the right-hand side shows the cost of innovation weighted by the discount rate and the risk of capital loss. Equation (21) shows a similar relationship for adaptation. I call (20) the “Northern valuation condition” VN and (21) the “MNE valuation condition” VM.

Dividing VN by VM solves the relative wage to be:

\[(22) \quad w = \frac{qA(1 - s\tau) + I}{A + I},\]

which does not depend on the extent of FDI. Firms take $w$ as given (or as a function of $\tau$) when making the decision to adapt. The rent gains from FDI depend on $w$, but the activity does not affect $w$. Thus, the effects that lead to an equilibrium extent of FDI derive from other parameters.
When combined with the two constant-measure relationships, the resource constraints (15) and (16) and the valuation conditions (20) and (21) provide equations that can solve for the endogenous variables \{E, w, n_N, n_M, \tau, and \alpha\}. The relative wage can be solved from (22). To focus analytics on the innovation rate and the extent of FDI, I eliminate E from the system of equations and substitute \(n_N = 1-n_M\).

Solving VN and VM for \(E/w\) and setting them equal to each other yields

\[
\frac{E}{q} + I(\rho +1) = (A + I)(\rho +1) \frac{1-s\tau}{1-\frac{q}{1-s\tau}},
\]

which can be rearranged to

\[
\frac{E}{q} = \frac{qA(1-s\tau) + I}{q(1-s\tau)-1}(\rho +1).
\]

Solving for \(E/q\) from the Southern resource constraint and plugging in yields the “joint valuation condition”, or VC:

\[
\frac{L_s}{s\tau + n_M} = \frac{qA(1-s\tau) + I}{q(1-s\tau)-1}(\rho +1).
\]

Fully differentiating (25) gives the derivative

\[
\frac{dn_M}{dh} \bigg|_{VC} = -\frac{qA(1-s\tau) + I}{q(1-s\tau)-1} \frac{L_s}{(\frac{s\tau}{1-\tau} + n_M)^2} < 0
\]

As the extent of FDI increases, the rate of innovation goes down. To understand the intuition for this result, consider equation (25). The left-hand side, which is equivalent to \(E/q\) solved from the Southern resource constraint, can be compared to \(E/q\) solved from the Southern resource constraint, can be compared to

\[
\pi^M = \frac{E}{q}(q(1-s\tau)-1).
\]

In fact, we can write
Clearly, as \( n_M \) increases, the MNE profits decline. This means that as more firms take advantage of lower factor costs, the returns decrease. The overall decline in expected value of an innovation causes firms to invest fewer resources in innovation.

Solving for \( E/q \) from both resource constraints and setting equal to each other yields the “joint resource constraint”, or RC:

\[
(28) \quad \frac{L_s}{s\tau + n_M} = \frac{1}{1-n_M} \left( L_N - uI \right) - \alpha A.
\]

Fully differentiating (28) yields

\[
(29) \quad \left. \frac{dn_M}{dt} \right|_{RC} = \frac{I}{1-n_M} + \frac{d\alpha}{dt} A
\]

\[
= \frac{1}{s\tau + n_F} + \frac{L_N - uI}{(1-n_M)^2} = \frac{d\alpha}{dn_M} A > 0
\]

From (19), \( \frac{d\alpha}{dn_M} = \frac{1}{(1-n_M)^2} > 0 \) and \( \frac{d\alpha}{dt} = -\frac{n_M}{1-n_M} > 0 \), thus the sign holds since

\[
L_N - u(I + A) > 0.
\]

In the RC, the rate innovation and the extent of FDI are positively correlated. The intuition here is straightforward, and follows Lai (1998) and Yang and Maskus (2000a). Northern production pulls resources directly from R&D, so as FDI increases more Northern resources are invested in innovation efforts.

The VC and RC lines can be graphed in \((t, n_F)\) space. Using the derivatives for (25) and (28), graph 1 shows an equilibrium point A for the innovation rate and the extent
of FDI. The lines are curved since the second derivatives have the opposite signs of (26) and (29).

Graph 1

The effects of intellectual property protection that strengthens trademarks against infringement can be seen by the shifts in the curves in graph 1. An increase in trademark protection lowers the infringement $\tau$. For the VC, consider how the curve shifts by taking the derivatives of each variable with respect to $\tau$. Differentiating (25) by $t$ and $\tau$ yields $\left. \frac{d\eta}{d\tau} \right|_{VC} < 0$. Holding the extent of FDI constant, the innovation rate increases as $\tau$ decreases. This implies a shift right in the VC curve. For the RC, fully differentiating (28) yields $\left. \frac{d\eta}{d\tau} \right|_{RC} > 0$, implying a shift right in the RC curve and thus the opposite result on innovation.

The overall effect on the innovation rate using equations (25) and (28) cannot be signed with any meaningful relationships between the variables. In Appendix A.1, I use Cramer's Rule to show that $d\eta/d\tau < 0$. In graph 1, the VC shifts further than the RC, and an increase in IPR protection that lowers $\tau$ leads to both an increase in FDI and an
increase in the innovation rate. This result compares favorably to the results in Lai (1998) and Yang and Maskus (2000a).

3. Imitation

The above model offers a simple depiction of trademark infringement in a general equilibrium model, with results that are familiar to the literature. For example, the increased extent of FDI following strengthened IPRs compares well to Yang and Maskus (2000a) similar increase in licensing. In particular, the ambiguous effect of increased intellectual property protection on the innovation rate further demonstrates how difficult it is to draw definite conclusions about IPRs. As discussed above, Helpman (1993) and Lai (1998) offer contrasting results using different assumptions. This paper extends the results to include trademark infringement.

