Skilled-Labor Intensity Differences across Firms, Product Quality, and Wage Inequality

Unjung Whang
University of Colorado at Boulder

November 2013

Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

© November 2013  Unjung Whang
Abstract

This paper proposes a theory to explain the relative wage-rate increase for skilled labor that results from trade liberalization that relies on within-sector reallocations of production resources (skilled and unskilled labor) across firms. Motivated by some stylized facts, in a model with firm heterogeneity, including firms that differ in their skill intensity even within a narrowly defined industry, firms with relatively high skill intensity that are more likely to be exporters, and a positive association between a firm’s skill intensity and its product quality, I develop a general equilibrium model where firms with a higher skill intensity endogenously choose a higher-quality product, and tend to be more profitable. In this framework, a reduction in trade costs allows members of the workforce to reallocate to more efficient firms that produce higher-quality products, using their skilled labor more intensively, resulting in a rising skill premium. The main sources of the increasing wage inequality that followed trade openness are a positive link between a firm’s skill intensity, its product quality, and quality competition.

Keywords: skill intensity differences, trade liberalization, heterogeneous quality, quality competition, wage inequality

JEL Classification: F12, F16, J31
1. Introduction

A large number of literatures have shown that wage inequality both within and between skill groups grows as a result of trade liberalization (see, e.g., Feenstra and Hanson 1995; Zhu and Trefler 2005; Yeaple 2005; Burstein and Vogel 2010; Helpman et al. (2010); Davis and Harrigan 2011).\(^1\) Recent theoretical works on trade openness and the exporter wage premium (that is, the wage differential between exporters and non-exporters) in the field of international trade have focused on a model of firm heterogeneity with labor market friction, while labor is assumed to be homogeneous (see, e.g., Egger and Kreickemeier 2009; Helpman and Itskhoki 2009; Helpman et al. 2010; Davis and Harrigan 2011). In spite of homogeneous workers, labor market imperfections allow ex-ante identical workers to be paid different wages across firms. As a result, within-group wage inequality occurs when trade liberalization, under the assumption of firm heterogeneity, affect firms differently. In this class of models, the exporter wage premium is mainly driven by the within-group wage differential between exporters and non-exporters.

Recent studies exploit the matched employer-employee data to show that the skill intensity varies across firms even within a narrowly defined industry. It also shows that firms with a higher level of skill intensity (i.e., a higher ratio of skilled to unskilled labor that produces a unit of output) are more likely to export, so that the exporter wage premium occurs simply due to their employing a higher proportion of skilled labor (see, e.g., Schank et al. 2007; Irarrazabal et al. 2009).\(^2\) In this context, the different composition of production resources across firms must be taken into account in order to accurately analyze the link between wage inequality and trade integration.

In addition, recent empirical research has shown that a product’s quality increases with a firm’s

---

\(^1\)Feenstra and Hanson (1995) and Zhu and Trefler (2005) propose multinational outsourcing as a possible reason for the rising wage inequality, while Yeaple (2005) and Burstein and Vogel (2010) emphasize skills-biased technical change in a model with firm heterogeneity to explain a positive link between trade openness and between-group wage inequality. Davis and Harrigan (2011) and Helpman et al. (2010) explain the within-group wage inequality by introducing a search and matching friction in a labor market with ex-ante identical workers.

\(^2\)See also Bernard et al. (2007), Crozet and Trionfetti (2011), and Harrigan and Reshef (2011) for evidence of firm heterogeneity with respect to factor intensity. Irarrazabal et al. (2009) show that the exporter wage premium is mainly caused by differences in workforce composition across firms. Similarly, Schank et al. (2007) use linked employer-employed data from Germany to confirm that the wage differential between exporters and non-exporters becomes small enough when employees’ observable and unobservable characteristics are controlled for.
skilled-labor intensity (see, e.g., Kyoji and Keiko 2010; Kugler and Verhoogen 2012). That is, firms that use their skilled workers relatively intensively produce higher-quality goods. These well-established, stylized facts on firm heterogeneity seem to imply that firms with a higher skill intensity specialize in a higher-quality product and tend to be more competitive/profitable. Consequently, they are more likely to become exporters in the presence of such trade costs as fixed export costs and variable trade costs. Since firms represented in a model with firm heterogeneity are unevenly affected by globalization, as trade costs fall production resources in a perfectly competitive labor market can be reallocated towards more efficient firms. In this context, the effect of trade liberalization on wage inequality cannot be fully described without considering both across-firm differences in skill intensity and firms’ competitiveness in product quality.

In this paper, I develop a model of firm heterogeneity to address how trade liberalization affects the within-sector relative wage of skilled workers (i.e., the skill premium), under the assumption of a perfectly competitive labor market. Firms differ in two dimensions: the skill intensity of production technology (that is, the differences in the ratio of skilled to unskilled labor used to produce one unit of output), and product quality. Instead of intrinsic productivity differences across firms, Melitz (2003), I assume that firms only differ in their skill intensity. Additionally, I assume that skilled workers are more productive in performing tasks that improve product quality. In this framework, each firm endogenously chooses its optimal product quality, given its own production technology. To be more specific, firms with a higher skill intensity choose to provide a better-quality product and become more competitive, in what is known as “quality competition”.

---

3See Kyoji and Keiko (2010) for the empirical evidence of a significant and positive association between the unit value of a product (as measured by quality) and a firms’ white-collar worker intensity using the factory level of the Japanese manufacturing sector.

4See Schott (2004), Bastos and Silva (2010), Baldwin and Harrigan (2011), Harrigan et al. (2011), and Johnson 2012 for a positive association between a firm’s product quality and its competitiveness.

5Irarrazabal et al. (2009) investigate the importance of firm’s workforce composition relative to intrinsic firm productivity as sources of the exporter’s productivity premium, and show that over 67% of the exporter productivity premium reflects differences in workforce composition rather than intrinsic firm productivity. In fact, the main results in my model are not affected by including Hicks-neutral productivity differences across firms in the sense that a more productive firm chooses a technology that uses higher skill intensive labor to produce a higher-quality product. For the sake of tractability, I focus on a model without heterogeneous productivity across firms.

6As noted in Baldwin and Ito (2011), I use the term “quality competition” when firms’ competitiveness depends on the quality-adjusted price (i.e., price/quality), where firms with a lower quality-adjusted price are more efficient.
The key driving force behind the increase in wage inequality that results from trade cost reductions is the reallocation of production resources towards more profitable firms, which use skilled labor relatively intensively in order to produce higher-quality products. A decrease in trade costs allows existing exporters to increase their production of high quality goods so as to make additional sales in foreign markets. This increased production is accompanied by an increase in the relative demand for skilled labor because these firms are relatively more skill intensive than average. On the other hand, the relatively unskilled-labor-intensive non-exporters that produce lower-quality products are forced to exit the market due to the increase in import competition. As a result, the inter-firm reallocation of the workforce towards more skilled-labor-intensive manufacturing leads to the increase in the relative demand for skilled workers. The two mechanisms, described above, imply that trade liberalization increases the within-sector wage inequality between the two skill groups. In contrast to the first two effects, which raise the skill premium, the relative skill intensity of firms that become new exporters, as a result of the reduced trade barriers, is ambiguous. For example, when the relative skill intensity of these new exporters is higher than average, the skill premium increases; otherwise, the skill premium will decline. But the last effect is not big enough to overturn the first two effects, so that the skill premium unambiguously increases as trade costs (both fixed and variable) fall.

The main results of this study are summarized as follows: The reduction in trade costs (fixed and/or variable trade costs) increases the within-sector wage inequality between skilled and unskilled workers. This pattern occurs regardless of the resource endowment differences between the trading countries. However, the effect of the lower trade costs on the skill premium is larger when a country trades with relatively skill-abundant countries than when it trades with relatively skill-scarce countries. Furthermore, when two asymmetric countries trade with each other, the skill premium in a relatively skill-scarce country is much higher and increases faster as trade costs fall than in that of a relatively skill-abundant counterpart. Lastly, when the barriers to trade are high enough, two-way trade between similar economies, in terms of factor endowments, is more prevalent than trade between asymmetric countries. This is consistent with the well-known stylized fact.
that the large volume of world trade is associated with trade among similar economies.

This paper is related to the literature that analyzes the effect of trade openness on within-sector wage inequality between skill groups (Burstein and Vogel 2010; Harrigan and Reshef 2011; Vannoorenberghe 2011; Sampson 2012, among others). The models presented in these studies consider across-firm differences in productivity with heterogeneous workers or with skill-biased heterogeneous firms, but they exclude the quality dimension. The standard model of firm heterogeneity without considering quality differentiation, including that of Melitz (2003), however, conflicts with recent empirical findings from both product- and firm-level data (see, for example, Verhoogen 2008; Bastos and Silva 2010; Baldwin and Harrigan 2011; Kugler and Verhoogen 2012). In these studies, the authors found a positive correlation between the quality of a firm’s product (that is, the unit value of a product) and its profitability. Since the differences in product quality in a model with firm heterogeneity is an important source of firms’ competitiveness, at least in some sectors, quality heterogeneity, in a market with vertically differentiated products, must be taken into consideration when establishing a relationship between the skill premium and trade liberalization. In this context, this paper also contributes to the literature on firms’ competitiveness and product quality where quality heterogeneity plays a critical role in shaping firms’ efficiency (see, for example, Schott 2004; Bastos and Silva 2010; Baldwin and Harrigan 2011; Harrigan et al. 2011; Johnson 2012).

The literature closest to my model is that of Harrigan and Reshef (2011). These studies present models where production technology, in terms of skill intensity, varies across firms. Harrigan and Reshef employ a skill-biased technology where they assume that intrinsic firm productivity is positively correlated with skill intensity. In their model, a reduction in trade costs ensures that only more productive firms with higher skill intensity tend to be exporters. Thus, the relative demand for skilled labor increases, resulting in the increase in wage inequality. This paper is similar to Harrigan and Reshef’s, with the exception that, in my model, the relatively high-skill-intensive firms become more efficient by endogenously choosing higher-quality products. Therefore, the

---

7 Harrigan and Reshef (2011) briefly gives an alternative interpretation of the model, adapting the “quality competition” framework. But they assume that product quality is exogenously given in the form of demand, and that it is
increase in wage inequality following trade liberalization can be explained by a positive link between a firm’s skilled-labor intensity and its product quality, and “quality competition”, where, assuming that skilled workers have comparative advantage in producing quality goods, a product’s quality levels are endogenously determined by individual firms.

The main purpose of this paper is to emphasize the importance of “quality competition”, at least in some (vertically) differentiated products markets, in accounting for the positive relationship between the within-sector skill premium and trade liberalization. Verhoogen (2008) explains the increase in within-plant wage inequality in Mexico as a result of the peso’s devaluation (resulting in expanding trade) by introducing a quality-upgrading mechanism in a partial equilibrium framework. To be more specific, Verhoogen argues that the more productive exporting firms tend to upgrade their product quality in the face of increased trade with rich northern countries. He also notes that upgrading product quality requires each worker be better trained and more productive, and thus higher-quality. Hence, this quality upgrading mechanism leads to an increase in the within-plant wage inequality, and thereby a rise in the within-sector skill premium. In contrast, in my model, the within-firm skill intensity is not affected by exporting activity. Rather, the within-sector skill premium increases because of the reallocation of production resources towards the higher-skill-intensive firms, and not because of within-plant skill upgrading. In fact, recent empirical findings support this argument: there is no effect of trade liberalization on firm-level skill upgrading (see, for example, Bernard and Jensen 1994; Trefler 2004; Vannoorenberghe 2011; Harrigan and Reshef 2011).

The remainder of the paper is organized as follows: Section 2 consists of two parts: First, a model of endogenous quality is described. This model’s endogenous quality exists in conjunction with across-firm differences in skill intensity in a closed economy, where a higher skill intensive firm chooses a higher product quality, and becomes more competitive, and only firms that produce high enough quality goods find it worthwhile to remain in the market. Second, I extend the model to an open economy, in which I analyze the effect of trade integration on wage inequality, where positively correlated with skill intensity, whereas in my model, the levels of product quality are determined endogenously by individual firms given their own skill intensity.
trade liberalization can be represented by a reduction in trade costs (fixed export and/or variable trade costs). In Section 2, a numerical exercise is also conducted for each autarky and open economy. Finally, Section 3 presents some concluding remarks.

2. The Theoretical Model

In this section, I develop a general equilibrium model of international trade, which builds on Melitz (2003), Bernard et al. (2007) and Harrigan and Reshef (2011), where a positive association between a firm’s skill intensity and its product quality, and quality competition play an important role in explaining the increase in the within-sector wage inequality between two skill groups as a result of an increasing openness to trade. In the following discussion, I focus on a one-sector version of the model in a vertically differentiated products market.

2.1. The Basic Set-up

2.1.1. Consumption

From the demand side, a representative consumer in a country cares about a product’s quality as well as its price. The consumer’s utility function takes the Dixit-Stiglitz CES preference over the consumption of the differentiated varieties. The utility of each consumer is given by

$$U = \left( \int_{\omega \in \Omega} [q(\omega)x(\omega)]^{\sigma-1} d\omega \right)^{\frac{1}{\sigma - 1}},$$

(1)

where $\omega$ denotes an individual variety in the potential set $\Omega$ of varieties that are available in the economy. $\sigma > 1$ is the elasticity of substitution between varieties. $q(\omega)$ and $x(\omega)$ denote quality and quantity of variety $\omega$ respectively.

Preferences are assumed to be identical across countries. Given the revenue $E$, a representative consumer chooses the consumption of each variety to maximize his utility. The corresponding

---

8I consider the costly trade model not only between identical countries, but also between different countries in terms of their resources endowment.
demand function for variety $\omega$ is

$$x(p(\omega), q(\omega)) = q(\omega)^{\sigma-1} p(\omega)^{-\sigma} \bar{p}^{\sigma-1} E, \tag{2}$$

$$\bar{x}(\hat{p}(\omega)) = [\hat{p}(\omega)]^{-\sigma} \bar{p}^{\sigma-1} E,$$

where $\bar{x}(\omega) = q(\omega) x(\omega)$ and $\hat{p}(\omega) = p(\omega)/q(\omega)$ are the quantity measured in units of utility and the quality-adjusted price of variety $\omega$ respectively. $\bar{P} = (\int_{\omega \in \Omega} [\hat{p}(\omega)]^{1-\sigma} d\omega)^{1/1-\sigma}$ is the aggregate price index of consumption and $E$ is total expenditure in this economy.

### 2.1.2. Production

Each variety of the differentiated good is produced by a monopolistically competitive firm that uses both skilled and unskilled labor. Unlike the standard model of heterogeneous firms along the lines of Melitz (2003), I assume that there are no intrinsic productivity differences across firms. Instead of having Hicks-neutral productivity differences, firms only differ in their skill-intensive technology. That is, firms discover their own skill intensity after paying a sunk entry cost.\(^9\)

Production involves both fixed and variable costs. A fixed production cost is assumed to be the same across firms in the same country, while the variable costs differ according to the firm-specific skill intensity and product quality. I assume that the function of the variable costs takes the Cobb-Douglas form and the marginal costs consist of two components: simple costs to assemble a physical output, and the additional costs that are related to product quality (quality-related work such as a technology-combined design). The total cost of firms that draw on skill intensity $\theta$ from a common distribution $G(\theta)$, where $\theta \in [0, 1]$, is

$$TC(\theta, q) = f w + s^\theta w^{1-\theta} [1 + \Psi(\theta, q)] x(\theta, q), \tag{3}$$

\(^9\)Within-sector productivity differences across firms can be substantially explained by differences in skill intensity (that is, the different proportions of skilled worker across firms). In a recent paper that uses firm-level data that matches employer and employee for Norwegian Manufacturing sector, the authors confirm the fact that over 67% of the exporter productivity premium reflects differences in skill intensiveness rather than in intrinsic firm efficiency (Irarrazabal et al. 2009). See also Crozet and Trionfetti (2011) for empirical evidence that factor intensities differ across firms even within the same industry.
where $f w$ denotes the fixed production cost in terms of units of labor, which is the same across firms in the same economy, but might be different across countries.\footnote{As in Harrigan and Reshef (2011), I assume that $w$ depends on the economy’s overall factor abundance and $w = \left( \frac{H}{H+L} \right) s + \left( \frac{L}{H+L} \right) w$, where $H$ and $L$ are the economy’s inelastic aggregate supplies of skilled and unskilled labor respectively. The cost function, therefore, is homogeneous of degree one in input prices.} $s$ and $w$ denote the wage rate of skilled and unskilled labor respectively. $x(\theta, q)$ denotes the quantity of a variety with quality level $q$ that is produced by a firm with skill level $\theta$.

