A Simple Model of Industrial Pollution

Environmental Economics
Econ 8545

Edward Morey

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I. The Basic Model

Consider a simple model in which the smoke produced by one industry imposes an externality on another industry.

Definition of Externality:

> There is an externality if an economic agent(s) does something that directly influences (not indirectly through market prices) some other economic agent(s), but the agent that produced the effect does not have the correct incentive to take the effect into account because there is no requirement, incentive, or penalty in place that causes that agent to fully account for the effect.

There are, for example, firm-firm externalities, firm-consumer externalities, consumer-firm externalities, consumer-consumer externalities and spillover-type externalities. I'll only consider firm-firm externalities.

Consider a world where smoke is produced in conjunction with beer and where this smoke adversely affects the production of flowers. In this context, consider policies for eliminating the inefficiency resulting from this externality.

Assume:

(A1) Three goods: beer, flowers and wheat

(A2) One homogeneous input: labor

(A3) A fixed supply of labor ≡ L

(A4) Smoke is a by- product of beer production but not flower or wheat production.

(A5) The smoke produced adversely affects the flower industry but not the wheat industry.

(A6) All the firms in a given industry (beer, flowers or wheat) have the same technology.

(A7) Each industry is competitive (i.e., each firm is a price taker for both its output and labor).

(A8) Now assume that the beer and flower firm are spatially configured such that each flower producer is subjected to s smoke. This assumption, while strong, is necessary if we are to consider the externality in terms of "representative" firms rather than on a firm by firm basis. (This assumption does not hold in the real world, and this has major implications for the efficient regulation of pollution)

Let $P_B \equiv$ price of beer

$P_F \equiv$ price of flowers

$P_w \equiv$ price of wheat

$p \equiv$ price of labor.

The equilibrium prices are those that equate supply and demand in all four markets (beer, flowers, wheat, and labor). Since only relative prices are important, normalize p = 1. Assumptions (4), (6), and (7) imply that each beer firm will produce the same amount of beer and smoke, and hire the same amount of labor..

Let $b \equiv$ amount of beer produced by a representative beer firm,

$s \equiv$ amount of smoke produced by a representative beer firm,

$l_b \equiv$ amount of labor allocated to beer production by the representative beer firm.

Assumptions (6), (7) and (8) imply that each flower firm will produce the same amount of flowers.

Let $f \equiv$ amount of flowers produced by a representative flower firm,

$l_f \equiv$ amount of labor used by a representative flower firm.

Assumptions (2), (5), (6) and (7) imply that each wheat firm will use the same amount of labor and produce the same amount of wheat.

Let $l_w \equiv$ amount of labor used by the representative wheat farmer,

$w \equiv$ amount of wheat produced by the representative wheat farmer.

Either there is, or isn't, smoke abatement technology in the beer industry.

(A9a) If no abatement technology exists in the beer industry,

$$s = s(b) \quad where \quad \frac{\partial s}{\partial b} > 0$$

Or,

(A9b) If abatement technology exists in the beer industry,

$$s = s(b, l_s) \quad where \quad \frac{\partial s}{\partial b} > 0, \; \frac{\partial s}{\partial l_s} < 0$$

where $l_s \equiv$ amount of labor allocated to smoke abatement by the representative beer firm.

Both cases 9a) and 9b) will be considered.

(A10) Either there is, or isn't smoke abatement technology in the flower industry. If there is, each flower firm can reduce the amount of smoke it is subjected to by allocating labor to smoke abatement. Rather than assuming abatement technology exists in the flower industry, assume the simpler case of no abatement capabilities in the flower industry.