To facilitate comparison within the literature, this section adds a product cycle in which the production knowledge in the latest innovations can also be transferred. That is, the South imitates goods at rate \( \mu \). For analytical tractability, I assume no FDI takes place. In this section, imitation is the sole method of technology transfer. The first part of this section introduces endogenous imitation to the baseline model with full IPR protection. The second part adds the risk of trademark infringement. The third part considers the present model in the context of exogenous imitation, resulting in relationships identical to Helpman (1993).

3.1 Endogenous Imitation with full IPR protection

As before, firms innovate new quality levels \( q^+ \) in the North and service Northern and Southern markets under particular trademarks. Consumers are willing to pay \( q \) as a premium for the intrinsic quality and the reputation of the good. Previous quality levels,
q- and below, are disseminated throughout the world, where they can be produced and sold at marginal cost $w_s = 1$. Southern firms can infringe on the trademark at rate $\tau$.

Now, however, Southern firms are also able to “imitate” the quality innovation. By investing their own resources in R&D, they can duplicate the quality of the good $q+$, essentially a transfer of production knowledge. I assume perfect imitation, so the good produced by a successful imitating firm provides the exact utility of the original.

The goods remain differentiated, however, by their trademarks. The imitating firm, call it the Follower, cannot sell the $q+$ good under the brand of the innovating firm (the Leader). Followers sell under a new trademark that is perceived by consumers to be of less value than the original trademark.

Different consumers assign different values to the trademarks. For simplicity, aggregate all consumers into two groups. The first group perceives the value of the original trademark to be higher than the Follower’s trademark, for reasons of reputation, brand loyalty, or first-mover advantage. Call these L-consumers, who prefer the Leader’s product. The second group of consumers, the F-consumers, is happy to purchase the $q+$ good under the “inferior” trademark *as long as it costs less*.7

As an example, consider pain relief medicine. A Northern firm innovates a new quality tablet to alleviate headaches, and earns a reputation for the trademark. A Southern firm then imitates the *exact* quality of the tablet, leaving no difference between the two, but must sell under a different trademark that does not enjoy the same reputation. Depending on the relative prices of the two goods, L-consumers would generally prefer

---

7 Landes and Posner (1987) support this assumption when they say “the fact that two goods have the same chemical formula does not make them of equal quality to even the most coolly rational consumer (pg. 275)”.

to pay a premium for the reputable trademark, while F-consumers would prefer a discount for the same drug.

To account for different groups of consumers, equations (1) and (2) must be re-written

\[
U^w = \int_0^\infty e^{-pt} \log u^w (t) dt,
\]

\[
\log u^w (t) = \sum_m \left[ g^m (j) x^w_{mj} (j) \right] dj.
\]

where \( w \in \{L, F\} \) indexes the type of consumer.\(^8\) I capture the value of trademark perception by assuming \( q^L > q^F \). L-consumers, who prefer the Leader’s trademark, assign a higher value to the good than F-consumers, although the actual quality level is the same. They receive utility from the trademark itself.

### 3.1.1 Price decisions

Under full IPR protection, the firms are secure that their trademarks cannot be infringed. After imitation, the Leader and Follower firms compete in prices for each good \( j \) that has been imitated. A measure \( \lambda \) of consumers, the L-consumers, are willing to pay \( q^L \) for a quality innovation, while a measure \( f \), or \( 1-\lambda \), prefer to pay \( q^F \) for the \( q^+ \) good when an imitated discount is available. The Follower firm competes against all potential producers of \( q^- \) by charging \( q^F w_s - \epsilon \), which at the limit is \( q^F \). The Leader firm practices limit pricing against the Follower firm by charging \( q^L \).

\(^8\) These utility functions are based on the ones derived by Glass (2000), in which consumers assign different values to the same goods. Glass, however, assumes consumers to differ along the actual quality innovation, so that two quality levels are sold in equilibrium. In the present model, consumers do not differ on quality, but in the value of the trademark. Only one quality-level is sold in equilibrium (except, of course, for the \( q^- \) goods sold illegally).
If prices are the same and no greater than $q^F$, all consumers will purchase the Leader’s product. The Leader can then capture the entire market by lowering its price to the Follower’s price. The Follower, however, will sell to the F-consumers by then lowering its own price in competition. The lowest possible price charged by the Leader firm that will allow non-negative profits is $w_N$, so by charging $p^F = w_N$ the Follower ensures sales to $1 - \lambda$ consumers.

The L-consumers, however, are willing to pay $q^L$ for the new product. If the Follower charges $w_N$, the Leader will charge $q^L$ and sell to measure $\lambda$ consumers. When the Leader sets price $p^L = q^L$, the Follower can raise its price to $q^F > w_N$ without losing market share. Thus, $\{p^L = q^L, p^F = q^F\}$ exists as a Nash equilibrium. If the Leader seeks to undercut the Follower, the Follower can lower its price to $w_N$ and capture the entire market, so the Leader will not deviate. On the other hand, the Follower cannot sell to any more consumers by changing its price, given the actions of the Leader, so it will charge the profit-maximizing $q^F$.

Note that three firm types are present in this section of the paper. Northern firms are those with a monopoly on the latest quality innovation. Leader firms are those innovators, formerly Northern firms, whose product has been imitated. Follower firms are the imitators with the under-appreciated trademark.