Regarding a simple assembly task, I assume that skilled workers have no productive advantage, so that the marginal cost related to this work increases with a firm’s skill intensity, assuming the wage ratio of skilled to unskilled worker is greater than one; that is, $s^\theta w^{1-\theta}$ increases with $\theta$. In addition, I assume that producing better-quality products requires higher costs and skilled workers show greater productivity in performing tasks that improve product quality (i.e., $\Psi_q(\theta, q) > 0$ and $\Psi_\theta(\theta, q) < 0$). This is a reasonable assumption in that producing a higher-quality product often requires a more complex technology and such a technology is difficult to follow unless the labor force is skilled enough.\footnote{See Abowd et al. (1996), Kyoji and Keiko (2010) and Kugler and Verhoogen (2012) for a positive relationship between a firm’s skill intensity and its product quality.} As a result, a firm with a higher skill intensity faces a higher marginal cost of simple assembly work, but a lower marginal cost of quality-associated work.

For the sake of tractability, I make the following parametric assumption regarding the functional form of $\Psi(\theta, q)$: the total cost of firms with skill intensity $\theta$ is

$$TC(\theta, q) = f w + s^\theta w^{1-\theta} \left[ 1 + \frac{q^\theta}{\theta^\alpha} \right] x(\theta, q),$$

where the parameter $\phi$ is the elasticity of the marginal cost with respect to quality, which is common across firms as well as countries. The parameter $\alpha$ measures the degree of skilled workers’ efficiency on the quality-related tasks.

A firm’s marginal-cost function, $s^\theta w^{1-\theta} \left[ 1 + \frac{q^\theta}{\theta^\alpha} \right]$, can be justified as follows: If there is no quality dimension in the model ($q = 0$), then the model is of the standard form, where it is reasonable to assume that skilled workers have no productivity advantage in assembling output without adding product quality (such as screwing nuts onto bolts). As long as skilled workers command a
higher wage (for example where \( s > w \)), this translates to a higher marginal cost, so that firms with a higher skill intensity are less efficient. In contrast, skilled workers have a comparative advantage in making quality goods (i.e., \( q > 0 \)); that is, skilled workers are relatively more productive in performing tasks that are related to product quality.\(^{12}\)

Firms in a monopolistic competition set a constant mark-up over their marginal costs. A firm with skill intensity \( \theta \) chooses its optimal product quality endogenously to solve\(^{13}\)

\[
\max_{\{q\}} \quad p(\theta, q)x(\theta, q) - f w - s^\theta w^{1-\theta} \left[ 1 + \frac{q^\phi}{\theta^\alpha} \right] x(\theta, q)
\]

s.t.  \( x(\theta, q) = q^{\sigma - 1} p(\theta, q)^{-\sigma} \tilde{P}^{\sigma - 1} E \)

and \( p(\theta, q) = \left( \frac{\sigma}{\sigma - 1} \right) s^\theta w^{1-\theta} \left[ 1 + \frac{q^\phi}{\theta^\alpha} \right] \).

Taking \( \tilde{P} \) and \( E \) as given, solving the maximization problem above gives the optimal choice of quality for firms with their skill intensity \( \theta \),\(^{14}\)

\[
q^*(\theta) = \left( \frac{1}{\phi - 1} \right) \frac{1}{\tilde{\sigma}} \theta^\frac{\phi}{\tilde{\sigma}},
\]

where \( \phi > 1 \) by the second-order condition for a maximum.\(^{15}\) The positive association between

\(^{12}\)Taking the derivative of the second term of marginal costs, \( (s^\theta w^{1-\theta}) \frac{\partial q^\phi}{\partial \theta} \) with respect to skill intensity \( \theta \) gives marginal cost decreases with skill intensity \( \theta \), assuming that \( \alpha \) is high enough. For more details regarding this assumption on \( \alpha \), see the following section.

\(^{13}\)One can think of this profit maximization as a two-stage process: All firms first choose their product quality \( q \) simultaneously after discovering their own skill intensity, \( \theta \). Second, firms simultaneously choose their prices and output levels given their product qualities. With monopolistic competition and a continuum of varieties, no one firm can influence the aggregate price and quality level so that firms take the aggregate quality-adjusted price index, \( \tilde{P} \), as given. The optimal price and quality chosen by firms with skill intensity \( \theta \) can be solved by working backwards from the second stage. In the second stage, a firm chooses the price and quantity, given its quality \( q \), so that the optimal price charged by firms with skill intensity \( \theta \) and quality \( q \) is a constant mark-up over their marginal cost:

\[
p(\theta, q) = \left( \frac{\sigma}{\sigma - 1} \right) s^\theta w^{1-\theta} \left[ 1 + \frac{q^\phi}{\theta^\alpha} \right].
\]

Knowing this price rule given each product quality, firms in the first stage choose their optimal quality in response to their own skill intensity, which is shown in equation (5).

\(^{14}\)Note that \( \tilde{P} = (\int_{\omega \in \Omega} \tilde{p}(\omega)^{1-\sigma} d\omega)^{1/1-\sigma} \). Although \( \tilde{P} \) is endogenous to the industry, firms take it as an exogenous variable since a firm’s size is negligible relative to the industry, as a whole, under the assumption of monopolistic competition.

\(^{15}\)See Cremer and Thisse (1994), Khandelwal (2010), and Schmitt (2002), where the authors assume the quadratic form of marginal cost \( MC \) with respect to quality, \( MC(\varphi, q) = \varphi + q^2 \) where \( \varphi \) and \( q \) are a firm-specific and a quality-specific component, respectively.
a firm’s skill intensity and its product quality is consistent with recent empirical findings. Indeed, using factory-level Japanese manufacturing data, Kyoji and Keiko (2010) show that the unit value of a product (that is, a product’s quality) is positively correlated with a firm’s white-color intensity.

Substituting the optimal quality schedule, \(q^*(\theta)\), into the marginal-cost function yields

\[
MC(\theta) = \left( \frac{\phi}{\phi - 1} \right) \theta w^{1-\theta}. \tag{7}
\]

Since \(\frac{\theta}{\phi - 1} > 1\), all firms in a country pay more than \(s^\theta w^{1-\theta}\) to produce one unit of output. That is, all firms have an incentive for adding quality, \(q^*(\theta) > 0\) for all \(\theta \in [0, 1]\). Notice that the price of a variety that is produced by firms with \(\theta\) is \(p(\theta) = \left( \frac{\sigma}{\sigma - 1} \right) MC(\theta)\). Since \(\sigma > 1\), firms with a higher skill intensity set a higher price because they manufacture the relatively better-quality product. The bottom line is that firms with a higher skill intensity \(\theta\) produce a higher quality variety and charge a higher price, which is consistent with a number of empirical findings.

2.2. The Closed Economy

In this section, I analyze the autarky equilibrium, where the wage ratio of skilled to unskilled worker (the skill premium) and the skill intensity threshold, below which firms exit the market, are endogenously determined. In the following section, I consider the costly trade equilibrium that exists in an open economy, and then investigate the effect of trade liberalization on wage inequality.

2.2.1. Firm Competition, Entry, Exit, and Free Entry Condition

Using the pricing rule, and the optimal quantity and quality schedule constructed in the previous section, the revenue of firms with skill intensity \(\theta\) is

\[
R(\theta) = \left( \frac{p(\theta)}{q(\theta)} \right)^{1-\sigma} \bar{p}^{\sigma-1} E = \lambda w^{1-\sigma} \left( \frac{s}{w} \right)^{\theta(1-\sigma)} \theta^{-\frac{a(\sigma-1)}{\phi}} \bar{p}^{\sigma-1} E, \tag{8}
\]

where \(\lambda = \left[ \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{\phi}{\phi - 1} \right) \left( \frac{1}{\phi - 1} \right)^{-1/\phi} \right]^{1-\sigma} > 0\) is constant. Note that the firm’s revenue is a function of product quality as well as price because firms compete on the quality-adjusted price,
$p/q$, rather than on price $p$.

The firm-specific skill intensity $\theta$ has two opposite effects on the firm’s revenue: (i) a higher $\theta$ leads to a higher revenue via the last term in equation (8), $\theta^{\frac{\alpha(\sigma-1)}{\sigma}}$, and (ii) the increase in $\theta$ reduces the firm’s revenue, because firms with a higher $\theta$ face a higher cost, which is captured by $(\frac{\sigma}{w})^{\theta(1-\sigma)}$, where we have a negative exponent due to $\sigma > 1$. These opposite effects occur because firms with a higher skill intensity increase their revenue by selling a relatively high-quality product at a higher price. At the same time, these firms pay a higher wage, thereby reducing their revenue.

When $\alpha > \phi ln(\frac{s}{w})$ the firm’s revenue increases with skill intensity $\theta$.\footnote{Taking the derivative of the firm’s revenue function with respect to $\theta$ gives $\frac{\partial R(\theta)}{\partial \theta} > 0$ if $\alpha > \phi ln(\frac{s}{w})\theta$ for $\theta \in [0, 1]$, so when $\alpha > \phi ln(\frac{s}{w})$ the firm’s revenue increases with skill intensity $\theta$ for all $\theta \in [0, 1]$.} Recent empirical studies of international trade support the view that firm’s skill intensity increases revenue through their production of higher-quality products: (i) a more skill intensive firm is more likely to be an exporter (e.g., Schank et al. 2007; Bas 2012), (ii) exporters pay higher wages because the labor force employed by exporting firms are relatively biased toward skilled labor (see, e.g., Irarrazabal et al. 2009), and (iii) a positive association between firm’s skill intensity and product quality, and more successful firms set higher prices, which implies “quality competition” (e.g., Verhoogen 2008; Kyoji and Keiko 2010; Bastos and Silva 2010; Baldwin and Harrigan 2011; Kugler and Verhoogen 2012). Hence, as suggested in a number of empirical findings, I assume that $\alpha > \phi ln(\frac{s}{w})$ holds, so that a firm’s revenue increases with firm-specific skilled-labor intensity $\theta$ and its resulting product quality. That is, firms with a higher $\theta$ produce a better quality variety and become more competitive. This inequality condition plays a key role in determining the rising wage inequality attributed to a reduction in trade costs because this assumption leads to a positive link between a firm’s skill intensity and its revenue, via its product quality.

Let $\theta^*$ denote a zero-profit skill intensity cut-off (i.e., skill intensity cut-off for surviving), so firms drawing a skill intensity below $\theta^*$ exit the market, while firms with a skill intensity above $\theta^*$ engage in production. There is an unbounded potential entrants and, in equilibrium, the expected
value of entry is equal to the sunk entry cost \( f_e \bar{w} \). Thus, the free entry condition is\(^{17}\)

\[
\bar{\pi} = \frac{\delta}{1 - G(\theta^*)} f_e \bar{w},
\]

where \( \bar{\pi} \) is the average firm’s profitability from successful entry. \( 1 - G(\theta^*) \) and \( \delta \) are the *ex ante* probability of successful entry and the exogenous probability of a firm’s failure respectively. The weighted-average skill intensity of successful firms is determined by the *ex post* probability distribution of \( \theta \) and the zero-profit threshold \( \theta^* \), which is as follows:

\[
\tilde{\theta}(\theta^*) = \frac{1}{1 - G(\theta^*)} \frac{1}{\sigma} \int_{\theta^*}^{1} \theta^{\sigma-1} g(\theta) d\theta \Bigg)^{1/(\sigma-1)}.
\]

Using firm’s profit function, \( \pi(\theta) = \frac{R(\theta)}{\sigma} - f\bar{w} \), the average firm profitability can be obtained by the weighted average skill intensity of successful entrants which is denoted by \( \tilde{\theta}(\theta^*) \):

\[
\bar{\pi} = \frac{R(\tilde{\theta}(\theta^*))}{\sigma} - f\bar{w}.
\]

Using the zero-profit condition, \( \frac{R(\theta^*)}{\sigma} = f\bar{w} \), the average firm profitability can be written as \( \bar{\pi} = f\bar{w} \left[ \frac{R(\tilde{\theta}(\theta^*))}{R(\theta^*)} - 1 \right] \). Based on the revenue function given by the equation (8), the average firm profitability is

\[
\bar{\pi} = f\bar{w} \left\{ \left( \frac{S}{\bar{w}} \right)^{\frac{a(\sigma-1)}{\sigma}} \left( \frac{\tilde{\theta}(\theta^*)}{\theta^*} \right) - 1 \right\}. \tag{12}
\]

From equations (9) and (12), we have

\[
\left( \frac{S}{\bar{w}} \right)^{\frac{a(\sigma-1)}{\sigma}} \left( \frac{\tilde{\theta}(\theta^*)}{\theta^*} \right) - 1 = \frac{\delta}{1 - G(\theta^*)} \frac{f_e \bar{w}}{f},
\]

where the weighted-average skill intensity for surviving firms, \( \tilde{\theta}(\theta^*) \), is given by equation (10).

The condition in equation (13) simply implies that the expected value of entry equals the sunk fixed

\(^{17}\)See Melitz (2003), Bernard et al. (2007), and Harrigan and Reshef (2011) for the free entry condition in detail. In fact, the free entry condition implies that the expected value of entry, \( \frac{1 - G(\theta^*)}{\delta} \bar{\pi} \), equals the sunk fixed entry cost, \( f_e \bar{w} \), that is, in effect, equivalent to equation (9).
entry cost. These two equations (10) and (13) show a negative association between the relative wage of skilled workers $s/w$ and the surviving cut-off $\theta^*$ in equilibrium, which is described in more detail later in this section.

2.2.2. Labor Market Equilibrium

I assume that the labor market is perfectly competitive, and is following Harrigan and Reshef (2011). Notice that the firm’s marginal cost in response to its optimal quality, $q^*$, is $MC(\theta, q^*(\theta)) = \left(\frac{\phi}{\phi - 1}\right)s^{\theta}w^{1-\theta}$. The labor demand for each type of worker (skilled and unskilled) to produce one unit of output can be obtained by applying Shepard’s Lemma. Hence, the skilled and unskilled labor demand per unit of output are

$$
\begin{align*}
  d_h(\theta, \frac{s}{w}) &= \left(\frac{\phi}{\phi - 1}\right)\theta \left(\frac{s}{w}\right)^{\theta-1}, \\
  d_l(\theta, \frac{s}{w}) &= \left(\frac{\phi}{\phi - 1}\right)(1-\theta) \left(\frac{s}{w}\right)^{\theta},
\end{align*}
$$

(14)

where $d_h$ and $d_l$ represent the unit labor demand for skilled and unskilled workers respectively. Hereafter, I use subscript $h$ to indicate skilled workers and $l$ to indicate unskilled workers. To produce one unit of good, a more skill-intensive firm (firms with a higher $\theta$) uses a relatively high proportion of skilled labor. This relationship can be easily captured by $\frac{d_h}{d_l}(\theta, \frac{s}{w}) = \left(\frac{\theta}{1-\theta}\right) \left(\frac{w}{s}\right)$, and where the relative share of skilled to unskilled workers needed to manufacture one unit of variety increases with $\theta$. Also note that the ratio of skilled to unskilled workers to produce a unit of output is inversely related to the relative wage of skilled workers ($s/w$).