Adopting assumptions (1) - (10) and a few more technical restrictions, the four relevant production functions for the three representative firms are

(1) $$b = b(l_b) \quad \frac{\partial b}{\partial l_b} > 0, \ \frac{\partial^2 b}{\partial l_b^2} < 0$$

(2) $$f = f(l_f, s) \quad \frac{\partial f}{\partial l_f} > 0, \ \frac{\partial^2 f}{\partial l_f^2} < 0 \quad and \quad \frac{\partial f}{\partial s} < 0$$

(3) $$w = w(l_w) \quad \frac{\partial w}{\partial l_w} > 0, \ \frac{\partial^2 w}{\partial l_w^2} < 0$$

and

(4a) $$s = s(b, l_s) \quad \frac{\partial s}{\partial b} > 0, \ \frac{\partial s}{\partial l_s} < 0$$

or

(4b) $$s = s(b) \quad \frac{\partial s}{\partial b} > 0$$

Assumptions (1) - (10) comprise a three industry model of industrial pollution, where all the firms in a given industry behave identically. **One might wonder why it was necessary to include the wheat industry in the analysis. The danger associated with excluding it will become obvious when we consider whether the ineficiency caused by the externality can be eliminated by subsidizing the flower industry.**

Given that all the firms in a given industry behave identically, the aggregate production functions are

(5) $$B = B(L_B) = N_B \cdot b(l_b)$$

where B ≡ total beer production,

L_B ≡ total amount of labor allocated to beer production,

N_B ≡ # of beer firms.

(6) $$F = F(L_F) = N_F \cdot f(l_f, s)$$

where F ≡ total flower production,

L_F ≡ total labor allocated to flower production,

N_F ≡ # of flower firms.

(7) $$W = W(L_W) = N_W \cdot w(l_w)$$

where W ≡ total wheat production,

L_W ≡ total labor allocated to wheat production,

N_W ≡ # of wheat firms.

and

(8a) $$S = S(B,\ L_S) = N_B \cdot s(b, l_s)$$

or

(8b) $$S = S(B) = N_B \cdot s(b)$$

where S ≡ total amount of smoke produced,

L_S ≡ total amount of labor allocated to smoke abatement.

The math will be simplified if we adopt the fiction that each industry's output is produced by just one firm where this firm is forced to behave as a price taker. The advantage of this fiction is that, with it, the analysis can be carried out in terms of the aggregate production functions and the aggregate allocation of labor. This fiction is adopted without loss of generality because all the firms in each industry behave identically. Note that this fiction would be inappropriate without assumption (8).

To complete the model, also assume that

(A11) There is a well behaved utility function for society.

$$(9) \qquad U = U(B,F,W) \quad \frac{\partial U}{\partial B} > 0,\ \frac{\partial^2 U}{\partial B^2} < 0,\ \frac{\partial U}{\partial F} > 0,\ \frac{\partial^2 U}{\partial F^2} < 0,\ \frac{\partial U}{\partial W} > 0,\ \frac{\partial^2 U}{\partial W^2} < 0$$

Equation (9) can be viewed as either a social welfare function or as the utility function for individual i, holding everyone else's utility level constant at zero. Implicit in equation (9) is the assumption that smoke does not enter as an exogenous variable in the direct utility functions.

If one adopts the fiction that society consists of just one individual, there will be no distinction between the efficient allocation and the socially optimal allocation.

(A12) There are no other distortions in the economy. This assumption allows us to ignore "Second-Best" issues. This assumption will be relaxed in a later section.

II. Conditions for Efficient Allocation

Define efficiency for the case where S = S(B, L_S). One efficient allocation is the allocation of L that maximizes the society's utility subject to the constraints of technology of production and smoke generation, and the total availability of labor.

max U(B,F,W)

subject to

$$B = B(L_B),\ F = F(L_F, S),\ W = W(L_W),\ S = S(B, L_S) \text{ and } L - L_B - L_F - L_W - L_S = 0$$

Forming the Lagrangian

$$\mathcal{L} = U\left\{B(L_B),\ F\left[L_F, S(B(L_B), L_S)\right],\ W(L_W)\right\} + \mu(L - L_B - L_F - L_W - L_S)$$

The first order conditions for efficiency are

(10) $$\frac{\partial \mathcal{L}}{\partial L_B} = \frac{\partial U}{\partial B}\frac{\partial B}{\partial L_B} + \frac{\partial U}{\partial F}\frac{\partial F}{\partial S}\frac{\partial S}{\partial B}\frac{\partial B}{\partial L_B} - \mu = 0$$