This leads to the following profit equations for each firm. Note that there is no difference between profits for goods sold in the North or the South, because with full IPR protection the markets are identical, except for size.

\[
\pi^L = \lambda E \left(1 - \frac{w}{q^L}\right)
\]
As above, I assume $q^F > w$ to ensure positive profits.

A Northern firm with a new quality innovation that has not been imitated competes only against the potential producers of the $q$-good. The price they charge in this competition depends on assumptions concerning pricing strategy. I consider four price schemes and show in table 3 how the results differ according to different assumptions. The remainder of this section uses a single assumption, and in Appendix A.3 I show the major equations for the other assumptions.

Recall that consumers differ in their willingness to pay for a trademarked good. If the price is higher than $q^F$, $F$-consumers would prefer to pay the competitive price for lower quality. This affects the firm’s pricing decision. If firms can perfectly discriminate in their pricing, they will charge $p^* = q^L$ for $L$-consumers and $p^* = q^F$ for $F$-consumers. This would involve firms selling the exact same product at different prices, under different packaging, side-by-side on the shelf. Although unlikely, this scenario is not implausible, and I show the results of the model using this assumption in the first column of table 3.

Alternatively, I could relax the assumption on quality valuation. If $F$-consumers are willing to pay $q^L$ for a quality innovation when no alternative to the original exists (that is, before imitation), Northern firms could charge the limit price $q^L$ and capture the entire market. I show the results using this scheme in the second column of table 3.

**Table 3: Results for various pricing strategies**

<table>
<thead>
<tr>
<th>Price discrimination</th>
<th>$p^* = q^L$</th>
<th>Whole-market</th>
<th>High-end pricing</th>
</tr>
</thead>
</table>

(33) \[ \pi^F = (1 - \lambda)E(1 - \frac{1}{q^F}) \]
Two further pricing strategies do not require any additional assumptions on the current model. The Northern firm, competing with the producers of the \( q \)-good, can either charge \( q^F \) and capture the entire market, or charge \( q^L \) and sell only to \( L \)-consumers. I call the former “whole-market” pricing, and the latter “high-end” pricing. The choice depends on the number of \( L \)-consumers. I show the results for both scenarios in columns 3 and 4, respectively, of table 3. The only major result affected by the assumptions on pricing strategy follows the derivative for the LN-line under high-end pricing. Since firms are already charging \( q^L \) and selling only to \( L \)-consumers, the profits do not change after imitation. Thus, in this scenario, a change in the innovation rate is not related though the Northern resource constraint to the measure of imitating firms.

With high-end pricing, the \( F \)-consumers, unwilling to pay \( q^L \) for an innovated good, purchase the \( q \)-good at marginal cost. Firms will not follow this strategy if \( \pi^N(q^F) > \pi^N(q^L) \), or \( \frac{q^L - q^F}{q^L q^F} > 1 - \lambda \). For the remainder of this section, I assume this inequality holds, and Northern firms charge \( p^* = q^F \). This yields the Northern profits

\[
(34) \quad \pi^N = E(1 - \frac{w}{q^F}).
\]

### 3.1.2 Research and Development
The costs and benefits of R&D for innovation do not change; Northern firms continue to invest resources at intensity $\iota$ with the labor cost $\iota$. For imitation, Southern firms invest resources at intensity $\mu$ with labor cost $M$ to gain $v^F$ if successful. Notice that imitation draws from Southern resources at marginal cost $w_s = 1$ as opposed to Northern resources as in section 2 above.

**Table 4: R&D summary for endogenous imitation**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Cost</th>
<th>Gain</th>
<th>Labor Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Innovation</td>
<td>$wI$</td>
<td>$\iota v^N$</td>
<td>$\iota I$</td>
</tr>
<tr>
<td>Imitation</td>
<td>$M$</td>
<td>$\mu v^F$</td>
<td>$\mu M$</td>
</tr>
</tbody>
</table>

I again assume free entry in innovation and imitation, leading to the following expressions:

(35) $v^N \leq wI \text{ c.s. } \iota > 0$

(36) $v^F \leq M \text{ c.s. } \mu > 0$

### 3.1.3 No-arbitrage

The risk of imitation affects a firm’s market return in the no-arbitrage conditions. Northern firms now face the additional risk $\mu dt$ of capital loss, but they become Leader firms at the same rate $\mu$. Leader and Follower firms only face the risk $t dt$ of capital loss. The value equations become

(37) $v^N = \frac{\pi^N + \mu v^L}{\rho + \mu + \iota}$

(38) $v^L = \frac{\pi^L}{\rho + \iota}$
\[ v^F = \frac{\pi^F}{\rho + \iota} \]

### 3.1.4 Resource Constraints

The resource constraints are altered by the inclusion of imitation. Northern labor is used for innovation and production by Northern and Leader firms. Northern firms, of measure \( n_N \), produce \( \frac{E}{q^F} \) goods, and Leader firms, of measure \( n_L \), produce \( \lambda \frac{E}{q^L} \) goods. Southern labor is used for imitation and production by Follower firms. Southern firms target imitation only at a measure \( n_N \) of goods that have not already been imitated, for full labor cost of \( \mu M n_N \). Follower firms, of measure \( n_F \), produce \((1-\lambda)\frac{E}{q^F}\) goods.

\[ L_N = \mathcal{I} + n_N \frac{E}{q^F} + n_L \lambda \frac{E}{q^L} \]

\[ L_S = \mu M n_N + n_F (1-\lambda) \frac{E}{q^F} \]

### 3.1.5 Constant Measures

As before, the measures of firm types must remain constant in the steady-state. Goods are produced by new Northern firms at rate \( \iota \), with production shifting to Leader firms or other Northern firms (when innovated over) at rate \((\iota+\mu)n_N\). Firms become Leader firms at rate \( \mu n_N \) and leave at rate \( \tau n_L \). Similarly, firms become Follower firms at rate \( \mu n_N \) and leave at rate \( \tau n_F \). These are summarized in table 5.