Based on the quantity demanded, optimal quality, and pricing, the total output of firms with $\theta$ is

$$
x(\theta) = \gamma w^{-\frac{\sigma}{\theta}} \left(\frac{s}{w}\right)^{-\frac{\theta}{\sigma}} \theta^{-\frac{\sigma}{\theta}} \bar{\sigma}^{\sigma - 1} E^{\sigma - 1},
$$

(15)

where $\gamma = \left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} \left(\frac{\phi}{\phi - 1}\right)^{-\sigma} \left(\frac{1}{\phi - 1}\right)^{(\sigma-1)/\phi}$ is a positive constant. Therefore, the total skilled
and unskilled labor demand of firms with $\theta$ are

$$D_h(\theta, \frac{s}{w}) = d_h\left(\theta, \frac{s}{w}\right) x(\theta) = \eta w^{-\sigma}\left(\frac{s}{w}\right)^{\theta (1-\sigma)} - \frac{\alpha (1-\sigma)}{\sigma - 1} \frac{\theta}{\sigma} \bar{p}^{\sigma - 1} E,$$

$$D_l(\theta, \frac{s}{w}) = d_l\left(\theta, \frac{s}{w}\right) x(\theta) = \eta w^{-\sigma}\left(\frac{s}{w}\right)^{\theta (1-\sigma)} - \frac{\alpha (1-\sigma)}{\sigma - 1} \frac{\theta}{\sigma} (1 - \theta) \bar{p}^{\sigma - 1} E,$$

where $\eta = \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \left(\frac{\phi}{\phi - 1}\right)^{(\sigma - 1)/\phi} > 0$ is constant.

Let $M$ denote the equilibrium mass of firms in this economy, and hence $M$ product varieties. The aggregate labor demand in the variable costs can be obtained by integrating the total labor demand of firms with the same skill intensity over $\theta \in [\theta^*, 1]$. This gives the aggregate labor demand for skilled and unskilled labor in the variable costs, which are as follows:

$$TD_h(\theta^*, \frac{s}{w}) = \int_{\theta^*}^{1} D_h(\theta, \frac{s}{w}) M \frac{g(\theta)}{1 - G(\theta^*)} d\theta,$$

$$TD_l(\theta^*, \frac{s}{w}) = \int_{\theta^*}^{1} D_l(\theta, \frac{s}{w}) M \frac{g(\theta)}{1 - G(\theta^*)} d\theta,$$

where $D_h(\theta, \frac{s}{w})$ and $D_l(\theta, \frac{s}{w})$ are given by equation (16). $\frac{g(\theta)}{1 - G(\theta^*)}$ is the ex post probability density function of surviving firms’ skill intensity, so $M \frac{g(\theta)}{1 - G(\theta^*)}$ refers to the mass of successful firms with the same $\theta$.

Next, I consider the aggregate labor demand for skilled and unskilled labor as part of the fixed costs (fixed entry and production costs). Let $M_e$ denote the equilibrium mass of new entrants in each period, thus the mass of successful entrants for production, $[1 - G(\theta^*)] M_e$, must be equal to the mass of firms, $\delta M$, who exit the market following the exogenous bad shock: $[1 - G(\theta^*)] M_e = \delta M$. Therefore, the mass of new entrants can be expressed as the fraction of successful production firms, which is $M_e = \frac{\delta}{[1 - G(\theta^*)]} M$. Note that the aggregate fixed costs consist of both the sunk fixed entry costs and the fixed production costs: $\left(f + \frac{\delta}{1 - G(\theta^*)} f_e\right) M w$. The aggregate demand for skilled
and unskilled labor involved in these fixed costs are\textsuperscript{18}

\[
FD_h = \left( \frac{H}{H+L} \right) M \left( f + \frac{\delta}{1 - G(\theta^*) f_e} \right),
\]

\[
FD_l = \left( \frac{L}{H+L} \right) M \left( f + \frac{\delta}{1 - G(\theta^*) f_e} \right).
\]

where $FD_h$ and $FD_l$ represent the aggregate demand for skilled and unskilled labor, corresponding to the fixed costs, respectively.

These two equations in (18) show the ratio of the total skilled to unskilled labor demand in the fixed costs is equal to the relative skilled labor abundance, that is, $FD_h / FD_l = H / L$. This ratio of labor demand in the fixed costs signifies that the ratio of skilled to unskilled labor demand in the variable costs also equals the relative skilled labor abundance $H / L$. Therefore, equations (16) and (17) give the relative labor market clearing condition which is as follows:

\[
\frac{TD_h(\theta^*, \frac{s}{w})}{TD_l(\theta^*, \frac{s}{w})} = \int_{\theta^*}^{1} D_h(\theta, \frac{s}{w}) g(\theta) d\theta \int_{\theta^*}^{1} D_l(\theta, \frac{s}{w}) g(\theta) d\theta = \left( \frac{w}{s} \right) \int_{\theta^*}^{1} \theta^{(1-\sigma)} \theta^{\frac{\alpha(\sigma-1)}{\sigma}} \theta g(\theta) d\theta = \frac{H}{L}.
\]

2.2.3. Autarky Equilibrium

In this section, I discuss the equilibrium conditions for autarky. In equilibrium, the threshold of skill intensity $\theta^*$ and the skill premium $s/w$ are determined jointly by equations (10), (13), and (19). All other aggregate variables are determined endogenously as a function of the parameters, $\theta^*$, and $s/w$, where the wage for unskilled labor is normalized to one, $w = 1$, as a numeraire. I close this section to derive the aggregate equilibrium variables: the total revenue/expenditure $E$, the mass of successful firms $M$, and the industry price index $\tilde{P}$.

Following Melitz (2003), the aggregate revenue $E$, in equilibrium, must be equal to the total

\textsuperscript{18}As in Harrigan and Reshef (2011), I assume that $w$ depends on the economy’s overall factor abundance, that is, $w = (\frac{H}{H+L}) s + (\frac{L}{H+L}) w$, where $H$ and $L$ are the economy’s inelastic aggregate supplies of skilled and unskilled labor respectively. The aggregate labor demand for each type of worker in the fixed costs, equation (18), can be obtained by taking the derivative of the total fixed costs, $\left( f + \frac{\delta}{1 - G(\theta^*) f_e} \right) Mw$, with respect to each $s$ and $w$. 

16
payment to both skilled and unskilled labor. That is, \( E = wL + sH \). The mass of producing firms is determined by the fact that the total revenue must equal, in equilibrium, the average firm’s revenue multiplied by the mass of firms. Thus the mass of successful firms is: \( M = \frac{E}{\bar{r}} \). Using the free entry condition (equations (9) and (11)), the average revenue can be expressed as the following equation:

\[
R(\tilde{\theta}(\theta^*)) = \bar{r} = \sigma \left[ f + \frac{\delta}{1 - G(\theta^*)}f_e \right] \bar{w}.
\]

Since \( \bar{w} = \left( \frac{H}{H+L} \right) s + \left( \frac{L}{H+L} \right) w \) the mass of successful firms is expressed by

\[
M = \frac{H + L}{\sigma \left[ f + \frac{\delta}{1 - G(\theta^*)}f_e \right]}.
\]  

(20)

Note that the mass of surviving firms is inversely associated with both fixed entry \((f_e)\) and production costs \((f)\). That is, the larger the fixed costs, the fewer the firms producing in equilibrium.

Finally, the industry price index can be derived from integrating each firm’s quality-adjusted price, multiplied by the equilibrium mass of firms over \( \theta \in [\theta^*, 1] \). Hence, the weighted average price index of the firms in equilibrium is

\[
\tilde{P} = \left[ \int_{\theta^*}^{1} \left( \frac{p(\theta)}{q(\theta)} \right) M \frac{g(\theta)}{1 - G(\theta^*)} d\theta \right]^{\frac{1}{1-\sigma}} = M^{\frac{1}{1-\sigma}} \left[ \int_{\theta^*}^{1} \left( \frac{p(\theta)}{q(\theta)} \right) \frac{g(\theta)}{1 - G(\theta^*)} d\theta \right]^{\frac{1}{1-\sigma}}
\]

\[
= M^{\frac{1}{1-\sigma}} \frac{p(\tilde{\theta})}{q(\tilde{\theta})} = \mu M^{\frac{1}{1-\sigma}} \tilde{\theta}^{-\frac{\sigma}{\sigma-1}} \tilde{\theta}^{-\frac{1}{\sigma-1}},
\]  

(21)

where \( \mu = \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\delta}{\delta-1} \right) \left( \frac{1}{\delta-1} \right)^{-1/\phi} \) is a positive constant. \( \tilde{\theta} \) is a function of \( \theta^* \), which is given in equation (10).

**Proposition 2.1.** There exists a unique autarky equilibrium in which the zero-profit skill intensity cut-off, \( \theta^* \), and the relative wage of skilled labor, \( \frac{s}{w} \), are determined.

**Proof.** See Appendix A.1.

\[ \square \]

**2.2.4. Numerical Exercise in Autarky**

Here, I numerically solve the model, and provide a visual interpretation of the autarky equilibria described in the previous section. To do so, I assume a normal distribution for \( \text{ex ante} \) firm skill.
intensity, $G(\theta)$, where the mean is 0.5.\footnote{I set the standard deviation at 0.15 so that skill intensity, $\theta$ is normally distributed over $[0,1]$. As an alternative distribution of skill intensity, I also use a uniform distribution over $[0,1]$ and the results are similar to normal distribution. In the following discussion, I focus on a normal distribution of skill intensity, which is consistent with the empirical evidence.} As presented in Bernard et al. (2007) and Harrigan and Reshef (2011), I use the following parameters: (1) $\sigma = 3.8$ for the elasticity of substitution, (2) $\delta = 0.025$ as a probability of exit due to a bad shock, (3) $f_e = 20$ for a sunk entry cost, and (4) $f = 1$ or 2 for a fixed production cost. For $\phi$ and $\alpha$, I set $\phi = 1.1 > 1$ and $\alpha = 1.5 > 0$ so that $\alpha > \phi \ln \left( \frac{s}{w} \right)$ holds in equilibrium, which ensures that firm’s revenue increases with $\theta$.\footnote{In the later section for an open economy, I use different sets of values for $\phi$ and $\alpha$ (e.g., $\phi = 1.5$ and $\alpha = 2$) to ensure that different values of $\phi$ and $\alpha$ do not affect the overall patterns of the skill premium and the zero-profit skill intensity cut-off as long as the condition, $\alpha > \phi \ln \left( \frac{s}{w} \right)$, holds. Indeed, ceteris paribus, the different sets of $\phi$ and $\alpha$ only give different scales of the skill premium and skill intensity cut-off, but they give the same patterns for these variables of interest.}

Figure 1 plots the firm’s revenue, product quality, and price against firm-specific skill intensity $\theta$, for which the endowment for skilled labor, $H$, and unskilled labor, $L$, are assumed to be 1500 and 2000, respectively. As previously noted, firms with a higher $\theta$ choose a higher product quality and price. They are also the firms that end up with a higher revenue and profit. Since a firm’s efficiency is inversely related to the quality-adjusted price, even firms whose products are sold at a higher price become more competitive, which is in conflict with the price-based competition in

![Figure 1: The Equilibrium Revenue, Quality, and Price in Autarky](image)
models that adhere to the spirit of Melitz (2003).

Table 1: The Equilibrium Zero-profit Skill Intensity Cut-off \((\theta^*)\) and Skill Premium \((s/w)\)

<table>
<thead>
<tr>
<th>((H/L))</th>
<th>(f = 1)</th>
<th>(f = 2)</th>
<th>((H/L))</th>
<th>(f = 1)</th>
<th>(f = 2)</th>
<th>((H/L))</th>
<th>(f = 1)</th>
<th>(f = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill Premium ((s/w))</td>
<td>2.98</td>
<td>3.23</td>
<td>2.17</td>
<td>2.37</td>
<td>1.73</td>
<td>1.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cut-off skill intensity ((\theta^*))</td>
<td>0.42</td>
<td>0.47</td>
<td>0.45</td>
<td>0.50</td>
<td>0.46</td>
<td>0.52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 presents the equilibrium skill premium \(s/w\) and the zero-profit skill intensity cut-off \(\theta^*\), which depend on both the fixed production costs \(f\) and the relative endowment of skilled labor \(H/L\). First, all else being equal, both the skill premium and the skill intensity cut-off are positively associated with the fixed costs of production. That is, the higher the fixed costs of production, the higher are both \(s/w\) and \(\theta^*\) in equilibrium. These relationships are also illustrated graphically in Figure 2, where the relative endowment of skilled labor is fixed as \(H/L = 0.75\). The intuition behind these results are natural: i) The relatively less efficient firms are forced to exit the market. This is because as the fixed production costs rise, so does \(\theta^*\) and; ii) Rising \(\theta^*\) leads to reallocation of production resources towards the relatively more skill intensive, competitive firms, so that the skill premium, \(s/w\), increase in response to the increase in the relative demand for skilled labor. Since the relative endowment of skilled labor, \(H/L\), is fixed, the increase in the skill premium stems from the increase in the zero-profit cut-off.

Second, ceteris paribus, the increase in the relative endowment of skilled labor, \(H/L\), leads to a decline in the skill premium, while it increases the threshold of skill intensity.\(^{21}\) Intuitively, the increase in \(H/L\) reduces the skill premium as a result of the less competitive labor market for skilled workers. The increase in the relative wage of unskilled workers forces less efficient firms (unskilled-labor intensive firms) out of the market, which raises the zero-profit skill intensity, \(\theta^*\).\(^{22}\) Theoretically, the relatively skill-abundant countries enjoy the relatively low wage inequality.

\(^{21}\)See also Figure 9 in Appendix A.1, which illustrates that the intersection of the two curves (these curves are based on the free entry condition and the labor market clearing condition) determines equilibrium \(\theta^*\) and \(s/w\) as shown in Table 1. As illustrated in Figure 9, the increase in the fixed production cost \(f\), all else being equal, shifts the free entry curve to the right, so that both \(s/w\) and \(\theta^*\) rise. On the other hand, all else being equal, the increase in the relative endowment of skilled labor, \(H/L\), shifts the labor market clearing condition to the right, so \(s/w\) falls while \(\theta^*\) rises.

\(^{22}\)Although the increase in \(\theta^*\) raises the skill premium, \(s/w\), this positive effect is relatively small enough to be
between skill groups, whereas firms in these countries face tougher competition than ones in the relatively skill-scare countries. The evolution of both the skill premium and the skill intensity cut-off in response to the relative endowment of skilled labor is also illustrated graphically in Figure 3, for which the fixed costs of production are $f = 2$.

dominated by the negative effect of $H/L$ on the skill premium, so that the increase in the relative endowment of skilled labor leads to a decline in the skill premium.

---

**Figure 2:** Zero-profit Skill Intensity and Skill Premium in Response to Different Fixed Costs

**Figure 3:** Zero-profit Skill Intensity and Skill Premium in Response to Different H/Ls
2.3. *An Open Economy between Symmetric Countries*

In this section, I consider an open economy model by introducing the fixed and variable costs of trade as in Bernard et al. (2007), Harrigan and Reshef (2011), and Melitz (2003). As in Melitz (2003), I assume that both countries are symmetric by assigning the same factor endowments $H$ and $L$ between countries. The symmetry assumption ensures that all countries have the same pattern of trade, the same return for both skilled and unskilled worker, and hence the same aggregate variables (the total revenue, the equilibrium mass of firms, and the industry price index). In the following discussion, without any loss of generality, I only consider that the world economy consists of two identical countries. In a later section, I extend the model to consider a costly trade between countries that differ in their relative factor endowment, $H/L$, by relaxing this symmetry assumption.

A firm must bear a fixed export cost $f_x\bar{w}$, measured by units of both skilled and unskilled labor, to sell its variety abroad. In addition, the firm faces iceberg-type variable trade costs, that is, $\tau > 1$ units of variety must be shipped for one unit to arrive in a foreign market. Given these types of trade costs, the only firms that are profitable enough can choose to export when they are in equilibrium. I analyze how trade liberalization (a reduction in trade costs) affects the skill intensity cut-off for domestic production, exporting, and the skill premium.