(11) $$\frac{\partial \mathcal{L}}{\partial L_F} = \frac{\partial U}{\partial F}\frac{\partial F}{\partial L_F} - \mu = 0$$

(12) $$\frac{\partial \mathcal{L}}{\partial L_W} = \frac{\partial U}{\partial W}\frac{\partial W}{\partial L_W} - \mu = 0$$

(13) $$\frac{\partial \mathcal{L}}{\partial L_S} = \frac{\partial U}{\partial F} \frac{\partial F}{\partial S} \frac{\partial S}{\partial L_S} - \mu = 0$$

and

(14) $$\frac{\partial \mathcal{L}}{\partial \mu} = L - L_B - L_F - L_W - L_S = 0$$

Given the properties of the utility function and the production functions, the sufficient conditions for a maximization are fulfilled. Assume there are no corner solutions and that the externality does not produce a nonconvexity.

Note that the efficiency requires full employment (equation (14)) and that $L_S > 0$ (equation (13)); that is, the efficiency requires that a positive amount of labor be allocated to smoke abatement.

The P.O. conditions can be rearranaged into a form that makes them more comparable to the P.O. conditions in a distortion-free world.

Setting (10) = (13) and solving for $\left(\frac{\partial U}{\partial B}\right) \Big/ \left(\frac{\partial U}{\partial F}\right)$ one obtains

(15) $$MRS_{BF} \equiv \left(\frac{\partial U}{\partial B}\right) \Big/ \left(\frac{\partial U}{\partial F}\right) = \frac{\left(\frac{\partial F}{\partial S}\right)\left(\frac{\partial S}{\partial L_S}\right)}{\frac{\partial B}{\partial L_B}} - \frac{\partial F}{\partial S}\frac{\partial S}{\partial B}$$

From (11) and (13) one obtains the result that the efficiency requires

(16) $$MRTS_{SL_F} \equiv \left(\frac{\partial F}{\partial S}\right) \Big/ \frac{\partial F}{\partial L_F} = 1 \Big/ \left(\frac{\partial S}{\partial L_S}\right)$$

Solve (16) for $\left(\frac{\partial F}{\partial L_F}\right)$ to obtain $\frac{\partial F}{\partial L_F} = \frac{\partial F}{\partial S}\frac{\partial S}{\partial L_S}$ Rearranging (16), one sees that the efficiency requires

$$\begin{bmatrix} \textit{the marginal product} \\ \textit{of labor allocated to flower} \\ \textit{production, in the} \\ \textit{production of flowers} \end{bmatrix} \equiv \frac{\partial F}{\partial L_F} = \frac{\partial F}{\partial S}\frac{\partial S}{\partial L_S} \equiv \begin{bmatrix} \textit{the marginal product of} \\ \textit{labor allocated to smoke} \\ \textit{abatement, in the} \\ \textit{production of flowers} \end{bmatrix}$$

Substitute $\left(\frac{\partial F}{\partial L_F}\right)$ for $\left(\frac{\partial F}{\partial S}\right)\left(\frac{\partial S}{\partial L_S}\right)$ in (15) to obtain the result that the efficiency requires

$$(17) \qquad MRS_{BF} \equiv \frac{\partial U}{\partial B}\bigg/\frac{\partial U}{\partial F} = \left[\frac{\partial F}{\partial L_F}\bigg/\frac{\partial B}{\partial L_B}\right] - \frac{\partial F}{\partial S}\,\frac{\partial S}{\partial B} \equiv MRT^{s}_{BF}$$

where MRT^{s}_{BF} is the marginal rate of transformation between beer and flowers from the perspective of society.