**Table 5: Constant Measures with Endogenous Imitation**

<table>
<thead>
<tr>
<th>Firm Type</th>
<th>In</th>
<th>Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N: )</td>
<td>( \iota )</td>
<td>((1+\mu)n_N)</td>
</tr>
<tr>
<td>( L: )</td>
<td>( \mu n_N )</td>
<td>( \tau n_L )</td>
</tr>
</tbody>
</table>
From the table, it is easy to see that \( n_L = n_F \), giving

\[(42)\]
\[
n_N = \frac{1}{1 + \mu}
\]
\[(43)\]
\[
n_F = \frac{\mu}{1 + \mu}
\]
\[(44)\]
\[
\mu = \frac{n_F}{n_N}
\]

### 3.1.6 Reduced-form Equations

Combining equations (32)-(39) gives the following value equations.

\[(45)\]
\[
\frac{E}{q^F} \left[ q^F - w + \frac{\mu \lambda}{\rho + 1} \frac{q^F}{q^L} (q^L - w) \right] = w I (\rho + \mu + 1)
\]
\[(46)\]
\[
\frac{E}{q^F} (q^F - 1) = M (\rho + 1)
\]

As before, the relative wage can be found by dividing (45) by (46), which yields

\[(47)\]
\[
\frac{q^F - q + \frac{\mu \lambda}{\rho + 1} (q^L - w)}{(1 - \lambda)(q^F - 1)} = \frac{w I}{M} \frac{\rho + \mu + 1}{\rho + 1}
\]

Solving this for the relative wage yields

\[(48)\]
\[
\frac{1}{w} \left[ \frac{I}{M} \frac{\rho + \mu + 1}{\rho + 1} + \frac{q^F (\rho + 1) + \mu \lambda q^F}{q^L (1 - \lambda)(\rho + 1)(q^F - 1)} \right] = \frac{q^F}{q^F - 1} \left[ \frac{\rho + 1 + \mu \lambda}{(1 - \lambda)(\rho + 1)} \right]
\]
or

\[(49)\]
\[
\frac{1}{w} = \frac{I}{M} \frac{q^F - 1}{q^F} \frac{\rho + \mu + 1}{\rho + \mu \lambda + 1} (1 - \lambda) + \frac{q^F (\rho + 1) + \mu \lambda q^F}{q^L (1 - \lambda)(\rho + 1 + \mu \lambda)}.
\]
From this equation, it is easy to show that \( d(1/w)/dt < 0 \), so that an increase in the innovation rate leads to a decrease in \( 1/w \), or an increase in the relative wage. Similarly, \( d(1/w)/dλ < 0 \), so an increase in the measure of L-consumers also leads to an increase in the relative wage. The derivative for the imitation rate is

\[
\frac{d(1/w)}{dμ} = \frac{ρ + 1}{(ρ + μλ + 1)^2} \left[ \frac{I}{M} \frac{q^F - 1}{q^F} (1 - λ) - λ \frac{q^L - q^F}{q^F} \right],
\]

which is positive or negative, depending on the value of \( q^F \) in relation to \( q^L \) and 1.

Substituting from the constant measures, the imitation value condition and the resource constraints provide a system of three equations for the endogenous variables \{E, \( ι \), \( n_F \}\}, with \( w \) determined separately by the innovation value condition.

**Equation (50):**

\[
(1 - λ) \frac{E}{q^F} (q^F - 1) = M (ρ + 1)
\]

**Equation (51):**

\[
L_N = ι + n_N \frac{E}{q^F} + n_F λ \frac{E}{q^L}
\]

**Equation (52):**

\[
L_S = n_F M + n_F (1 - λ) \frac{E}{q^F}
\]

Solving for \( E = \frac{M}{1 - λ} \frac{ρ + 1}{q^F - 1} \) from the imitation value condition and substituting into the resource constraints yields the following two equations for the variables \{\( ι \), \( n_F \}\).

**Equation (53):**

\[
L_N = ι + \frac{M}{1 - λ} \frac{ρ + 1}{q^F - 1} \left[ 1 - n_F \frac{q^L - λq^F}{q^L} \right]
\]

**Equation (54):**

\[
L_S = n_F M \left[ 1 + \frac{ρ + 1}{q^F - 1} \right]
\]

Fully differentiating these two equations characterizes a steady-state equilibrium for the economy.
As in section 2, the second derivatives are the opposite signs of the first derivatives.

Plotting these lines in \((\iota, n_F)\) space yields a graph similar to graph 1:

Graph 2

3.2 Endogenous imitation with trademark infringement

Consider now the effects of imperfect IPR protection. As in section 2, firms can infringe on the trademarks of the \(q^+\) goods. Suppose this happens at the same rate \(\tau\), and affects both trademarks equally. Consumers make the same decision as before, and firms engage in the same pricing game. The most consumers are now willing to pay for the Leader’s good is \((1-\tau)q^L\), and the most for the Follower’s good is \((1-\tau)q^F\). Again, any price-cutting strategy by the Leader firm would result in the follower charging \(w_N\) to capture the full market, so the Leaders sets price \((1-\tau)q^L\). To maximize profit, the
Follower firms charges the maximum price \((1-\tau)q^F\), since no lower price will increase its market share. Thus, \(\{p^L = (1-\tau)q^L, p^F = (1-\tau)q^F\}\) holds as a Nash equilibrium.