2.3.1. *Firm Competition*

With monopolistic competition, the export price of firms with $\theta$ is a constant mark-up over the marginal costs, which now includes the iceberg-type variable trade costs $\tau$:

$$p_x(\theta) = \tau p_d(\theta) = \tau \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{\phi}{\phi - 1}\right) s^\theta w^{1-\theta}, \quad (22)$$

where $p_x$ and $p_d$ denote the export cost of insurance and freight (c.i.f.) and the domestic price, respectively. Hereafter, I use subscript $d$ to indicate the domestic-related variable and $x$ to indicate the export-related variable. Since the aggregate variables (for example, the total expenditure $E$
and the aggregate price index \( \tilde{P} \) between two identical countries are the same, the total revenue of firms with \( \theta \) is

\[
R(\theta) = \begin{cases} 
R_d(\theta) & \text{if it serves only the domestic} \\
(1 + \tau^{1-\sigma}) R_d(\theta) & \text{if it exports,}
\end{cases}
\]  

(23)

where the domestic revenue \( R_d(\theta) \) is given in equation (8). Note that the export revenue, denoted by \( R_x(\theta) \), is:

\[
R_x(\theta) = \tau - \sigma R_d(\theta).
\]

(23)

The presence of fixed costs for both domestic production and exporting implies that the profit of firms with \( \theta \) from domestic sales, \( \pi_d(\theta) \), and foreign sales, \( \pi_x(\theta) \), are

\[
\pi_d(\theta) = \frac{R_d(\theta)}{\sigma} - f_d \bar{w},
\]

\[
\pi_x(\theta) = \frac{R_x(\theta)}{\sigma} - f_x \bar{w},
\]

(24)

where \( f_d \) and \( f_x \) denote the fixed costs for domestic production and exporting, respectively.

There are now two skill intensity cut-offs: i) the skill intensity cut-off for domestic sales (that is, the marginal skill intensity of surviving firms), \( \theta^*_d \), below which firms are forced to exit the market, and ii) the export skill intensity cut-off, \( \theta^*_x \), above which firms are profitable enough to send their products to foreign markets. These two skill intensity cut-offs and equation (24) give the zero-profit conditions for both domestic production and exporting: \( R_x(\theta^*_x) = \sigma f_x \bar{w} \) and \( R_d(\theta^*_d) = \sigma f_d \bar{w} \).

Using two zero-profit conditions and equation (8), \( R_x(\theta) = \tau^{1-\sigma} R_d(\theta) \) implies

\[
\left( \frac{s}{w} \right)^{-\theta^*_x} (\theta^*_x)^{\frac{\alpha}{\sigma}} = \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}} \left( \frac{s}{w} \right)^{-\theta^*_d} (\theta^*_d)^{\frac{\alpha}{\sigma}},
\]

\[
\left( \frac{s}{w} \right)^{-,(\theta^*_x - \theta^*_d)} \left( \frac{\theta^*_x}{\theta^*_d} \right)^{\frac{\alpha}{\sigma}} = \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}}.
\]

(25)

Equation (25) shows a relationship between two thresholds of skill intensity in equilibrium. Note that \( \left( \frac{s}{w} \right)^{-\theta} (\theta)^{\frac{\alpha}{\sigma}} \) increases with \( \theta \).\(^{23}\) For values of \( \tau \left( \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}} > 1 \), therefore, the skill intensity

\(^{23}\)Under the assumption of \( \alpha > \phi ln(\frac{s}{w}) \), which ensures that firms with a higher skill intensity produce a higher-
cut-off for exporting is greater than the zero-profit skill intensity cut-off, that is $\theta^*_x > \theta^*_d$. This implies that some domestic firms are not efficient enough to serve the export market, so firms below $\theta^*_x$ but above $\theta^*_d$ will produce only for the domestic market. As suggested in a number of empirical findings, which show that firms with the relatively high skill intensity are more likely to be exporters, I assume that the condition, $\tau\left(\frac{f}{f_x}\right)^{1/\phi} > 1$, holds, so I focus on the case that $\theta^*_x > \theta^*_d$.

2.3.2. Firm Entry, Exit, and the Free Entry Condition

The free entry condition implies that the expected value of entry is equal to the sunk entry cost. The expected value of entry consists of two components: the expected profit from domestic and foreign sales. Thus, the free entry condition is

$$
\frac{1-G(\theta^*_d)}{\delta} \pi_d + \frac{1-G(\theta^*_x)}{\delta} \pi_x = f e \bar{w},
$$

(26)

where the average profitability is equal to the profit of a firm with the weighted average skill intensity (that is, $\pi_d = \pi_d(\theta^*_d)$ and $\pi_x = \pi_d(\theta^*_x)$). The weighted-average skill intensity in each market is

$$
\hat{\theta}_d(\theta^*_d) = \left[ \frac{1}{1-G(\theta^*_d)} \int_{\theta_d}^{1} \theta^{\sigma-1} g(\theta) d\theta \right]^{1/\sigma-1},
$$

$$
\hat{\theta}_x(\theta^*_x) = \left[ \frac{1}{1-G(\theta^*_x)} \int_{\theta_x}^{1} \theta^{\sigma-1} g(\theta) d\theta \right]^{1/\sigma-1}.
$$

(27)

Using the fact that $\pi_d = \frac{R_d(\theta_d)}{\sigma} - f e \bar{w}$, $\pi_x = \frac{R_x(\theta_x)}{\sigma} - f_x \bar{w}$, $R_d(\theta^*_d) = \sigma f e \bar{w}$, $R_x(\theta^*_x) = \sigma f x \bar{w}$, and equation (8), we can rewrite the free entry condition (equation (26)) as a function of the skill quality product and become more profitable, it is easy to show that $(\frac{f}{f_x})^{-\theta} (\theta)^{\alpha/\phi}$ increases with $\theta$.

24The expected value of entry for the domestic market (exporting) can be obtained by multiplying the ex ante probability of successful entry (exporting) by the expected domestic (exporting) profitability, and discounted by the exogenous probability of failure; that is, $\frac{1-G(\theta^*_d)}{\delta} \pi_d$ and $\frac{1-G(\theta^*_x)}{\delta} \pi_x$. 

23
premium, the two skill intensity cut-offs, and the two weighted-average skill intensities:

\[
(1 - G(\theta_d^*)) f \left[ \left( \frac{S}{W} \right)^{(\theta_d - \theta_d^*)} \left( \frac{\theta_d}{\theta_d^*} \right) \frac{a(\sigma - 1)}{\phi} - 1 \right] + \\
(1 - G(\theta_x^*)) f_x \left[ \left( \frac{S}{W} \right)^{(\theta_x - \theta_x^*)} \left( \frac{\theta_x}{\theta_x^*} \right) \frac{a(\sigma - 1)}{\phi} - 1 \right] = \delta f_e, \tag{28}
\]

where the weighted average skill intensity for domestic and exporting firms, \( \hat{\theta}_d \) and \( \hat{\theta}_x \), is given in equation (27).

### 2.3.3. Labor Market Equilibrium

The labor market equilibrium in an open economy will be similar to one in the autarky case. From the quantity demanded, the price level for each domestic and foreign market, and the optimal quality schedule, I give the total output of a firm with \( \theta \) for domestic and foreign sales as follows:

\[
x_d(\theta) = \gamma W^{-\sigma} \left( \frac{S}{W} \right)^{-\theta\sigma} \theta^{\theta(\sigma - 1)} \tilde{P}^{\sigma - 1} E, \tag{29}
\]

\[
x_x(\theta) = \tau^{-\sigma} x_d(\theta), \tag{30}
\]

where \( \gamma = \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \left( \frac{\phi}{\phi - 1} \right)^{-\sigma} \left( \frac{1}{\phi - 1} \right)^{(\sigma - 1)/\phi} \) is a positive constant. Using the unit labor demand for skilled and unskilled labor: \( d_h(\theta, \frac{S}{W}) = \left( \frac{\phi}{\phi - 1} \right) \theta^{\frac{S}{W} - 1} \) and \( d_l(\theta, \frac{S}{W}) = \left( \frac{\phi}{\phi - 1} \right) (1 - \theta) \left( \frac{S}{W} \right)^{\theta} \).

I obtain the total skilled and unskilled labor demand of firms with \( \theta \) that only serves the domestic market:

\[
D_{d,h}(\theta, \frac{S}{W}) = \eta W^{-\sigma} \left( \frac{S}{W} \right)^{\theta(1 - \sigma) - 1} \theta^{\theta(\sigma - 1)} \tilde{P}^{\sigma - 1} E, \tag{31}
\]

\[
D_{d,l}(\theta, \frac{S}{W}) = \eta W^{-\sigma} \left( \frac{S}{W} \right)^{\theta(1 - \sigma)} \theta^{\theta(\sigma - 1)} (1 - \theta) \tilde{P}^{\sigma - 1} E,
\]

where \( \eta = \left( \frac{\sigma}{\sigma - 1} \right)^{-\sigma} \left( \frac{\phi}{\phi - 1} \right)^{1 - \sigma} \left( \frac{1}{\phi - 1} \right)^{(\sigma - 1)/\phi} > 0 \) is constant. Following the same line of reasoning as for labor demand for domestic production, the total skilled and unskilled labor demand per
firm with $\theta$ for export markets is

\[
D_{x,h}(\theta, \frac{s}{w}) = \tau^{-\sigma} D_{d,h}(\theta, \frac{s}{w}), \\
D_{x,l}(\theta, \frac{s}{w}) = \tau^{-\sigma} D_{d,l}(\theta, \frac{s}{w}).
\]  

(32)

Let $M$ denote the equilibrium mass of incumbent firms, as in the previous section, and hence the mass of exporting firms, which is denoted by $M_x$, is equal to $[1 - G(\theta^*_x)]/[1 - G(\theta^*_d)]M$.\textsuperscript{25} The aggregate labor demand in the variable costs can be obtained by integrating over the total labor demand of the mass of firms with the same $\theta$. This gives the aggregate labor demand for skilled and unskilled labor corresponding to variable costs, which are

\[
TD_h(\theta^*_d, \theta^*_x, \frac{s}{w}) = \int_{\theta^*_d}^{1} D_{d,h}(\theta, \frac{s}{w})M \frac{g(\theta)}{1 - G(\theta^*_d)} d\theta + \tau^{-\sigma} \int_{\theta^*_x}^{1} D_{d,l}(\theta, \frac{s}{w})M \frac{g(\theta)}{1 - G(\theta^*_d)} d\theta, \\
TD_l(\theta^*_d, \theta^*_x, \frac{s}{w}) = \int_{\theta^*_d}^{1} D_{d,l}(\theta, \frac{s}{w})M \frac{g(\theta)}{1 - G(\theta^*_d)} d\theta + \tau^{-\sigma} \int_{\theta^*_x}^{1} D_{d,l}(\theta, \frac{s}{w})M \frac{g(\theta)}{1 - G(\theta^*_d)} d\theta.
\]  

(33)

As shown in the autarky case, the ratio of total skilled to unskilled labor demand in the fixed costs equals the ratio of the skilled to unskilled labor endowment, $FD_h/FD_l = H/L$, where $FD_h$ and $FD_l$ represents the aggregate skilled and unskilled labor demand in fixed costs which are as follows:

\[
FD_h = \left( \frac{H}{H+L} \right) M \left( f + \frac{1 - G(\theta^*_x)}{1 - G(\theta^*_d)} f_x + \frac{\delta}{1 - G(\theta^*_d)} f_e \right), \\
FD_l = \left( \frac{L}{H+L} \right) M \left( f + \frac{1 - G(\theta^*_x)}{1 - G(\theta^*_d)} f_x + \frac{\delta}{1 - G(\theta^*_d)} f_e \right).
\]  

(34)

Since $FD_h/FD_l = H/L$, the ratio of aggregate skilled to unskilled labor demand in the variable costs also equals the relative skilled-labor abundance, $H/L$. Equation (31), (33), and $M_x = [1 - G(\theta^*_x)]/[1 - G(\theta^*_d)]M$ gives the relative labor-market clearing condition,

\textsuperscript{25}Given that the \textit{ex ante} probability of the producing firms is $1 - G(\theta^*_d)$, $[1 - G(\theta^*_x)]/[1 - G(\theta^*_d)]$ indicates the \textit{ex ante} probability of being an exporter, conditional on successful entry.
To close this section, I determine the condition for costly trade equilibrium. Two equilibrium skill intensity cut-offs, $\theta^*_d$ and $\theta^*_x$, and the skill premium $s/w$ are derived jointly from equations (25), (27), (28), and (35). All other aggregate variables in equilibrium can be expressed by $s/w$ and $\theta^*_d$, and $\theta^*_x$ by choosing a wage of the unskilled labor equals to one, $w = 1$, as a numeraire. In what follows, I determine the aggregate equilibrium variables: the total revenue $E$, the mass of the incumbent firms $M$, and the industry price index $\tilde{P}$.

As shown in autarky, the aggregate revenue $E$ in equilibrium must equal the total payment for the production of resources (skilled and unskilled labor), that is, $E = wL + sH$. Since the total revenue in this economy must be equal to the revenue of the average firm multiplied by the mass of successful firms, that is, $E = M\bar{r}$, the mass of the producing incumbent firms is

$$M = \frac{E}{\bar{r}} = \frac{H + L}{\sigma[f + \frac{1 - G(\theta^*_d)}{1 - G(\theta^*_d)}f_x + \frac{\delta}{1 - G(\theta^*_x)}f_e]}.$$ (36)

To derive the average revenue $\bar{r}$, I use the free entry condition, that is, the expected value of free entry equals the sunk fixed entry costs: $\frac{1 - G(\theta^*_d)}{\sigma} \left[ \frac{\tau}{\sigma} - \bar{w} \left[ f + \frac{1 - G(\theta^*_d)}{1 - G(\theta^*_d)}f_x \right] \right] = f_e\bar{w}$. Here I also use the equation, $\bar{w} = \left( \frac{H}{H + L} \right) s + \left( \frac{L}{H + L} \right) w$ to obtain equation (36).

Lastly, the aggregate price index can be expressed as the weighted average price index of each
domestic and exporting firm, which is as follows:

\[
\check{P} = \left[ M \int_{\theta_d^*}^{1} \left( \frac{p(\theta)}{q(\theta)} \right)^{1-\sigma} \frac{g(\theta)}{1-G(\theta_d^*)} d\theta + M_x \tau^{1-\sigma} \int_{\theta_x^*}^{1} \left( \frac{p(\theta)}{q(\theta)} \right)^{1-\sigma} \frac{g(\theta)}{1-G(\theta_x^*)} d\theta \right]^{\frac{1}{1-\sigma}} \\
= \left[ M \left( \frac{p(\tilde{\theta}_d)}{q(\tilde{\theta}_d)} \right)^{1-\sigma} + M_x \tau^{1-\sigma} \left( \frac{p(\tilde{\theta}_x)}{q(\tilde{\theta}_x)} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
= \left[ M \lambda w^{1-\sigma} \left( \frac{s}{w} \right) \tilde{\theta}_d(1-\sigma) \tilde{\theta}_d^{\alpha(\sigma-1)} \tilde{\theta}_d^{\phi-1} + \frac{1-G(\theta_x^*)}{1-G(\theta_d^*)} M \tau^{1-\sigma} \lambda w^{1-\sigma} \left( \frac{s}{w} \right) \tilde{\theta}_x(1-\sigma) \tilde{\theta}_x^{\alpha(\sigma-1)} \tilde{\theta}_x^{\phi-1} \right]^{\frac{1}{1-\sigma}}
\]

(37)

where \( \lambda = \left[ \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\phi}{\phi-1} \right)^{-1/\phi} \right]^{1-\sigma} \) is a positive constant. The weighted average skill intensity for each market, \( \tilde{\theta}_d \) and \( \tilde{\theta}_x \) are given in equation (27).

**Proposition 2.2.** There exists a unique costly trade equilibrium in which the zero-profit skill intensity cut-off, \( \theta_d^* \), the skill intensity cut-off for exporting, \( \theta_x^* \), and the relative wage of skilled labor, \( \frac{s}{w} \), are determined.