From (11) and (12), one obtains the standard result that the efficiency requires

$$(18) \qquad MRS_{WF} \equiv \frac{\partial U}{\partial W}\bigg/\frac{\partial U}{\partial F} = \left[\frac{\partial F}{\partial L_F}\bigg/\frac{\partial W}{\partial L_W}\right] \equiv MRT_{WF}$$

The allocation will be efficient when equations (16) − (18) and (14) are fulfilled. Summarizing,

$$MRTS_{SL_F} \equiv \frac{\partial F}{\partial S}\bigg/\frac{\partial F}{\partial L_F} = 1\bigg/\frac{\partial S}{\partial L_S} \qquad (16)$$

$$MRS_{BF} \equiv \frac{\partial U}{\partial B} \Big/ \frac{\partial U}{\partial F} = \left[\frac{\partial F}{\partial L_F} \Big/ \frac{\partial B}{\partial L_B} \right] - \frac{\partial F}{\partial S} \frac{\partial S}{\partial B} \equiv MRT^{s}_{BF} \tag{17}$$

$$MRS_{WF} \equiv \frac{\partial U}{\partial W} \Big/ \frac{\partial U}{\partial F} = \left[\frac{\partial F}{\partial L_F} \Big/ \frac{\partial W}{\partial L_W} \right] \equiv MRT_{WF} \tag{18}$$

and

$$L - L_B - L_F - L_W - L_S = 0 \tag{14}$$

We will use these conditions to determine whether a number of different forms of economic organization (e.g. pure competition with a tax on beer, etc) will achieve efficiency. When checking, we won't need to worry about the labor market clearing (equation (14)). The competitive assumption of prices adjusting until S = D in all four markets guarantees this result. (Do we need to worry about market clearing if we are solving for a pigovian tax in a specific example?)

In closing, note that if U(B,F,W) is viewed as the social welfare function, the allocation of labor that

max $U(B,F,W)$

subject to

$$B = B(L_B), \quad F = F(L_S,S), \quad W = W(L_W), \quad S = S(B,L_S)$$
$$and\ L - L_B - L_F - L_W - L_S = 0$$

is the allocation that maximizes social welfare; i.e. it is both efficient and equitable. Denote this allocation by (L_B^*, L_F^*, L_W^*, L_S^*). If S = S(B, L_S), L_B^*, L_F^*, L_W^*, and L_S^* can be obtained as the solution to equations (14), (16), (17) and (18). This allocation is unique. More generally, there are an infinite number of efficient allocations, each of which is the solution to a member of the class problems.

$$\max\ u^i(b^i,f^i,w^i)\ s.t.\ \bar{u}^j(b^j,f^j,w^j) = 0 \quad j = 1,2,..i-1,i+1,...,N.$$
$$B = B(L_B),\quad F = F(L_S,S),\quad W = W(L_W),\quad S = S(B,L_S)$$
$$L - L_B - L_F - L_W - L_S = 0,\ \sum_{j=1}^{N} b^j = B,\ \sum_{j=1}^{N} f^j = F\ and\ \sum_{j=1}^{N} w^j = W$$

where, for example, f^i is the quantity of flowers consumed by individual i.

Every vector $(\bar{u}^1, \bar{u}^2, ...\bar{u}^{i-1}, \bar{u}^{i+1}, ...\bar{u}^i)$ generates its own unique efficient allocation of labor. Denote $(\hat{L}_B, \hat{L}_F, \hat{L}_W, \hat{L}_S)$ as an arbitrarily chosen member of this set of efficient allocations. One member of this set is $(L_B^*, L_F^*, L_W^*, L_S^*)$.

After this point, I will not distinguish between the efficient and optimal allocation and use the * to denote the efficient/optimal allocation. There is no difference if u is the SWF or if there is just one individual.

Digression:

1. Before proceeding to check whether pure competition will achieve efficiency when the external smoke effect exists and $S = S(B, L_S)$, note that if $S = S(B)$ i.e. no abatement technology exists then the efficiency conditions are equations (17), (18) and (14) but not equation (16). Equation (16) involves L_S and L_S doesn't exist if $S = S(B)$.

2. If there is no external effect, then either

$$\frac{\partial F}{\partial S} = 0$$ (smoke does not affect flower production)

or

$$\frac{\partial S}{\partial B} = 0$$ (the amount of smoke received by the flower industry is independent of what happens in the beer industry implies that either there is no smoke or the smoke is an act of nature)

or both.