The profit equations under this new pricing scheme are given by:

\[
\pi^N = E(1 - \sigma \tau - \frac{w}{q^F}) \\
\pi^L = \lambda E(1 - \sigma \tau - \frac{w}{q^L}) \\
\pi^F = (1 - \lambda) E(1 - \sigma \tau - \frac{1}{q^F})
\]

There is no change in the R&D activity, so I reproduce (35) and (36) here.

\[
\nu^N \leq wI \text{ c.s. } \iota > 0 \\
\nu^F \leq M \text{ c.s. } \mu > 0
\]

The no-arbitrage conditions are also unchanged, so I reproduce (37) - (39) here.

\[
\nu^N = \frac{\pi^N + \mu \nu^L}{\rho + \mu + \iota} \\
\nu^L = \frac{\pi^L}{\rho + \iota} \\
\nu^F = \frac{\pi^F}{\rho + \iota}
\]

The resource constraints are altered by the presence of infringing firms. Firms use the same resources for R&D in innovation as before. Northern firms produce \(E^N/q^F\) goods to sell in the North and \((1-\tau) \frac{E^S}{(1-\tau)q^F}\) goods to sell in the South, for a total labor use of \(n_N E/q^F\). Leader firms produce \(\lambda E^N/q^L\) goods to sell in the North and
\[ \lambda (1-\tau) \frac{E^s}{(1-\tau)q^L} \] goods to sell in the South, for a total labor use of \( \lambda n_F q^L \). Thus, the Northern resource constraint can be written

\[ (65) \quad L_N = u + n_N \frac{E}{q^F} + \lambda n_F \frac{E}{q^L} \]

Southern firms use the same resources for R&D in imitation as before. Follower firms now produce \((1-\lambda)E^N/q^F\) goods for to sell in the North, and \((1-\lambda)(1-\tau)\frac{E^s}{(1-\tau)q^F}\) goods for sell in the South. Infringing firms produce \(\lambda \tau \frac{E^s}{(1-\tau)q^L} + (1-\lambda)\tau \frac{E^s}{(1-\tau)q^L}\) goods, making the resource constraint

\[ (66) \quad L_S = \mu M n_N + n_F (1-\lambda) \frac{E}{q^F} + \frac{\tau}{1-\tau} s E \lambda q^F + (1-\lambda) q^L \]

As before, I can solve for the relative wage from the valuation conditions

\[ (67) \quad \left[ I(\rho + \mu + 1) + \frac{E}{q^F} \left( \frac{q^L (\rho + \mu + \mu \lambda) q^F}{(\rho + 1) q^L} \right) \right] = E (1-\tau) \left( \frac{\rho + 1 + \mu \lambda}{\rho + 1} \right) \]

which, plugging in for \(E\) and inverting, gives

\[ (68) \quad \frac{1}{w} = (1-\lambda) \frac{I}{M} \left( \frac{\rho + 1 + \mu}{\rho + 1 + \mu \lambda} \right) \frac{1-1/ q^F - s \tau}{1-s \tau} + \frac{q^L (\rho + 1 + q^F \mu \lambda)}{q^F q^L (\rho + 1 + \mu \lambda)(1-s \tau)} \]

The imitation valuation condition and the two resource constraints give a system of three equations for \( \{E, \tau, n_F\} \). Substituting \( E = \frac{M}{1-\lambda q^F (1-s \tau) - 1} \frac{\rho + 1}{\rho + 1 + \mu \lambda} \) into the resource constraints yields

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9 Continuing “whole-market” pricing for Northern firms.
(70) \[ L_N = \rho + 1 \left( 1 - n_F \frac{q^L - \lambda q^F}{q^L} \right) \]

(71) \[ L_S = n_F M + n_F M \frac{\rho + 1}{1 - \lambda q^F (1 - s\tau)} + \frac{\rho + 1}{1 - \tau} \frac{M}{q^F (1 - \lambda) q^F (1 - s\tau)} \frac{\lambda q^F + (1 - \lambda) q^L}{q^L q^F} \]

Fully differentiating these equations yields the following derivatives

(72) \[ \frac{dn_F}{dt} \bigg|_{LN} = \frac{I + M}{1 - \lambda q^F (1 - s\tau)} \left( 1 - n_F \frac{q^L - \lambda q^F}{q^L} \right) > 0 \]

(73) \[ \frac{dn_F}{dt} \bigg|_{LS} = -\frac{n_F M [q^F (1 - s\tau) - 1] + n_F M + \frac{s\tau}{1 - \tau} M \left( \frac{\lambda}{1 - \lambda} + 1 \right)}{M [\rho + tq^F (1 - s\tau)]} < 0. \]

Since the second derivatives are opposite signs, when graphed in \((n_F, t)\) space the LN and LS lines are identical to graph 2.

Consider a change in the IPR regime that lowers the rate of infringement \(\tau\).