**Proof.** See Appendix A.2. ■

**Proposition 2.3.** As trade costs, \( \tau \) and/or \( f_x \), fall, the skill premium, \( \frac{s}{w} \), increases.

**Proof.** See Appendix A.3. ■

Whether the skill intensity cut-off for domestic production and exporting increases or decreases in response to a reduction in trade costs is illustrated in Figure 4. As the trade costs fall, the zero-profit skill intensity cut-off, \( \theta_d^* \), increases (the least efficient firms are forced to exit in the face of increased import competition), while the skill intensity cut-off for exporting, \( \theta_x^* \), decreases (more firms tend to be exporters as a result of the reduced trade costs). Furthermore, these two skill intensity cut-offs converge as the variable cost of trade approaches one \( \tau = 1 \). That is, \( \theta_d^* = \theta_x^* \) once any barriers to trade disappear (i.e., \( \theta_d^* = \theta_x^* \) when \( \tau = 1 \) and \( f = f_x \)).\(^{27}\) As also shown in Figure 4, the zero-profit skill intensity cut-off for autarky, denoted by \( \theta_A^* \), is lower than the

\(^{27}\)When \( \tau \left( \frac{f}{s} \right)^{1/(\sigma-1)} \) equals one (e.g., \( \tau = 1 \) and \( f = f_x \)), the two skill intensity thresholds must be equal, that is \( \theta_d^* = \theta_x^* \). See Equation (25) for this relationship between the two skill intensity cut-offs.
zero-profit skill intensity cut-off for free trade, $\theta^*_F$. Since free trade induces an increase in import competition, fewer firms will survive under free trade compared to under a condition of autarky.

![Diagram of cut-offs](image)

Figure 4: Autarky, Zero Profit and Exporting Skill Intensity Cut-offs, and Free Trade Cut-off

2.3.5. A Numerical Exercise of Costly Trade Under the Symmetric Assumption

The basic set-up for the parameters is the same as in the previous section for autarky. In addition, the fixed export costs, $f_x$, and the variable trade costs $\tau$ are used to find the zero-profit skill intensity cut-off, $\theta^*_d$, the skill intensity cut-off for exporting, $\theta^*_x$, and the skill premium, $s/w$, in equilibrium. I set $f_x = 2$, which is the same as the fixed costs for domestic production $f$. I first exploit $\phi = 1.1$ and $\alpha = 1.5$, and then I use different values of $\phi$ and $\alpha$ (e.g., $\phi = 1.5$ and $\alpha = 2$) to show how different sets of $\phi$ and $\alpha$ will affect the skill premium $s/w$ and the two skill intensity cut-offs, $\theta^*_d$ and $\theta^*_x$.

Figure 5 plots the skill premium, the two skill intensity cut-offs, and the weighted average skill intensities for both surviving and exporting firms against the variable costs of trade $\tau$. For these plots, I use different sets of $\phi$ and $\alpha$: I set $\phi = 1.1$ and $\alpha = 1.5$ for Panels A and B, and $\phi = 1.5$ and $\alpha = 2$ for Panels C and D.

Panels A and C in Figure 5 show that the relative wage of skilled labor decreases with an increase in the variable trade costs $\tau$, that is, there is an increase in wage inequality as a result of a reduction in the variable costs of trade. When $\tau$ is large enough, no firms can export, therefore the skill premium, $s/w$ will be the same as the one under the condition of autarky equilibrium. As the variable trade costs fall, the relative wage of skilled labor rises because of the reallocation of workers towards relatively high skill intensive firms. To be more specific, the inter-firm reallocation of resources has an effect on the skill premium through three different channels. First, a fall in the
trade costs makes exporting easier, and allows existing exporters to increase their labor demand so as to increase production and make additional sales in export markets. This is accompanied by an increase in the relative demand for skilled workers because they are, on average, relatively more skill intensive. As shown in Panels B and D of Figure 5, the weighted average skill intensity of exporting firms is higher than that of one of the surviving firms. Second, the relatively unskilled-labor intensive non-exporters are forced to exit the market in the face of the increased import competition from successful foreign firms that export to the unsuccessful firms’ domestic market. Under the condition of the perfectly competitive labor market, their labor force would be reallocated towards more efficient, relatively high skill intensive firms, so that the relative demand for skilled workers increases, and hence the wage inequality also increases. These two effects unambiguously increase the between-group wage inequality.

In contrast to the first two effects, the relative skill intensity of firms, that become new exporters as a result of lowering barriers to trade, is ambiguous. As a matter of fact, if the cost of trade are initially low, then the relative skill intensity of these new exporters would be higher than average, so that the skill premium would increase as a result of a fall in trade costs. On the other hand, if the initial level of trade barriers were high, the average skill intensity of firms newly entering the export market would be lower than that of surviving firms, which would result in a reduction in wage inequality. Panel B and D in Figure 5 illustrate the third effect, which depends on the initial level of trade openness. If the initial level of variable trade costs is higher than 1.15 (i.e., $\tau > 1.15$), the third effect also leads to an increase in the skill premium; otherwise, the skill premium decreases as the trade costs fall. However, the latter effect, which might decrease the skill premium, would not be big enough to overturn the first two effects, so that the skill premium unambiguously increases as the variable costs of trade fall. The overall effect of a reduction in the variable costs of trade on the skill premium is shown in Panels A and C of Figure 5.

As illustrated in Panels B and D of Figure 5, as $\tau$ falls, $\theta_x^*$ declines, while $\theta_d^*$ increases. A reduction in trade costs allows firms to export relatively easily, so that the skill intensity cut-off for exporting decreases. On the other hand, the relatively inefficient firms are forced out of the
market in the face of the increased import competition that results from the reduced trading costs, so the zero-profit skill intensity cut-off \( \theta^*_d \) increases. As noted in Figure 4, these two skill intensity cutoffs, \( \theta^*_d \) and \( \theta^*_x \), converge to the same level, above which all producing firms will be exporters when \( \tau = 1 \) and \( f = f_x \) (free trade).

As mentioned earlier, the increasing trend of the relative wage of skilled to unskilled workers following the reduction of trade costs is not affected by the values of \( \phi \) and \( \alpha \) as long as the following condition, \( \alpha > \phi \ln \left( \frac{s}{w} \right) \), holds. As illustrated in Panels A and C of Figure 5, different sets of \( \phi \) and \( \alpha \) only lead to different scales of skill premium. To be more specific, the skill premium rises from 2.37 to 2.64 in response to the reduction of the variable trade costs \( \tau \) from 1.4 to 1, when \( \phi = 1.1 \) and \( \alpha = 1.5 \); whereas the skill premium, \( s/w \), increases by about 12% from 2.43 to 2.72, when the economy moves from autarky (that is, when \( \tau > 1.45 \)) to free trade (\( \tau = 1 \)), in the case of \( \phi = 2 \) and \( \alpha = 1.5 \). In the same fashion, the patterns of two skill intensity cut-offs are also not affected by different sets of \( \phi \) and \( \alpha \): the zero-profit skill intensity cut-off increases, while its cut-off for exporting decreases as trade costs fall, which are shown in Panels B and D of Figure 5.
Figure 5: The Evolution of the Skill Premium and Two Skill Intensity Cut-offs corresponding to the Variable Trade Costs

Note: Panels A and B of this figure plot the skill premium, the two skill intensity cut-offs, and the weighted average skill intensity for domestic production and exporting against the variable trade costs, $\tau$, when $\phi = 1.1$ and $\alpha = 1.5$. Panels C and D plot $s/w$, $\theta_d^*$, $\theta_x^*$, $\tilde{\theta}(\theta_d^*)$, and $\tilde{\theta}(\theta_x^*)$ against the variable trade costs in the case of $\phi = 1.5$ and $\alpha = 2$. For remaining parameters, I set $(H, L) = (1500, 2000)$, $\sigma = 3.8$, $\delta = 0.025$, $f_e = 20$, and $f = f_x = 2$. 
Since an arbitrary set of parameters for $\phi$ and $\alpha$ is used to show the pattern of variables ($s/w$, $\theta_d^*$ and $\theta_x^*$) in response to the reduction of trade costs, it remains unclear how the skill premium and the two skill intensity cut-offs are affected by each parameter, $\phi$ and $\alpha$. As a result, I numerically examine the partial effects of $\phi$ and $\alpha$ on the skill premium and the zero-profit skill intensity cut-off. Table 2 presents that, all else being equal, the increase in the quality elasticity of the marginal costs $\phi$ reduces the zero-profit skill intensity cut-off, $\theta_d^*$, and leads to a decrease in the skill premium. Note that $\phi > 1$. The intuition behind this result stems from the fact that the higher level of $\phi$ makes high skill intensive firms, that produce high-quality varieties, relatively less efficient compared to low skill intensive firms, and this allows relatively low skill intensive firms to enter the market. Therefore, the zero-profit skill intensity cut-off decreases, resulting in the decline in the skill premium.

Table 2: Partial Effects of $\phi$ and $\alpha$ on the skill premium and skill intensity cut-offs

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 1.1$</th>
<th>$\alpha = 1.2$</th>
<th>$\alpha = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi = 1.1$</td>
<td>$\phi = 1.2$</td>
<td>$\phi = 1.5$</td>
</tr>
<tr>
<td></td>
<td>Free Trade ($\tau = 1$)</td>
<td>Free Trade ($\tau &gt; 1.65$)</td>
<td>Free Trade ($\tau = 1$)</td>
</tr>
<tr>
<td>$s/w$</td>
<td>3.15</td>
<td>2.83</td>
<td>3.01</td>
</tr>
<tr>
<td>$\theta_d^*$</td>
<td>0.59</td>
<td>0.55</td>
<td>0.58</td>
</tr>
<tr>
<td>$\theta_x^*$</td>
<td>0.59</td>
<td>1</td>
<td>0.58</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\alpha = 1.5$</th>
<th>$\alpha = 1.7$</th>
<th>$\alpha = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi = 1.1$</td>
<td>$\phi = 1.2$</td>
<td>$\phi = 1.5$</td>
</tr>
<tr>
<td></td>
<td>Free Trade ($\tau = 1$)</td>
<td>Free Trade ($\tau &gt; 1.40$)</td>
<td>Free Trade ($\tau = 1$)</td>
</tr>
<tr>
<td>$s/w$</td>
<td>2.63</td>
<td>2.37</td>
<td>2.76</td>
</tr>
<tr>
<td>$\theta_d^*$</td>
<td>0.54</td>
<td>0.50</td>
<td>0.56</td>
</tr>
<tr>
<td>$\theta_x^*$</td>
<td>0.54</td>
<td>1</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Note: For the remaining parameters, I set $(H,L) = (1500, 2000)$, $\sigma = 3.8$, $\delta = 0.025$, $f_e = 20$, and $f = f_x = 2$.

All else being equal, on the other hand, the zero-profit skill intensity cut-off increases with $\alpha$. Thus, the skill premium also increases with $\alpha$. Note that $\alpha$ measures the degree of skilled workers’ efficiency on quality-related tasks. With a higher $\alpha$, skilled workers become more efficient relative to unskilled workers, so that less competitive firms, which use unskilled-labor more intensively, are driven out of the market. As a result, the increase in $\alpha$ raises the skill premium as well as the
marginal skill intensity for surviving firms, which are shown in Table 2.

Instead of reducing the variable trade costs, I examine how the fixed export costs affect the skill intensity cut-offs, and hence wage inequality. Figure 6 plots the skill premium, the two skill intensity cut-offs, and the two weighted-average skill intensities for domestic production and exporting firms against the level of the fixed trade costs $f_x$, while maintaining the same level of variable trade costs $\tau$.\footnote{For Figure 6, I set $H = 1500, L = 2000, f_c = 20, f = 2, \tau = 1.1, \phi = 1.1, \text{and } \alpha = 1.5.$} As illustrated in Figure 6, the export skill intensity cut-off $\theta_x^*$ declines as the fixed trade costs fall, while the zero-profit skill intensity cut-off $\theta_d^*$ increases as the fixed costs of trade decreases.

![Figure 6: The Relationship Between the Skill Premium and the Fixed Costs of Trade](image)

With regard to the skill premium, I show a similar trend when I reduce the fixed exporting costs, $f_x$, instead of lowering the variable trade costs, $\tau$, which is also shown in Figure 6. As with the effect of the variable trade costs, the decline in the fixed costs for exporting ambiguously raises the skill premium. The intuition behind of this result is in line with the case of lowering the variable costs of trade. As outlined earlier, as the fixed export costs fall, the existing exporters
increase their labor demand, while the relatively inefficient non-exporters are driven out of the markets. These two mechanisms lead to the reallocation of production resources towards firms that have relatively higher than average skill intensity, so that the wage inequality between groups increases. In contrast, the average skill intensity of firms that become new exporters may be lower than the average skill intensity of surviving firms when the initial level of fixed costs are relatively low (that is, when \( f_x < 2.2 \) as shown in Figure 6); as a result, this drives the skill premium down. Nevertheless, the latter effect, that forces the skill premium to fall, cannot overturn the former effects, so that the between-group wage inequality unambiguously increases.

### 2.4. An Open Economy between Asymmetric Countries

Lastly, I extend the model to consider a situation of costly trade between asymmetric countries that differ in their relative factor endowment, \( H/L \). As in the previous section, the world economy consists of two countries: country \( A \) and \( B \). The only difference between these countries is their comparative endowments of skilled and unskilled labor: \( H_A, L_A, H_B \) and \( L_B \).

#### 2.4.1. Firm Competition

Given firms’ pricing rules, the optimal quality and quantity of good produced, the equilibrium revenue of firms with \( \theta \) in the domestic market is

\[
R_d^i(\theta) = \lambda w_i^{1-\sigma} \left( \frac{s_i}{w_i} \right)^{\theta (1-\sigma) / \theta} \theta^{\alpha (\sigma-1) / \phi} \bar{P}^{\sigma-1} E_i \quad \text{for } i \in \{A, B\},
\]

where \( \lambda = \left[ \left( \frac{\sigma}{\sigma-1} \right) \left( \frac{\phi}{\phi-1} \right)^{-1/\phi} \right]^{1-\sigma} > 0 \) is constant. \( R_d^i(\theta) \) denotes the domestic revenue of firms with \( \theta \) in country \( i \). The export revenues of firms with \( \theta \) in country \( A \) and \( B \) are

\[
R_x^A(\theta) = \lambda \tau A \left[ \frac{w_A}{s_A} \right]^{\theta (1-\sigma) / \theta} \theta^{\alpha (\sigma-1) / \phi} \bar{P}^{\sigma-1} E_A,
\]

\[
R_x^B(\theta) = \lambda \tau B \left[ \frac{w_B}{s_B} \right]^{\theta (1-\sigma) / \theta} \theta^{\alpha (\sigma-1) / \phi} \bar{P}^{\sigma-1} E_B.
\]
Note that, in equilibrium, the export revenue is proportional to the domestic revenue. The relationship between these two revenues can be obtained from equations (38) and (39), which is

\[ R^A_x(\theta) = \tau^{1-\sigma} \left( \frac{\bar{P}_B}{\bar{P}_A} \right)^{\sigma-1} \left( \frac{E_B}{E_A} \right) R^A_d(\theta), \]

and

\[ R^B_x(\theta) = \tau^{1-\sigma} \left( \frac{\bar{P}_A}{\bar{P}_B} \right)^{\sigma-1} \left( \frac{E_A}{E_B} \right) R^B_d(\theta). \]

The export revenue of firms in home country A, relative to the domestic revenue, decreases with variable trade costs \( \tau \) and the relative market size of the home country, \( E_A/E_B \), while it increases with the relative price level in the foreign market \( \bar{P}_B/\bar{P}_A \).