In which case the efficiency conditions are equations (18), (14) with L_s=0 and

(19) $$MRS_{BF} \equiv \frac{\partial U}{\partial B} \bigg/ \frac{\partial U}{\partial F} = \left[\frac{\partial F}{\partial L_F} \bigg/ \frac{\partial B}{\partial L_B} \right] \equiv MRT_{BF}^{s\ p}$$ *if there is no externality*

Compare equations (19) and (17). They are identical except for the externality term

$$\left[-\frac{\partial F}{\partial S}\,\frac{\partial S}{\partial B} \right]$$

III. Competitive Equilibrium and Efficiency

Assume no government intervention, that there is no mergers between beer firms and flower firms, and that flower firms do not bribe beer firms to reduce their smoke production. These assumptions, while restrictive, still admit many real world externalities. Bribery will be considered in Section IV, merger in section V, and government tax intervention in section VI. **In the absence of these effects, competition will not achieve efficiency if an external production effect exists; that is, the market will fail.** This can be demonstrated as follows:

Utility maximization implies that in competitive equilibrium

(20) $$MRS_{BF} \equiv \frac{\partial U}{\partial B} \Big/ \frac{\partial U}{\partial F} = \frac{P_B}{P_F}$$

and that

(21) $$MRS_{WF} \equiv \frac{\partial U}{\partial W} \Big/ \frac{\partial U}{\partial F} = \frac{P_W}{P_F}$$

In competitive equilibrium, each firm will be maximizing its profits. If π_B is the profits in the beer industry, $\pi_B = P_B B(L_B) - L_B$ (remember that p = 1).

Profits in the beer industry will be maximized when

(22) $$P_B = 1 \Big/ \frac{\partial B}{\partial L_B} \equiv MPC_B$$

For the flower and wheat industry, profit maximization implies

(23) $$P_F = 1 \Big/ \frac{\partial F}{\partial L_F} \equiv MPC_F$$

and

(24) $$P_W = 1 \Big/ \frac{\partial W}{\partial L_W} \equiv MPC_W$$

From (22) and (23), one obtains the result that in competitive equilibrium

(25) $$\frac{P_B}{P_F} = \frac{\left[1 \Big/ \frac{\partial B}{\partial L_B}\right]}{\left[1 \Big/ \frac{\partial F}{\partial L_F}\right]} = \left(\frac{\partial F}{\partial L_F} \Big/ \frac{\partial B}{\partial L_B} \right) \equiv MRT_{BF}^{p}$$

where MRT_{BF}^{p} is the marginal rate of transformation between beer and flower production from the perspective of an unregulated beer industry. The p superscript denotes “private”.

Combining (20) and (25), it follows that in competitive equilibrium

$$(26) \qquad MRS_{BF} \equiv \left(\frac{\partial U}{\partial B}\right) \Big/ \left(\frac{\partial U}{\partial F}\right) = \left(\frac{\partial F}{\partial L_F}\right) \Big/ \left(\frac{\partial B}{\partial L_B}\right) \equiv MRT_{BF}^{p}$$

Equation (26) is inconsistent with equation (17) and equation (17) is a necessary condition for efficiency; i.e. the market will fail because the competitive system will not account for the external effect.

Note that the market fails independent of whether $S = S(B, L_S)$ or $S = S(B)$. Also note that if the external effect is absent ($\partial F/\partial S=0$, or $\partial S/\partial B=0$, or both), it is straightforward to prove that pure competition will achieve efficiency. With no external effects, efficiency is fulfilled if equations (14) (18) and (19) hold. Equation (26) implies the fulfillment of (19). Equations (21), (23) and (24) imply the fulfillment of (18). Market clearing will fulfill equation (14). Summarizing, **the existence of the production external effect causes the competitive allocation to be inefficient.**

I proceed by first addressing the issue of whether there is any potential for this economic system to correct itself without government intervention, and then address the issue of eliminating the inefficiency with government tax policy.

IV. Bribery

Section III demonstrated that the competitive equilibrium (in the absense of bribes, mergers or government intervention) will be inefficient. This raises two questions with respect to bribes:

1. Does the flower industry have an incentive to bribe the beer industry to reduce the beer industry's output of smoke?

   And,

2. Can the inefficiency caused by the external effect be eliminated by such a bribe?

Let's address the second question first.