Holding the measure of firms constant, and differentiating (70) with respect to \(t\) and \(\tau\), yields

(74) \[ \frac{dh}{d\tau} \bigg|_{LN} = -\frac{M}{1 - \lambda [q^F (1 - s\tau) - 1]^2} \frac{\rho + 1}{\lambda \left( 1 - n_F \frac{q^L - \lambda q^F}{q^L} \right)} < 0 \]

Similarly, holding the rate of innovation constant and differentiating (70) with respect to \(n_F\) and \(\tau\) yields

(75) \[ \frac{dn_F}{d\tau} \bigg|_{LN} = \frac{sq^F q^L}{q^F (1 - s\tau)} \frac{1 - n_F q^L - \lambda q^F}{q^L} > 0 \]

A decrease in \(\tau\) shifts the LN curve right.
Comparable differentiation for the LS curve shows

\[
\frac{dh}{d\tau} = \left. \frac{\frac{sM (\rho + 1)}{q^F (1 - s\tau) - 1} [N]}{n_F M [q^F (1 - s\tau) - 1] + n_F M + \frac{s\tau M (\frac{\lambda}{1 - \lambda} q^F + 1)}{1 - \tau}} \right|_{LS} < 0
\]

where \( [N] = \frac{q^F n_F}{q^F (1 - s\tau) - 1} + \left( \frac{1}{(1 - \tau)^2} + \frac{\tau}{1 - \tau} \right) \frac{s q^F}{q^F (1 - s\tau) - 1} \left( \frac{\lambda}{1 - \lambda} q^F + 1 \right) \)

and

\[
\frac{dn_F}{d\tau} = \left. \frac{\frac{sM (\rho + 1)}{q^F (1 - s\tau) - 1} [N]}{M [\rho + \tau q^F (1 - s\tau)]} \right|_{LS} < 0
\]

A decrease in \( \tau \) shifts the LS curve right.

**Graph 3**

If imitation is the mode of technology transfer, then an strengthening of IPRs leads to an unambiguous increase in the innovation rate. Basically, it captures the idea that a firm will invest more resources, and achieve more innovations, if the value to any particular innovation increases. As \( \tau \) decreases, the profits for both Northern firms and Leader firms increase, which means the value to innovation increases.

**3.3 Exogenous Imitation**
This section facilitates comparison of the present model to Helpman (1993) by assuming an exogenous imitation rate that can be directly affected by the IPR regime.\textsuperscript{10} Imitation serves as the only manner in which technology is transferred to the South. I assume that Southern firms obtain the production knowledge at the Poisson arrival rate $\mu \Delta t$, where $\mu$ is determined by the level of IPR protection. That is, any increase in the strength of IPRs leads to one-for-one decrease in $\mu$. For simplicity, I assume that imitated goods receive the same trademark as the original innovation, or close enough that consumers do not distinguish between the two. This diffuses all profits, since without differentiation Bertrand competition drives all prices to marginal cost.

The value equation and the profit equation for innovating firms remain the same, giving

\begin{equation}
\pi^N = E(1 - sT - \frac{W}{q})
\end{equation}

as in (57) above. Since the firm loses all economic profits through both innovation and imitation, with no potential return, the value equation becomes

\begin{equation}
\nu^N = \frac{\pi^N}{\rho + \mu + t}
\end{equation}

which, with (60) and (78), yields

\begin{equation}
E(1 - sT - \frac{W}{q}) = wI
\end{equation}

The Northern resource constraint simplifies to

\begin{equation}
L_N = U + n_N \frac{E}{q}
\end{equation}

since no Leader firms exist. The Southern resource constraint simplifies to

\textsuperscript{10} This section also compares to Lai (1998), section 4.
which is identical to (16) above with Southern firms in place of MNEs.

Only measures for Northern and Southern firms remain, and they can be reduced to relationships between the innovation and imitation rates by the following:

\[
\begin{align*}
(83) \quad n_N &= \frac{1}{1 + \mu} \\
(84) \quad n_S &= \frac{\mu}{1 + \mu}.
\end{align*}
\]

Combining (80) – (84) yields the following system of equations for the endogenous variables \{E, w, \tau\}.

\[
\begin{align*}
(85) \quad E(1 - \frac{w}{q} - s\tau) - w(\rho + \mu + \tau) &= 0 \\
(86) \quad L_N - \frac{1}{1 + \mu} \frac{E}{q} &= 0 \\
(87) \quad L_S - \frac{E}{q} \left[\frac{s\tau + \mu(1-\tau)}{(1-\tau)(1 + \mu)}\right] &= 0
\end{align*}
\]

Fully differentiating the above system, and applying Cramer’s Rule, yields the following result:\[11\]

\[
\begin{align*}
(88) \quad \frac{d\tau}{d\mu} &= 2q(1-\tau)(1 + \mu) \left[\frac{E}{q(1 + \mu)^2} \frac{s\tau + (1-\tau)}{q(1-\tau)(1 + \mu)} \right] > 0. \\
&= (1-\tau)\mu + s\tau \left[\frac{E}{q(1 + \mu) + I}\right]
\end{align*}
\]

As the rate of imitation goes down, which can be considered a result of tighter IPRs, the rate of innovation decreases, just as in Helpman (1993).

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\[11\] Appendix A.4 shows the method.
4 Welfare effects

This section investigates the potential welfare consequences of trademark enforcement. The baseline model of section 2 emphasizes the lost return to R&D in weak IPRs. Infringement lowers the profits of innovating firms and reduces the signaling power of the trademark for consumers who desire the quality goods. The only welfare gains, however, are in the profits of infringing firms that obtain economic rents using improper labeling.\textsuperscript{12}

Of the four pricing schemes discussed in section 3.1.1, the only one that yields consumer surplus is the “whole-market” strategy used throughout the section. By definition, with perfect price discrimination firms are able to capture the full willingness to pay by consumers. If I relax the assumption on quality valuation, in which firms charge $q^L$ for all consumers, F-consumers actually face a pseudo-welfare loss since they would otherwise only wish to pay $q^F$ for the good. With “high-end” pricing, L-consumers pay their full valuation $q^L$, and F-consumers pay the perfectly competitive price for a good they value at $w_s$.