In the presence of fixed costs for both domestic production (\( f \)) and for exporting (\( f_x \)), a firm’s profit from domestic sales, denoted by \( \pi_d(\theta) \), and foreign sales \( \pi_x(\theta) \) are

\[ \pi^i_d(\theta) = \frac{R^i_d(\theta)}{\sigma} - f_i w_i \quad \text{for} \quad i \in \{A, B\}, \]

and

\[ \pi^i_x(\theta) = \frac{R^i_x(\theta)}{\sigma} - f_i w_i \quad \text{for} \quad i \in \{A, B\}. \]

Using the zero-profit condition of marginal firms in each market, that is \( R^i_d(\theta^*_d) = \sigma f_i w_i \) and \( R^i_x(\theta^*_x) = \sigma f_i w_i \), equation (38), (39) and (40) give

\[ \left( \frac{s_A}{w_A} \right)^{-\left( \theta^*_x - \theta^*_d \right)} \left( \frac{\theta^*_x}{\theta^*_d} \right)^{\frac{\sigma}{\sigma-1}} = \tau \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \left( \frac{\bar{P}_A}{\bar{P}_B} \right) \left( \frac{E_A}{E_B} \right)^{\frac{1}{\sigma-1}}, \]

and

\[ \left( \frac{s_B}{w_B} \right)^{-\left( \theta^*_x - \theta^*_d \right)} \left( \frac{\theta^*_x}{\theta^*_d} \right)^{\frac{\sigma}{\sigma-1}} = \tau \left( \frac{f_x}{f} \right)^{\frac{1}{\sigma-1}} \left( \frac{\bar{P}_B}{\bar{P}_A} \right) \left( \frac{E_B}{E_A} \right)^{\frac{1}{\sigma-1}}. \]

These two equality conditions in equation (42) show a relationship between two skill intensity cut-offs. Applying the same logic in the previous section under the symmetric assumption, the skill intensity cut-off for exporting firms is greater than the zero-profit skill intensity cut-off, that is, \( \theta^*_i > \theta^* \) where \( i \in \{A, B\} \), when the right hand side of equation (42) is greater than one. For the remainder of the paper, I assume that this condition holds so that only the most profitable firms
tend to be exporters, which is consistent with a substantial amount of empirical evidence.\footnote{Theoretically, it is possible that the skill intensity cut-off for the domestic sales is larger than the export skill intensity cut-off, $\theta_{d}^{*} > \theta_{x}^{*}$, when two countries have different endowments (and hence different $\bar{P}$ and $E$) and $\tau \left( \frac{L}{Z} \right)^{1/\sigma} = 1$. This implies that the least efficient firms only serve the export market, while the most profitable firms serve both domestic and foreign markets. This possibility is shown in Table 3 in the following numerical exercise.}

### 2.4.2. Firm Entry, Exit, and the Free Entry Condition

Since the free entry condition, that is, the expected value of entry equals the sunk fixed entry cost, is basically the same as in the case of symmetric countries, I quickly move through this section by expressing equations that are necessary to determine the equilibrium variables of interest. The free entry condition in each country $A$ and $B$ in equilibrium follows equation (28), which is as follows:

\[
[1 - G(\theta_{d}^{*})] \frac{1}{W_{i}} \left( \frac{\theta_{d}^{i} - \theta_{d}^{*}}{\theta_{d}^{*}} \right)^{(1-\sigma)} \left( \frac{\theta_{d}^{i}}{\theta_{d}^{*}} \right)^{\frac{\alpha(\sigma-1)}{\sigma}} - 1 + \left[ 1 - G(\theta_{x}^{*}) \right] f_{x} \left[ \frac{\theta_{x}^{i} - \theta_{x}^{*}}{\theta_{x}^{*}} \right] \left( \frac{\theta_{x}^{i}}{\theta_{x}^{*}} \right)^{\frac{\alpha(\sigma-1)}{\sigma}} - 1 = \delta f_{e} \quad \text{for } i \in \{A, B\},
\]

where the weighted average skill intensities for surviving and exporting firms in each country $A$ and $B$, denoted by $\bar{\theta}_{d}^{i}$ and $\bar{\theta}_{x}^{i}$, are given as

\[
\bar{\theta}_{d}^{i}(\theta_{d}^{*}) = \left[ \frac{1}{1 - G(\theta_{d}^{*})} \int_{\theta_{d}^{*}}^{1} \theta^{\sigma-1} g(\theta) d\theta \right]^{1/(\sigma-1)} \quad \text{for } i \in \{A, B\},
\]

\[
\bar{\theta}_{x}^{i}(\theta_{x}^{*}) = \left[ \frac{1}{1 - G(\theta_{x}^{*})} \int_{\theta_{x}^{*}}^{1} \theta^{\sigma-1} g(\theta) d\theta \right]^{1/(\sigma-1)} \quad \text{for } i \in \{A, B\}.
\]

The free entry condition in equation (43) seems to be independent of the aggregate variables (that is, the price index and the total expenditure) of a trading counterpart. It should be noted, however, that the skill intensity cut-off for domestic production and exporting, $\theta_{d}^{*}$ and $\theta_{x}^{*}$, in equation (43) is a function of the relative market size and the price level of a trading partner under the assumption of asymmetricity, which is indicated in equation (42).
2.4.3. Labor Market Equilibrium

The labor market equilibrium conditions are similar to those in the previous section for the identical country case. The total skilled and unskilled labor demand of firms with $\theta$ corresponding to domestic sales are the same as in equation (31) with the appropriate country index $i \in \{A, B\}$ on the wage and the aggregate variables:

$$D^i_{d,h}(\theta, w_i, s_i) = \eta w_i^{-\sigma} \left( \frac{s_i}{w_i} \right)^{\theta(1-\sigma)-1} \theta^{\frac{\alpha(\sigma-1)}{\sigma}} \hat{P}_i^{\sigma-1} E_i \text{ for } i \in \{A, B\},$$

$$D^i_{d,l}(\theta, w_i, s_i) = \eta w_i^{-\sigma} \left( \frac{s_i}{w_i} \right)^{\theta(1-\sigma)} \theta^{\frac{\alpha(\sigma-1)}{\sigma}} (1-\theta) \hat{P}_i^{\sigma-1} E_i \text{ for } i \in \{A, B\},$$

(45)

where $\eta = \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} \left( \frac{\phi}{\phi-1} \right)^{1-\sigma} \left( \frac{1}{\phi-1} \right)^{(\sigma-1)/\phi} > 0$ is constant. $D^i_{d,h}(\theta, w_i, s_i)$ denotes the total labor demand for skilled workers by firms with the same $\theta$ in country $i$ in response to domestic sales. $D^i_{d,l}(\theta, w_i, s_i)$ represents the total labor demand for unskilled labor of firms with $\theta$ in country $i$ corresponding to domestic sales.

The total demand for skilled and unskilled labor of the exporting firms depends on a trading counterpart’s relative price index and expenditure as well as on the variable costs of trade. Thus, the total skilled and unskilled labor demand per firm with $\theta$ corresponding to export sales are

$$D^i_{x,h}(\theta) = \tau^{-\sigma} \left( \frac{\hat{P}_B}{\hat{P}_A} \right)^{\sigma-1} \left( \frac{E_B}{E_A} \right) D^A_{d,h}(\theta), \quad D^i_{x,l}(\theta) = \tau^{-\sigma} \left( \frac{\hat{P}_B}{\hat{P}_A} \right)^{\sigma-1} \left( \frac{E_B}{E_A} \right) D^A_{d,l}(\theta),$$

(46)

$$D^i_{x,l}(\theta) = \tau^{-\sigma} \left( \frac{\hat{P}_A}{\hat{P}_B} \right)^{\sigma-1} \left( \frac{E_A}{E_B} \right) D^B_{d,h}(\theta), \quad D^i_{x,l}(\theta) = \tau^{-\sigma} \left( \frac{\hat{P}_A}{\hat{P}_B} \right)^{\sigma-1} \left( \frac{E_A}{E_B} \right) D^B_{d,l}(\theta).$$

$D^i_{x,j}(\theta)$, where $i \in \{A, B\}$ and $j \in \{h, l\}$, denotes the total labor demand for $j$ type of workers of firms with $\theta$ in country $i$ in response to foreign sales. Note that the relative labor demand of export to domestic sales relies on the relative difficulty of the foreign market (that is, the relative market size and price level) as well as the variable trade costs.

Following the same procedure described in the symmetric-country case, the aggregate demand for skilled and unskilled workers in each country $A$ and $B$ corresponding to the variable input costs
can be expressed by

\[
TD_h^A = \eta \frac{M^A}{1 - G(\theta_d^{A+})} \bar{w}_A^{-\frac{1}{\sigma}} \bar{p}_A^{\frac{1}{\sigma} - 1} E_A \left[ \int_{\theta_d^{A+}}^{1} \left( \frac{s_A}{w_A} \right)^{\theta (1 - \sigma) - 1} \theta^{\frac{a(\sigma-1)}{\theta}} \theta g(\theta) d\theta \right] 
\]

\[
TD_h^B = \eta \frac{M^B}{1 - G(\theta_d^{B+})} \bar{w}_B^{-\frac{1}{\sigma}} \bar{p}_B^{\frac{1}{\sigma} - 1} E_B \left[ \int_{\theta_d^{B+}}^{1} \left( \frac{s_B}{w_B} \right)^{\theta (1 - \sigma) - 1} \theta^{\frac{a(\sigma-1)}{\theta}} \theta g(\theta) d\theta \right] 
\]

The aggregate skilled and unskilled labor demand in country \( i \in \{A,B\} \) corresponding to the sunk fixed costs are

\[
FD_h^i = \left( \frac{H_i}{H_i + L_i} \right) M^i \left[ f + \frac{1 - G(\theta_d^{i+})}{1 - G(\theta_d^{i+})} f_x + \frac{\delta}{1 - G(\theta_d^{i+})} f_e \right] \quad \text{for} \ i \in \{A,B\},
\]

\[
FD_i^i = \left( \frac{L_i}{H_i + L_i} \right) M^i \left[ f + \frac{1 - G(\theta_d^{i+})}{1 - G(\theta_d^{i+})} f_x + \frac{\delta}{1 - G(\theta_d^{i+})} f_e \right] \quad \text{for} \ i \in \{A,B\},
\]

where \( M^i \) denotes the mass of incumbent firms in country \( i \).

In a perfectly competitive labor market, the supply of skilled (unskilled) labor in equilibrium is equal to the sum of skilled (unskilled) labor demand in both the variable and fixed costs. Therefore, I have the following labor market clearing conditions in each country:

\[
H^i = TD_h^i + FD_h^i \quad \text{for} \ i \in \{A,B\},
\]

\[
L^i = TD_i^i + FD_i^i \quad \text{for} \ i \in \{A,B\}.
\]
2.4.4. Costly Trade Equilibrium under the Asymmetric Assumption

To close this section, I need to determine the equilibrium aggregate variables in each country: the total revenue $E$, the mass of successful firms $M$, and the industry price index $\tilde{P}$. Under the asymmetric assumption between countries, all aggregate variables differ between countries.

The aggregate revenue $E$ in equilibrium must equal the total payment to production resources (skilled and unskilled labor):

$$E_i = w_i L_i + s_i H_i \quad \text{for } i \in \{A, B\}. \quad (50)$$

Following the same logic of the previous section, the mass of incumbent firms in country $i$ is

$$M^i = \frac{E_i}{\tilde{r}_i} = \frac{H_i + L_i}{\sigma[f + \frac{1 - G(\theta^B_x)}{1 - G(\theta^A_x)}f_A + \frac{\delta}{1 - G(\theta^B_d)}f_x]} \quad \text{for } i \in \{A, B\}. \quad (51)$$

Lastly, the aggregate price index can be expressed as the weighted average price index of each domestic and exporting firms, which is as follows:

$$\tilde{P}_A = \left[ \frac{\lambda M^A w_A^{1-\sigma}}{M^A} \left( \frac{s_A}{w_A} \right)^{\frac{\theta^A_d}{\theta^A_x} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1}{\phi - 1} \right)^{1-\sigma}} + \frac{1 - G(\theta^B_x)}{1 - G(\theta^A_x)} M^A \lambda A^{1-\sigma} w_A^{1-\sigma} \left( \frac{s_B}{w_B} \right)^{\frac{\theta^B_d}{\theta^B_x} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1}{\phi - 1} \right)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}},$$

$$\tilde{P}_B = \left[ \frac{\lambda M^B w_B^{1-\sigma}}{M^B} \left( \frac{s_B}{w_B} \right)^{\frac{\theta^B_d}{\theta^B_x} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1}{\phi - 1} \right)^{1-\sigma}} + \frac{1 - G(\theta^A_x)}{1 - G(\theta^B_x)} M^A \lambda A^{1-\sigma} w_A^{1-\sigma} \left( \frac{s_A}{w_A} \right)^{\frac{\theta^A_d}{\theta^A_x} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1}{\phi - 1} \right)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}, \quad (52)$$

where $\lambda = \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{1}{\phi - 1} \right)^{1-\sigma}$ is a positive constant. Notice that $\frac{1 - G(\theta^B_x)}{1 - G(\theta^A_x)} M^B$ refers to the mass of country $B$’s exporting firms that ship their varieties to country $A$. This term, thus, is included to determine country $A$’s aggregate price index. The weighted-average skill intensities for domestic and exporting firms, $\tilde{\theta}^A_d$ and $\tilde{\theta}^A_x$, are given in equation (44).

The equilibrium under the asymmetric assumption consists of a vector of 7 variables in each country $A$ and $B$: $\{w_i, s_i, \theta^A_d, \theta^A_x, \tilde{\theta}^A_d, \tilde{\theta}^A_x, \tilde{P}_i\}$ for $i \in \{A, B\}$. These equilibrium variables are determined by 7 equations for each country: the equations that links the two skill intensity cut-offs (equation (42)), the free entry condition (equation (43)), the weighted-average skill intensities...
(equation (44) for the domestic and foreign market, the labor market clearing conditions (equation (51) for each skilled and unskilled labor) and the aggregate price index (equation (52)). The remaining aggregate variables, $M^i$ and $E^i$, are determined as functions of these vectors of the equilibrium variables (equation (50) and (51)).

**Proposition 2.4.** As trade costs, $\tau$ and/or $f_x$, fall, the relative wage of skilled to unskilled worker in both countries increases.

**Proof.** Proven numerically in the following section.

2.4.5. Numerical Exercise of Costly Trade between Two Different Countries

The basic set-up for the parameters is the same as in the previous section for the symmetric-country case: $\sigma = 3.8, \delta = 0.025, f_e = 20, f = f_x = 2, \phi = 1.1,$ and $\alpha = 1.5$. The equilibrium variables for interest depend on the degree of trade liberalization as well as on a trading partner’s endowment of skilled and unskilled labor. Table 3 summarizes the evolution of the skill premium, the skill intensity cut-offs for both domestic production and exporting in response to variable trade costs, and the country’s resource endowment.

As shown in Table 3, the relative wage of the skilled worker, $s/w$, in both countries increases with the reduction in variable trade costs $\tau$. That is, when two asymmetric countries trade with each other, the reduction of variable trade costs increases the skill premium in both countries. The increased wage inequality that followed globalization is due to the reallocation of production resources towards more profitable firms, which produce higher-quality products using higher skill intensive technology.

Table 3 also indicates that two skill intensity cut-offs move in opposite directions as trade costs fall. That is, the skill intensity cut-off for domestic production increases, while the export skill intensity cut-off decreases as trade costs fall. Lowering trade barriers allows firms to export relatively easily, which results in a decline of the skill intensity cut-off for exporting. On the other hand, the reduction in trade barriers leads to an increase in import competition, so that the least efficient firms are forced to exit the market, which causes the zero-profit skill intensity cut-off to
Another interesting fact, presented in Table 3, is that a high enough trade barrier between two countries creates a regime where one-way trade, from the relatively skill-abundant to the relatively skill-scarce country, occurs. For instance, when country $A$ is more skilled labor abundant than country $B$ (that is, $A(H,L) = (1500,2000) \& B(H,L) = (1300,2000)$) and the variable trade cost is high enough (so, $\tau = 1.33$), the export skill intensity cut-off in country $B$ is closed to one, $\theta^{B*}_x = 1$, whereas the export skill intensity cut-off in country $A$ is 0.79, that is $\theta^{A*}_x = 0.79$. This implies that some firms with $\theta \in [0.79, 1]$ in country $A$ are efficient enough to send their varieties to country $B$, while no firms in country $B$ benefit from selling their products in country $A$.

Furthermore, when the barrier to trade is high enough, two-way trade between similar economies, in terms of factor endowment, is more prevalent than trade between asymmetric countries. This phenomenon is consistent with the well-known stylized fact that the large volume of world trade is associated with trade among similar economies (e.g., Linder’s hypothesis).

When two different countries trade with each other, the skill premium in the relatively skill-abundant country is much higher and increases faster, as trade costs fall, than in the relatively skill-scare country. For instance, the skill premium in country $A$ is increased by 11% from 2.37 (autarky) to 2.63 (free trade); whereas it increases 10% from 2.23 (autarky) to 2.45 (free trade) in country $B$ when country $A$ trades with country $B$, where $A : (H,L) = (1500,2000)$ and $B : (H,L) = (1700,2000)$. This relationship is illustrated in Figure 8 as well as in Table 3. This conflicts with the well-known Stolper-Samuelson theorem in Heckscher-Ohlin-type trade theory.