Denote

(27) $TB = M - \beta \cdot S\,(B(L_B), L_S) \qquad \beta \geq 0$

where

TB ≡ the total bribe (in \$) paid by the flower industry,

M ≡ the amount paid by the flower industry to the beer industry if $S = 0$,

and

β ≡ how much the bribe decreases everytime smoke production increases by one unit; i.e. β is the per-unit bribe.

M and β are obviously choice variables for the flower industry but let's momentarily adopt the fiction that

M and β are imposed exogenously. Given this, is there an exogenous β that will cause competitive equilibrium, modified by the bribe, to be efficient? The answer is yes and can be demonstrated as follows. Given M and β, profits in the flower industry are

(28) $\pi_F = P_F \cdot F(L_F, S(B(L_B), L_S)) - L_F - M + \beta \cdot S(B(L_B), L_S)$

and the necessary condition for π maximization is

(29) $$\frac{\partial \pi_F}{\partial L_F} = P_F \cdot \frac{\partial F}{\partial L_F} - 1 = 0 \quad \Rightarrow \quad P_F = 1 \Big/ \frac{\partial F}{\partial L_F}$$

The wheat industry is unaffected by the bribe so profit maximization in the wheat industry implies

(30) $$P_W = 1 \Big/ \frac{\partial W}{\partial L_W} = MPC_W$$

In the beer industry, profits are

(31) $\pi_B = P_B \cdot B(L_B) - L_B - L_S + M - \beta \cdot S(B(L_B), L_S)$

and the necessary conditions for profit maximization are

(32) $$\frac{\partial \pi_B}{\partial L_B} = P_B \cdot \frac{\partial B}{\partial L_B} - 1 - \beta \frac{\partial S}{\partial B} \frac{\partial B}{\partial L_B} = 0$$

and

(33) $$\frac{\partial \pi_B}{\partial L_S} = -1 - \beta \frac{\partial S}{\partial L_S} = 0$$

In the bribery case, the beer industry can be thought of as selling two products: beer and smoke reduction. Remembering that efficiency requires

$$\frac{\partial U}{\partial B} \Big/ \frac{\partial U}{\partial F} = \left[\frac{\partial F}{\partial L_F} \Big/ \frac{\partial B}{\partial L_B} \right] - \frac{\partial F}{\partial S} \frac{\partial S}{\partial B} \qquad (17)$$

and that in competitive equilibrium (even with the bribe)

$$MRS_{BF} = \frac{\partial U}{\partial B} \Big/ \frac{\partial U}{\partial F} = \frac{P_B}{P_F}, \qquad (20)$$

the equilibrium, with the bribe, will not be efficient unless (substituting (20) into (17))

(34) $$\frac{P_B}{P_F} = \left[\frac{\partial F}{\partial L_F} \Big/ \frac{\partial B}{\partial L_B} \right] - \frac{\partial F}{\partial S} \frac{\partial S}{\partial B}$$

In addition, we know that in equilibrium

$$P_F = 1 \Big/ \frac{\partial F}{\partial L_F} \tag{29}$$

which is a necessary condition for profit maximization in the flower industry.

Substituting (29) into (34) and solving for P_B, one obtains

(35) $$P_B^* = \left[1 \Big/ \frac{\partial B}{\partial L_B}\right] - \left[\frac{\partial F}{\partial S} \Big/ \frac{\partial F}{\partial L_F}\right] \frac{\partial S}{\partial B}$$

Therefore, efficiency condition (17) will be fulfilled if $P_B = P_B^*$. One can solve for the β that will achieve this by substituting (35) into (32). Solving for β^* one obtains

(36) $$\beta^* = - \frac{\left(\dfrac{(\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial S}\right)}{\left(\dfrac{(\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial L_F}\right)} = - MRTS_{SL_F}^F (L_B^* L_S^*, L_F^*) > 0$$

where (L_B^*, L_F^*, L_W^* and L_S^*) is the efficienct allocation of labor (for more details see page 14).