Consider the situation prior to imitation, when Northern firms maintain a monopoly on their innovation but face the problem of infringement in the Southern market. With “whole-market” pricing, L-consumers are paying only $q^F$ for a good they value at $q^L$. Consequently, they enjoy consumer surplus. Graph 4 shows demand and supply curves in this pricing scenario. The horizontal axis graphs the full continuum of goods from zero to one. The proportion $\tau$ (labeled “t” in the graph) of these goods are knock-offs, for which consumers are only willing to pay the marginal cost 1. The rest of
the goods, proportion 1-τ, are legitimate goods, for which L-consumers are willing to pay \( q^L \) and F-consumers are willing to pay \( q^F \). Both the Northern firms and the infringing firms supply the goods at price \((1-\tau)q^F\).

**Graph 4 Consumer surplus with trademark infringement**

![Graph 4 Consumer surplus with trademark infringement](image)

The areas between the F-demand curve and the supply curve are equal to each other. For infringed goods, F-consumers face a \( \tau(1-\tau)q^F \) welfare loss, but for legitimate goods they gain \((1-\tau)[q^F-(1-\tau)q^F]\) consumer surplus, for an overall gain of zero. L-consumers face the same welfare loss on infringed goods, but gain \((1-\tau)[q^L-(1-\tau)q^F]\) in consumer surplus. The overall welfare gain for L-consumers is \((1-\tau)(q^L-q^F) + (1-\tau)^2q^F\).

Notice that after imitation, the supply curve for L-consumers shifts to \((1-\tau)q^L\), taking away all consumer surplus.

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12 Welfare gains could be accrued if trademark infringement lowered production in the North, opening resources for innovation, but these benefits are small relative to the costs consumers face when paying a
The relationship between consumer surplus and the infringement rate is given by

\[ \frac{d(c.s.)}{d(1-\tau)} = q^L - q^F + 2(1-\tau)q^F > 0. \]

As IPRs are strengthened, raising \((1-\tau)\), the consumer surplus for L-consumers increases.

In this situation, IPRs are welfare-enhancing.

4. Conclusion

This paper introduces trademark infringement into a dynamic, general equilibrium setting. I elaborate the conception of intellectual property rights beyond an incremental rise in imitation costs. An increase in the strength of intellectual property protection increases the rate at which firms shift production to the South. It also increases the innovation rate, regardless of whether technology is transferred by FDI or through imitation. Trademark enforcement may enhance welfare by broadening the gap between the amount some consumers are willing to pay for a good and the actual price charged.
Works Cited


Appendix A.1 Cramer’s Rule for the sign of $dι/dτ$

In section 2, equations (15), (16), (20), and (21) provide a system of four equations for the four endogenous variables $\{E, w, ι, n_M\}$. Fully differentiating these equations for these variables and the rate of infringement $τ$ yields the following:

\[
\begin{bmatrix}
1 - sτ - \frac{w}{q} & -\frac{E}{q} - I(ρ + 1) & -wI & 0 \\
1 - sτ - \frac{1}{q} & -I(ρ + 1) & -wI - A & 0 \\
-(1 - n_M) & 0 & -I & \frac{E}{q} \\
-\frac{sτ}{1 - τ} - \frac{n_M}{q} & 0 & -n_M A & -\frac{E}{q} - ιA
\end{bmatrix}
\begin{bmatrix}
dE \\
dW \\
dτ \\
dn_M
\end{bmatrix}
= \begin{bmatrix}
sE \\
sE \\
0 \\
\frac{sE}{q(1 - τ)^2}
\end{bmatrix}

Using Cramer’s Rule, the sign for $dt/dτ$ can be found with $\frac{dt}{dτ} = \frac{|A|}{|A|}$, where

\[
|A| = -sE \left[ \frac{E}{q} (\frac{sτ}{1 - τ} + 1) + (1 - n_M)ιA \right] - \left( \frac{E}{q(1 - τ)} \right)^2 s \left[ I(ρ + 1) \frac{w - 1}{q} + E \frac{(1 - sτ - 1)}{q} \right] < 0
\]

and

\[
|A| = \left[ \frac{E}{q} (wI + A) + (ρ + 1)ιA \left[ \frac{E}{q} (\frac{sτ}{1 - τ} + 1) + (1 - n_M)ιA \right] + \left[ \frac{E}{q} (n_M A + I) + ιA \left[ I(ρ + 1) \frac{w - 1}{q} + E \frac{(1 - sτ - 1)}{q} \right] \right] > 0.
\]

Thus, $dt/dτ < 0$.

Appendix A.2 Adaptation costs from Southern resource constraint
Section 2 in the main paper assumes that adaptation uses Northern resources. In this appendix, I show that the major results do not change if adaptation uses Southern resources.