---

30When one observes both exports and imports of the same good, this pattern of flow is described as two-way or intra-industry trade.
Table 3: Equilibrium Zero-profit, \((\theta^*_d)\), Export Skill Intensity Cut-off, \((\theta^*_x)\), and Skill Premium \((s/w)\) between Asymmetric Countries

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\text{Country A} )</td>
<td>(\text{Country B} )</td>
<td>(\text{Country A} )</td>
<td>(\text{Country B} )</td>
<td>(\text{Country A} )</td>
<td>(\text{Country B} )</td>
</tr>
<tr>
<td>Skill Premium ((\frac{s}{w}))</td>
<td>(\tau = 1)</td>
<td>2.631</td>
<td>2.844</td>
<td>2.630</td>
<td>2.630</td>
<td>2.631</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.1)</td>
<td>2.596</td>
<td>2.833</td>
<td>2.606</td>
<td>2.606</td>
<td>2.617</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.2)</td>
<td>2.519</td>
<td>2.729</td>
<td>2.525</td>
<td>2.525</td>
<td>2.530</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.3)</td>
<td>2.438</td>
<td>2.592</td>
<td>2.423</td>
<td>2.423</td>
<td>2.410</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.33)</td>
<td>2.420</td>
<td>2.579</td>
<td>2.403</td>
<td>2.403</td>
<td>2.391</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.37)</td>
<td>-</td>
<td>-</td>
<td>2.386</td>
<td>2.386</td>
<td>2.374</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.4)</td>
<td>-</td>
<td>-</td>
<td>2.379</td>
<td>2.379</td>
<td>-</td>
</tr>
<tr>
<td>Domestic skill intensity Cut-off ((\theta^*_d))</td>
<td>(\tau = 1)</td>
<td>0.549</td>
<td>0.536</td>
<td>0.546</td>
<td>0.546</td>
<td>0.543</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.1)</td>
<td>0.516</td>
<td>0.502</td>
<td>0.514</td>
<td>0.514</td>
<td>0.512</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.2)</td>
<td>0.503</td>
<td>0.492</td>
<td>0.502</td>
<td>0.502</td>
<td>0.501</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.3)</td>
<td>0.501</td>
<td>0.491</td>
<td>0.502</td>
<td>0.502</td>
<td>0.501</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.33)</td>
<td>0.501</td>
<td>0.491</td>
<td>0.501</td>
<td>0.501</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.37)</td>
<td>-</td>
<td>-</td>
<td>0.501</td>
<td>0.501</td>
<td>0.500</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.4)</td>
<td>-</td>
<td>-</td>
<td>0.501</td>
<td>0.501</td>
<td>-</td>
</tr>
<tr>
<td>Export skill intensity Cut-off ((\theta^*_x))</td>
<td>(\tau = 1)</td>
<td>0.544</td>
<td>0.542</td>
<td>0.546</td>
<td>0.546</td>
<td>0.549</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.1)</td>
<td>0.598</td>
<td>0.607</td>
<td>0.603</td>
<td>0.603</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.2)</td>
<td>0.674</td>
<td>0.711</td>
<td>0.687</td>
<td>0.687</td>
<td>0.702</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.3)</td>
<td>0.764</td>
<td>0.891</td>
<td>0.802</td>
<td>0.802</td>
<td>0.838</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.33)</td>
<td>0.794</td>
<td>0.996</td>
<td>0.844</td>
<td>0.844</td>
<td>0.895</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.37)</td>
<td>-</td>
<td>-</td>
<td>0.911</td>
<td>0.911</td>
<td>0.997</td>
</tr>
<tr>
<td></td>
<td>(\tau = 1.4)</td>
<td>-</td>
<td>-</td>
<td>0.975</td>
<td>0.975</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: For the remaining parameters, I set \(\sigma = 3.8, \delta = 0.025, f_e = 20, f = f_x = 2, \phi = 1.1, \) and \(\alpha = 1.5.\)
Figure 7: The Evolution of Country A’s Variables corresponding to $\tau$

Figure 7 plots four different variables of interest against variable trade costs $\tau$. They are the relative wage of skilled labor, the zero-profit and export skill intensity cut-offs, and the real wage of skilled labor. Panel A in Figure 7 illustrates country A’s skill premium increases in response to a reduction in trade costs, regardless of its trading partner’s endowment of skilled and unskilled labor. In addition, the impact of the lower trade costs on the skill premium is larger when a country trades with a relatively skill-abundant country than when it trades with a relatively skill-scare one.

Panels B and C in Figure 7 show that the two skill intensity cut-offs move in opposite directions as $\tau$ falls and that these trends occur regardless of the trading partner’s labor endowment. Lastly, Panel D illustrates that the real wage of skilled labor increases with the reduction of variable trade costs. As $\tau$ falls, the growth rate of the real wage of skilled workers is larger when a country trades with a relatively skill-abundant country, which is closely related to the pattern of the skill intensity cut-offs.

Note: For these plots, I use the following parameter values: $\sigma = 3.8$, $\delta = 0.025$, $f_c = 20$, $f = f_e = 2$, $\phi = 1.1$, and $\alpha = 1.5$. Also I set $A(H, L) = (1500, 2000)$ for a country A’s endowment of skilled and unskilled labor.

31 The effect of the reduction of trade costs on the real wage of unskilled workers is relatively small compared to the impact on the real wage for skilled labor. The real wage for country A’s skilled labor is calculated by $\frac{\Delta s_{A}}{f_{s_{A}}}$. 

43
premium.

Figure 8: The Evolution of the Skill Premium corresponding to \( \tau \): \( \left( \frac{H_A}{L_A} \right) < \left( \frac{H_B}{L_B} \right) \)

3. Concluding Remarks

This paper has constructed a general equilibrium model to demonstrate that trade liberalization is accompanied by wage inequality where differences in skill intensity across firms and “quality competition” in a differentiated products market play a key role in determining the pattern of wage inequality following a reduction in trade costs. As trade costs fall, the relative demand for skilled labor increases, resulting in the rise in wage inequality. This is, because production resources are reallocated to the relatively higher-skill-intensive firms, which are more competitive/profitable as they choose to produce higher-quality products. To account for the link between the skill premium and trade liberalization, some well-established stylized facts in the line of firm heterogeneity are used as a component of the theoretical framework. They include: i) differences in skill intensity across firms, ii) the more skill intensive firms are the more likely they are to be exporters, and iii) the positive association between a firm’s skill intensity and its product quality. The rise in wage inequality occurs regardless of whether factor endowments differ between trading countries.
effect of lowering trade costs on the skill premium is larger, however, when a country trades with the relatively skill-abundant country rather than with the relatively skill-scarce one. Furthermore, when countries differ in their resource endowment, the skill premium in the relatively skill-scarce country is much higher and increases at a faster rate, as trade costs fall, than in the relatively skill-abundant country.

This paper has focused on a single sector model to illustrate that the within-sector relative wage of skilled labor increases as a result of a trade liberalization, that relies on across-firms reallocations of productive resources in a perfectly competitive labor market. Within-industry wage inequality, however, cannot be fully described the overall wage inequality in a country where production resources may be allocated between sectors of the economy as well as within sectors. In fact, Redding et al. (2012), found that about two-thirds of Brazil’s overall wage inequality can be explained by the within-sector wage inequality. In addition, as noted in Goldberg and Pavcnik (2007), the lack of labor reallocation across sectors is found in most studies on the impact of trade liberalization on developing countries. Nevertheless, to fully understand a link between globalization and wage inequality, one must consider between-industry allocation of resources as a source of the remaining wage inequality. To incorporate multiple industries in a model of within-sector heterogeneity, it may be necessary to consider either neoclassical trade theory or a search and matching mechanism under the assumption of labor market friction in the spirit of firm heterogeneity.

An increasing number of studies also use linked employer-employee data to explore the wage distribution over a continuum of different types of workers. This allows researchers to capture the different effects of trade integration on within-group wage inequality, which is an important issue for policy makers. This, however, cannot be addressed in the model of two types of worker, so it would be informative to extend the research to a model that considers a continuum of workers.

Lastly, the present study has focused on between-skill-group wage inequality, but did not consider the possibility of wage dispersion within skill groups by assuming a perfectly competitive labor market; hence, the effect of trade openness on overall wage inequality cannot be fully demon-
strated. In line with the recent empirical findings, which show a significant correlation between the exporter wage premium conditional on workers’ skill level and trade liberalization (see., e.g., Baumgarten 2013), however, one should possibly extend a model of firm heterogeneity by introducing labor market frictions so that the effect of trade openness on within-sector wage inequality can be accounted for by both within- and between-skill-group channels.

References


Sampson, Thomas. “Selection into Trade and Wage Inequality,” CEP Discussion Papers dp1152, Centre for Economic Performance, LSE June 2012.


A. Appendix

A.1. Proof of Proposition 2.1

To prove this, two curves defined by the equilibrium conditions (13) and (19) must be uniquely intersected in the \((s/w, \theta^*)\) space. For convenience, I rewrite two equilibrium conditions (the free entry condition and the labor market clearing condition) as follows:

\[
FE\left(\frac{s}{w}, \theta^*\right) = \left\{ \left(\frac{s}{w}\right)^{\left(\bar{\theta}(\theta^*)-\theta^*(1-\sigma)\right)\left(\frac{\bar{\theta}(\theta^*)}{\theta^*}\right)\left(\frac{\alpha(\sigma-1)}{\varphi}\right)-1} \right\} - \frac{\delta}{1-G(\theta^*)} \frac{f_e}{f} = 0, \tag{A.1}
\]
\[ LE\left( \frac{s}{w}, \theta^* \right) = \left( \frac{w}{s} \right) \frac{1}{\Phi_\theta\left( \frac{s}{w} \right)} \left( \frac{s}{w} - \frac{s}{w} \right)^{\alpha(1-\sigma)} \theta^{\alpha(1-\sigma)} - \frac{H}{L} = 0, \tag{A.2} \]

where the weighted average skill intensity \( \tilde{\theta}(\theta^*) \) is given by equation (10).

First, I show that the free entry condition (equation (A.1)) gives a negative association between \( s/w \) and \( \theta^* \) using the implicit function theorem.

\[
\frac{\partial FE(s/w, \theta^*)}{\partial (s/w)} = (1 - \sigma)(\tilde{\theta}(\theta^*) - \theta^*) \left( \frac{s}{w} \right)^{(\tilde{\theta}(\theta^*) - \theta^*)(1-\sigma)-1} \left( \frac{\tilde{\theta}(\theta^*)}{\theta^*} \right)^{a(\sigma-1)} < 0.
\]

where the inequality holds because the constant term, \( 1 - \sigma \), is negative. By the second fundamental theorem of calculus, taking the derivative with respect to \( \theta^* \) gives

\[
\frac{\partial FE(s/w, \theta^*)}{\partial \theta^*} = \xi(s/w, \theta^*) \left\{ \tilde{\theta}'(\theta^*) \left[ \frac{\alpha}{\tilde{\theta}(\theta^*)} - \ln \left( \frac{s}{w} \right) \phi \right] - \left[ \frac{\alpha}{\theta^*} - \ln \left( \frac{s}{w} \right) \phi \right] \right\} - \delta \frac{f_e}{f(1 - G(\theta^*))^2} < 0,
\]

where \( \xi(s/w, \theta^*) = \frac{\alpha}{\phi} \left( \frac{s}{w} \right)^{(\tilde{\theta}(\theta^*) - \theta^*)(1-\sigma)} \left( \frac{\tilde{\theta}(\theta^*)}{\theta^*} \right)^{a(\sigma-1)} \) is positive. \( \tilde{\theta}'(\theta^*) \) denotes the derivative of the weighted average skill intensity with respect to \( \theta^* \). By the second fundamental theorem of calculus, taking the derivative of equation (10) with respect to \( \theta^* \) gives:

\[
\tilde{\theta}'(\theta^*) = \frac{1}{1 - \sigma} \left( \int_{\theta^*}^1 \frac{g(\theta)}{1 - G(\theta^*)} d\theta \right)^{\frac{1}{\sigma-1}} \left( \frac{\theta^*}{1 - \theta} \right)^{\sigma-1} \frac{g(\theta^*)}{1 - G(\theta^*)}.
\]

Since \( \sigma > 1 \), \( \tilde{\theta}'(\theta^*) \) is negative. Thus, the inequality, \( \frac{\partial FE(s/w, \theta^*)}{\partial \theta^*} < 0 \), holds because of the assumption that I take in the previous section, \( \alpha > \ln \left( \frac{s}{w} \right) \phi \), which implies that the firms revenue/profitability increases with the skill intensity as well as with product quality. Therefore, two equilibrium variables \( (s/w \text{ and } \theta^*) \) have a negative relationship, which is established by using the implicit function theorem:

\[
\frac{d\left( \frac{s}{w} \right)}{d\theta^*} = -\frac{\partial FE\left( \frac{s}{w}, \theta^* \right)/\partial \theta^*}{\partial FE\left( \frac{s}{w}, \theta^* \right)/\partial \left( \frac{s}{w} \right)} < 0.
\]

Second, the free entry condition (A.1) implies that \( s/w \to \infty \) as \( \theta^* \to 0 \), while \( s/w \to 0 \) as \( \theta^* \to \)
as $\theta^*$ goes to zero, the skill premium $s/w$ has to increase as much as possible so that
$FE(s/w, \theta^*) = 0$. As $\theta^*$ increases, on the other hand, $s/w$ must decrease as quickly as possible for
$FE(s/w, \theta^*) = 0$ so that $s/w$ goes to zero.

To complete this proof, I need to show that the labor market clearing condition gives a positive
link between $\theta^*$ and $s/w$. Unfortunately, equation (A.2) cannot be solved analytically using the
implicit function theorem. To establish a positive link between $\theta^*$ and $s/w$, I follow the method
introduced by Harrigan and Reshef (2011) where they effectively prove it. I begin with the labor
market clearing condition, equation (19):

$$\int_{\theta^*}^{1} \frac{D_h(\theta, \frac{s}{w})g(\theta)d\theta}{D_l(\theta, \frac{s}{w})g(\theta)d\theta} = \frac{H}{L}. $$

Since $\frac{D_h(\theta, \frac{s}{w})}{D_l(\theta, \frac{s}{w})} = \frac{\theta}{1-\theta} \left(\frac{w}{s}\right)$, which is from equation (16), I can rewrite equation above as follows:

$$\left(\frac{w}{s}\right) \int_{\theta^*}^{1} \frac{\theta}{1-\theta} \frac{D_l(\theta, \frac{s}{w})g(\theta)d\theta}{D_l(\theta, \frac{s}{w})g(\theta)d\theta} = \frac{H}{L}. $$

where $\left(\frac{\theta}{1-\theta}\right)$ represents the ratio of skilled to unskilled labor employed by firms with $\theta$. By
defining $\Psi(\theta, s/w, \theta^*) = \frac{D_l(\theta, \frac{s}{w})}{D_l(\theta, \frac{s}{w})g(\theta)d\theta}$, the labor market clearing condition can be rewritten as:

$$\left(\frac{w}{s}\right) \int_{\theta^*}^{1} \frac{\theta}{1-\theta} \Psi(\theta, s/w, \theta^*)g(\theta)d\theta = \frac{H}{L}. $$

where $\Psi(\theta, s/w, \theta^*)$ can be interpreted as the share of unskilled labor that works for firms with $\theta$.
The above Equation indicates that the relative endowment of skilled labor ($H/L$) is equal to the
average of the firm level skill ratios weighted by the firm’s unskilled labor share. Now I examine
how the increase in $\theta^*$ affects the skill premium, $s/w$. The increase in skill intensity $\theta^*$ implies
that the least profitable firms, which are less skill intensive than average, exit so that the weighted
average of the skill ratio of the surviving firms increases (i.e., $\int_{\theta^*}^{1} \left(\frac{\theta}{1-\theta}\right) \Psi(\theta, s/w, \theta^*)g(\theta)d\theta \succ$).