Altering the assumption on adaptation costs changes the resource constraints (15) and (16) to

\[ L_N = \mathcal{U} + n_N \frac{E}{q} \]

and

\[ L_S = \frac{s \tau}{1 - \tau} \frac{E}{q} + n_F \frac{E}{q} + \alpha A n_N. \]

Also, the valuation conditions (20) and (21) change to

\[ E(1 - \frac{w}{q} - s \tau) = w I (\rho + t) \]

and

\[ E(1 - \frac{1}{q} - s \tau) = (w I + A) (\rho + t). \]

These equations do not easily reduce to the VN and VM equations in section 2, so I use Cramer’s Rule to develop a result for \( \frac{dt}{d\tau} \).

Fully differentiating the system yields

\[
\begin{bmatrix}
1 - s \tau - \frac{w}{q} & -\frac{E}{q} - I (\rho + t) & -w I & 0 \\
1 - s \tau - \frac{1}{q} & -(A + I) (\rho + t) & -w (A + I) & 0 \\
-(1 - n_M) & 0 & -I - An_M & \frac{E}{q} - \tau A \\
\frac{s \tau}{1 - \tau} - n_M & 0 & 0 & -\frac{E}{q}
\end{bmatrix}
\begin{bmatrix}
dE \\
dw \\
dt \\
dn_M
\end{bmatrix} = 
\begin{bmatrix}
sE \\
sE \\
0 \\
\frac{E}{(1 - \tau)^2}
\end{bmatrix}
\frac{d\tau}{d\tau}
\]
The expression for \( \frac{d\iota}{d\tau} \) derives from \( \frac{dn}{d\tau} = \frac{|B_i|}{|B|} \) where

\[
|B_i| = -sE \left( \frac{E}{q} - A(\rho + t) \right) \left( \frac{E}{q} - tA\left(1 + \frac{s\tau}{1 - \tau}\right) + tA\left(1 - n_M\right) \right)
\]

\[
- \left( \frac{E}{q} - tA \right) - \frac{s}{(1 - \tau)^2} E \left( I(\rho + t) \frac{w - 1}{q} + (1 - s\tau)\left( \frac{E}{q} - A(\rho + t) \right) + \frac{1}{q} \left( A(\rho + t)w - \frac{E}{q} \right) \right)
\]

and

\[
|B| = w(A + I) \frac{E}{q} \left( \frac{E}{q} - tA\left(1 + \frac{s\tau}{1 - \tau}\right) + tA\left(1 - n_M\right) \right)
\]

\[
+ \frac{E}{q} \left( I + n_M A \right) \left( I(\rho + t) \frac{w - 1}{q} + (1 - s\tau)\left( \frac{E}{q} - A(\rho + t) \right) + \frac{1}{q} \left( A(\rho + t)w - \frac{E}{q} \right) \right)
\]

If \( wA(\rho + t) > E/q > A(\rho + t) \) then \( |B_i| < 0 \) and \( |B| > 0 \), thus \( d\iota/d\tau < 0 \). Altering the assumption on the adaptation cost does not change the main result of the model.

**Appendix A.3 Results of different pricing schemes**

**A.3.1 Price Discrimination**

If a firm is able to perfectly discriminate in pricing, its profit equation is

\[
\pi^N = \lambda E \left( \frac{q^L}{q^F} - w \right) + (1 - \lambda) E \left( \frac{q^F}{q^L} - w \right).
\]

Combined with the valuation conditions (35) – (39), this yields the relative wage

\[
\frac{1}{w} = \frac{1}{P} \left[ \frac{I}{M} \frac{q^F}{q^L} - 1 - \lambda + \lambda \frac{q^F}{q^L} \right] + \frac{1 - \lambda}{q^F (\rho + t + \mu \lambda)}
\]

**A.3.2 All consumers pay \( q^L \)**
If all consumers are willing to pay $q^L$ for a new innovation, Northern firms earn the profits

$$\pi^N = \frac{E}{q^L} (q^L - w).$$

Again, combining with the valuation conditions yields

$$\frac{1}{w} = \frac{1}{q^L} + \frac{I}{M} \frac{1-\lambda}{P} \frac{q^F - 1}{q^F}.$$

### A.3.3 High-end pricing

If the assumption $w \frac{q^L - q^F}{q^L q^F} > 1 - \lambda$ does not hold, then firms engage in high-end pricing. This gives them the profit equation

$$\pi^N = \lambda \frac{E}{q^L} (q^L - w)$$

and yields the relative wage

$$\frac{1}{w} = \frac{1}{q^L} + \frac{I}{M} \frac{1-\lambda}{\lambda} \frac{q^F - 1}{q^F}.$$

### Appendix A.4 Exogenous Innovation

Fully differentiating (85) – (87) yields

\[
\begin{bmatrix}
1 - \frac{w}{q} s\tau - \frac{E}{q} - I(\rho + \mu + t) & -wI \\
-\frac{t - 1}{1 + \mu} q & 0 & -I - \frac{\mu}{(t + \mu)^2} q \\
-\frac{1}{q} s\tau + t(1 - \tau) & 0 & \frac{E}{q} \frac{\mu(1 - \tau) - s\tau}{(1 - \tau)(t + \mu)^2}
\end{bmatrix}
\begin{bmatrix}
dE \\
dw \\
d\mu
\end{bmatrix}
= \begin{bmatrix}
-\frac{wI}{t} \frac{E}{(t + \mu)^2} q \\
\frac{E}{t} \frac{\mu(1 - \tau) + s\tau}{q (1 - \tau)(t + \mu)^2}
\end{bmatrix}
\begin{bmatrix}
d\mu
\end{bmatrix}
\]

Applying Cramer’s Rule to the above gives the expression for $dt/d\mu$ found in (88).