To keep the same level of the left hand side of equation above, the relative wage of skilled to
unskilled labor \((s/w)\) should be higher. Thus I confirm that the zero profit cut-off, \(\theta^*\), is positively associated with the skill premium \(s/w\) along the labor market clearing equation. The intuition behind the positive link between \(\theta^*\) and \(s/w\) is that an increase in \(\theta^*\) leads to an incipient relative excess in demand for skilled labor, which results in raising the relative skilled worker wage. Now I confirm that the equilibrium variables \(\theta^*\) and \(s/w\) are determined uniquely.

![Graph](image)

**Figure 9: Free Entry and Labor Market Equilibrium Curves in Autarky**

Figure 9 illustrates the existence of the equilibrium skill intensity cut-off, \(\theta^*\), and the skill premium, \(s/w\), at which the free entry condition (equation (10) and (13)) and the labor market clearing condition (equation (19)) are intercepted. For Figure 9, I assume that all parameters are the same as noted in the numerical exercise (section 2.3.4). In addition, I assume that the relative abundance of skilled labor \(H/L\) is either 0.5 or 0.75 and that the fixed production cost, \(f\), is either 1 or 2.\(^{32}\)

As shown in Figure 9, the decline of fixed cost \(f\) shifts the free entry curve to the left and the increase in the relative abundance of skilled labor \(H/L\) shifts the labor market curve to the right.

---

\(^{32}\)As noted in the section for the numerical exercise, I assume that the skill intensity is normally distributed with \(\mu = 0.5\) and \(s.d = 0.15\) so that the skilled intensity \(\theta\) is distributed over \([0, 1]\).
These are all four possible equilibria, which depend on the value of $H/L$ and $f$. The equilibrium skill intensity cut-off and the skill premium, in each equilibrium, are also shown in Table 1.

A.2. Proof of Proposition 2.2

Conveniently, I rewrite the costly trade equilibrium conditions under the symmetric assumption as follows: First equation (A.3) links the equilibrium variables of interest ($s/w$, $\theta_d^*$ and $\theta_x^*$), which depend on trade costs. Equations (A.4) and (A.5) represent the free entry condition and labor market clearing condition respectively.

$$
\left(\frac{s}{w}\right)^{-\left(\theta_x^* - \theta_d^*\right)} \left(\frac{\theta_x^*}{\theta_d^*}\right)^{\frac{\alpha}{\sigma}} = \tau \left(\frac{f_s}{f}\right)^{\frac{s}{S}}. \tag{A.3}
$$

Equations (A.4) and (A.5) represent the free entry condition and labor market clearing condition respectively.

$$
FE(s/w, \theta_d^*) = [1 - G(\theta_d^*)] \left[ \left(\frac{s}{w}\right)^{(\tilde{\theta}_d(\theta_d^*) - \theta_d^*)^{(1-\sigma)}} \left(\frac{\tilde{\theta}_d(\theta_d^*)}{\theta_d^*}\right)^{\frac{\alpha(\sigma-1)}{\sigma}} - 1 \right] + \left[1 - G(\theta_x^*)\right] f_s \left[ \left(\frac{s}{w}\right)^{(\tilde{\theta}_x(\theta_x^*) - \theta_x^*)^{(1-\sigma)}} \left(\frac{\tilde{\theta}_x(\theta_x^*)}{\theta_x^*}\right)^{\frac{\alpha(\sigma-1)}{\sigma}} - 1 \right] - \delta f_e. \tag{A.4}
$$

$$
LE(s/w, \theta_d^*) = \frac{\left(\frac{w}{\tau}\right) \left[ \int_{\theta_d^*}^{1} \left(\frac{s}{w}\right)^{(1-\sigma)} \theta^\frac{\alpha(\sigma-1)}{\sigma} \theta g(\theta) d\theta + \tau^{-\sigma} \int_{\theta_x^*}^{1} \left(\frac{s}{w}\right)^{(1-\sigma)} \theta^\frac{\alpha(\sigma-1)}{\sigma} \theta g(\theta) d\theta \right]}{\int_{\theta_d^*}^{1} \left(\frac{s}{w}\right)^{(1-\sigma)} \theta^\frac{\alpha(\sigma-1)}{\sigma} (1 - \theta) g(\theta) d\theta + \tau^{-\sigma} \int_{\theta_x^*}^{1} \left(\frac{s}{w}\right)^{(1-\sigma)} \theta^\frac{\alpha(\sigma-1)}{\sigma} (1 - \theta) g(\theta) d\theta} - \frac{H}{L}. \tag{A.5}
$$

where the weighted average skill intensity for each market, $\tilde{\theta}_d(\theta_d^*)$ and $\tilde{\theta}_x(\theta_x^*)$ are given by equations (27). Note that the export skill intensity cut-off is a function of both the skill premium and the zero-profit skill intensity cut-off, that is, $\theta_x^*(s/w, \theta_d^*)$ given by equation (A.3). Based on equation (A.3), it can easily be shown that $\frac{\partial \theta_x^*}{\partial (s/w)} > 0$ and $\frac{\partial \theta_x^*}{\partial \theta_d^*} > 0$.

In the first step, I show that the free entry condition given by equation (A.4) has a downward slope in the $(s/w, \theta_d^*)$ space using the implicit function theorem: $\frac{d(\frac{s}{w})}{d\theta_d^*} = -\frac{\partial FE(\frac{s}{w}, \theta_d^*)/\partial \theta_d^*}{\partial FE(\frac{s}{w}, \theta_d^*)/\partial (\frac{s}{w})} < 0$. A
tedious amount of manipulation gives
\[
\frac{\partial F(s/w, \theta^*_d)}{\partial s/w} = \left[1 - G(\theta^*_d)\right]f(1 - \sigma)\left[\bar{\theta}_d(\theta^*_d) - \theta^*_d\right] \frac{(\bar{\theta}_d(\theta^*_d) - \theta^*_d)}{\theta^*_d} \left(\frac{\theta^*_d}{\theta^*_d}\right)^{\frac{a(\sigma-1)}{\theta^*_d}} \\
- g(\theta^*_d) \frac{\partial \theta^*_d}{\partial (s/w)} f_x \left[\frac{(s/w)}{\theta^*_d} \left(\frac{(\bar{\theta}_d(\theta^*_d) - \theta^*_d)/(1 - \sigma)}{(\theta^*_d)}\right) \frac{a(\sigma-1)}{\theta^*_d} - 1\right] \\
+ \psi(s/w, \theta^*_x) \left[\theta^*_d(\theta^*_x) \frac{\partial \theta^*_d}{\partial (s/w)} f_x \left[\theta^*_d(\theta^*_x) \frac{\partial \theta^*_d}{\partial (s/w)} f_x \left[\frac{(s/w)}{\theta^*_d} \left(\frac{\theta^*_d}{\theta^*_d}\right)^{\frac{a(\sigma-1)}{\theta^*_d}} - 1\right]\right] - \frac{\alpha}{\theta^*_d} - \ln \left(\frac{s/w}{\theta^*_d}\right) \phi \right] < 0,
\]

where \(\psi(s/w, \theta^*_x) = [1 - G(\theta^*_x)]f_x \left(\frac{s}{\theta^*_x}\right) \left(\frac{(\bar{\theta}_d(\theta^*_x) - \theta^*_x)/(1 - \sigma)}{(\theta^*_x)}\right) \frac{a(\sigma-1)}{\theta^*_x}\) is positive. Note that \(\phi > 1, \sigma > 1, \alpha > 0, \) and \(\bar{\theta}_d(\theta^*_x) < 0.\) The inequality, \(\frac{\partial F(s/w, \theta^*_d)}{\partial (s/w)} < 0,\) holds, due to the assumption of \(\alpha > \ln(\frac{s}{\theta^*_d})\phi.\)

\[
\frac{\partial F(s/w, \theta^*_d)}{\partial \theta^*_d} = - g(\theta^*_d) f_x \left[\frac{(s/w)}{\theta^*_d} \left(\frac{(\bar{\theta}_d(\theta^*_d) - \theta^*_d)/(1 - \sigma)}{(\theta^*_d)}\right) \frac{a(\sigma-1)}{\theta^*_d} - 1\right] + \\
\nu(s/w, \theta^*_x) \left[\theta^*_d(\theta^*_x) \frac{\partial \theta^*_d}{\partial \theta^*_x} f_x \left[\frac{(s/w)}{\theta^*_d} \left(\frac{\theta^*_d}{\theta^*_d}\right)^{\frac{a(\sigma-1)}{\theta^*_x}} - 1\right]\right] - \frac{\alpha}{\theta^*_d} - \ln \left(\frac{s/w}{\theta^*_d}\right) \phi \right] < 0,
\]

where \(\nu(s/w, \theta^*_x)\) and \(\psi(s/w, \theta^*_x)\) are both positive. Since \(\frac{\partial \theta^*_d}{\partial \theta^*_d} > 0\) and \(\alpha > \ln(\frac{s}{\theta^*_d})\phi,\) \(\frac{\partial F(s/w, \theta^*_d)}{\partial \theta^*_d} < 0.\) By the implicit function theorem, the free entry condition implies a negative relationship between \(\theta^*_d\) and \(s/w,\) that is, \(\frac{d(\theta^*_d)}{d(\theta^*_d)} = - \frac{\partial F(s/w, \theta^*_d)}{\partial \theta^*_d}/\frac{\partial \theta^*_d}{\partial (s/w)} < 0.\) In addition, applying the same logic in the proof of the proposition \(2.1, s/w \to \infty\) as \(\theta^*_d\) approaches to zero, while \(s/w\) approaches to zero as \(\theta^*_d\) goes to one.

\[\nu(s/w, \theta^*_x) = \left(\frac{\sigma-1}{\theta^*_x}\right) [1 - G(\theta^*_x)] f_x \left(\frac{s}{\theta^*_x}\right) \left(\frac{(\bar{\theta}_d(\theta^*_x) - \theta^*_x)/(1 - \sigma)}{(\theta^*_x)}\right) \frac{a(\sigma-1)}{\theta^*_x}\] and \(\psi(s/w, \theta^*_x) = \left(\frac{\sigma-1}{\theta^*_x}\right) [1 - G(\theta^*_x)] f_x \left(\frac{s}{\theta^*_x}\right) \left(\frac{(\bar{\theta}_d(\theta^*_x) - \theta^*_x)/(1 - \sigma)}{(\theta^*_x)}\right) \frac{a(\sigma-1)}{\theta^*_x}.
\]
Second, to complete the proof, I need to show a positive relationship between $\theta^*_d$ and $s/w$. Applying the same logic presented in the proof of proposition 2.1, the equation (A.5) can be rewritten as:

$$\left(\frac{w}{s}\right)\left[\int_{\theta^*_d}^{1} \left(\frac{\theta}{1-\theta}\right) \Psi(\theta, \frac{s}{w}, \theta^*_d) g(\theta) d\theta + \tau^{-\sigma} \int_{\theta^*_d}^{1} \frac{\theta}{1-\theta} \Psi(\theta, \frac{s}{w}, \theta^*_d) g(\theta) d\theta\right] = \frac{H}{L}.$$

Now it can be easily shown that the zero-profit skill intensity $\theta^*_d$ is positively associated with the skill premium $s/w$ on the $(\theta^*_d, s/w)$ space using the same logic as in the proof of proposition 2.1. Thus equilibrium exists and is unique in the costly trade between symmetric countries. ■

A.3. Proof of Proposition 2.3

Now, I prove that the skill premium $s/w$ increases as trade costs (i.e., $\tau$ and/or $f_x$) fall. To do this, I examine the shifts in each $FE$ and $LE$ curve in response to a reduction in trade costs. Without any loss of generality, I compare two curves in the autarky regime with ones in the costly trade condition. In this way, one can analyze the effect of lowering trade costs on the skill premium. First, I compare the free entry condition under autarky with the free entry condition with costly trade. Conveniently, I rewrite the free entry equation in costly trade, which is from equation (28).

$$(1 - G(\theta^*_d)) f \left[ \left(\frac{s}{w}\right) \left(\frac{\hat{\theta}_d - \theta^*_d}{\hat{\theta}_d}\right)^{(1-\sigma)} \left(\frac{\hat{\theta}_d}{\theta^*_d}\right)^{\frac{\alpha(\sigma-1)}{\phi}} - 1 \right] +

(1 - G(\theta^*_x)) f_x \left[ \left(\frac{s}{w}\right) \left(\frac{\hat{\theta}_x - \theta^*_x}{\hat{\theta}_x}\right)^{(1-\sigma)} \left(\frac{\hat{\theta}_x}{\theta^*_x}\right)^{\frac{\alpha(\sigma-1)}{\phi}} - 1 \right] = \delta f_e. \quad (A.6)$$

Note that equation (A.6) reduces to $$(1 - G(\theta^*_d)) f \left[ \left(\frac{s}{w}\right) \left(\frac{\hat{\theta}_d - \theta^*_d}{\hat{\theta}_d}\right)^{(1-\sigma)} \left(\frac{\hat{\theta}_d}{\theta^*_d}\right)^{\frac{\alpha(\sigma-1)}{\phi}} - 1 \right] = \delta f_e,$$

which is the first term on the left-hand side of equation (A.6), in the closed economy. When trade costs are low enough for some firms to engage in exporting, the first term in the left-hand side of equation A.6 must be smaller than the one in autarky, because the second term in the left-hand side is positive and the right-hand side of equation A.6, $\delta f_e$, does not depend on trade costs. Notice
that the first term in the left-hand side of equation (A.6) will decrease as $s/w$ increases, while $\theta_d^*$ is fixed. Thus, the free entry curve ($FE$) will shift upward (that is, shifts to the right) as trade costs fall.

Second, the upward-sloping labor market curve ($LE$) also shifts upward (that is, shifts to the left) as a result of the reduction in trade costs. For convenience, I rewrite the labor market equilibrium condition under costly trade that is given in the proof of proposition 2.2:

$$
\left(\frac{w}{s}\right) \left[ \int_{\theta_d^*}^{1} \left( \frac{\theta}{1-\theta} \right) \Psi(\theta, \frac{s}{w}, \theta_d^*) g(\theta) d\theta + \tau^{-\sigma} \int_{\theta^*_d}^{1} \left( \frac{\theta}{1-\theta} \right) \Psi(\theta, \frac{s}{w}, \theta_d^*) g(\theta) d\theta \right] = \frac{H}{L},
$$

where $\Psi(\theta, \frac{s}{w}, \theta_d^*)$ denotes the share of unskilled workers employed by firms with $\theta$ out of the total endowment of unskilled labor. Now consider either the movement from autarky to costly trade or the reduction of trade costs under the condition of costly trade. Note that autarky regime reduces equation (A.7) to

$$
\left(\frac{w}{s}\right) \left[ \int_{\theta_d^*}^{1} \left( \frac{\theta}{1-\theta} \right) \Psi(\theta, \frac{s}{w}, \theta_d^*) g(\theta) d\theta \right] = \frac{H}{L}.\quad (A.7)
$$

Either the movement from autarky to the costly trade or the reduction of trade costs increases the second integral in the left-hand side of equation (A.7). Thus $\left(\frac{w}{s}\right) \left[ \int_{\theta_d^*}^{1} \left( \frac{\theta}{1-\theta} \right) \Psi(\theta, \frac{s}{w}, \theta_d^*) g(\theta) d\theta \right]$ must be decreased with lowering trade costs because the right hand side of equation (A.7), $H/L$, is constant. Holding $\theta_d^*$ fixed, the skill premium $s/w$ must increase in order to keep the left hand side of equation (A.7) constant because of the definition of $\Psi(\theta, \frac{s}{w}, \theta_d^*)$: as the relative price of a skilled worker increases, $\Psi(\theta, \frac{s}{w}, \theta_d^*)$ decreases. As a result, the $LE$ curve also shifts up, as trade costs fall. Since both the $FE$ and $LE$ curves shift upward the relative wage of skilled labor $s/w$ will be higher when trade costs fall.

Figure 10 illustrates that the skill premium increases as trade costs fall. When trade costs decrease, both the free trade $FE$ and the labor market curve, $LE$, shift upward, which results in increasing wage inequality, $s/w$. 

\[\blacksquare\]
Figure 10: FE and LE curves in autarky and costly trade