β^* also implies that the efficiency condition (16) is achieved in equilibrium; i.e, (16) can be obtained by plugging (36) into (33) and solving for $1 \Big/ \frac{\partial S}{\partial L_S}$

Since β^* does not enter into equation (29) or (30), efficiency condition (18) will also hold in equilibrium. **In words, attainment of efficiency at the optimal allocation, L_B^*, L_F^*, L_W^*, L_S^* requires that the per-unit bribe, β, is set equal to minus the marginal rate of technical substitution between smoke and labor in the production of flowers evaluated at the optimal allocation of labor.**

Summarizing this first part of section IV, the answer to the second question (page 19) is yes, **the inefficiency caused by the external effect can be eliminated with an appropriately chosen bribe rate.**

Now let's address the first question in two parts. Does the flower industry have an incentive to bribe the beer industry to reduce their output of smoke? And if so, will the bribe they choose eliminate the inefficiency caused by the external effect?

The answer to the first part of this two-part question is yes. Assume that the flower industry knows the beer industry's technology for producing beer and smoke. Given this, they can determine the beer industry's supply function for smoke.

(37) $S = \varnothing(P_B, \beta)$

Given this supply function, profits in the flower industry are

(38) $\pi_F = P_F \cdot F(L_F, \varnothing(P_B, \beta)) - L_F - M + \beta \cdot \varnothing(P_B, \beta)$

and the necessary conditions for profit maximization in the flower industry are

$$\frac{\partial \pi_F}{\partial L_F} = P_F \cdot \frac{\partial F}{\partial L_F} - 1 = 0 \qquad \Rightarrow \qquad P_F = 1 \Big/ \frac{\partial F}{\partial L_F}$$

and

(39) $$\frac{\partial \pi_F}{\partial \beta} = P_F \cdot \frac{\partial F}{\partial S} \frac{\partial S}{\partial \beta} + \beta \cdot \frac{\partial S}{\partial \beta} + \varnothing(P_B, \beta) = 0$$

Note that β is now a choice variable for the flower industry.

Equation (39) tells us that the profits maximization in the flower industry requires $\beta \neq 0$, i.e. the flower industry has an incentive to bribe the beer industry to reduce their output of smoke. The first term in this equation is the additional revenue generated from the sales of flowers when β is increased by an incremental unit. The sum of the other two terms is the additional bribery costs incurred when β is increased by an incremental unit.

Denote $\tilde{\beta}$ as the market equilibrium profit maximizing β. Will $\tilde{\beta}$ eliminate the inefficiency caused by the externality? The answer is yes. One way to prove this is to show that β^* (equation 36) is the profit maximizing per-unit bribe $\tilde{\beta}$ in the flower industry. I leave the proof to the reader.

Summarizing, **there is a bribe that will eliminate the inefficiency, and the flower industry has an incentive to offer this bribe.** Given this result, one must ask why all producer-producer external effects are not "corrected" with bribes. In fact, very few seem to be corrected in this manner. Why?

The above analysis explicitly assumed that the smoke recipient has knowledge of the polluting industry's technology and implicitly assumed that the cost of negotiating the bribe and monitoring the beer industry's smoke output are both zero. These assumptions don't usually hold in the real world. The recipient industry does not know the other industry's supply function for smoke and the costs of negotiating and monitoring might be quite high.

Bribing the polluter to reduce pollution also implies the recipient industry's acceptance of the polluting industry's right to pollute. Offering a bribe therefore reduces their chances of successfully suing for damages. These factors, taken together, suggest that the inefficiency associated with the external effect are, often, not likely to be eliminated with a bribe.

V. Merger

The first result that I want to demonstrate in this section is that if beer and flower firms merge, such that all the smoke produced in a conglomorate only affects flower producers in that conglomerate, then the competitive equilibrium output will be efficient. Each of these beer/flower conglomerates is assumed to remain a price taker.

After the mergers, profits in the merged industry will be

(40) $\pi_{B+F} = P_B \cdot B(L_B) + P_F \cdot F(L_F, S(B(L_B), L_S)) - L_B - L_F - L_S$

and the necessary conditions for profit maximization are

(41) $$\frac{\partial \pi_{B+F}}{\partial L_B} = P_B \cdot \frac{\partial B}{\partial L_B} + P_F \frac{\partial F}{\partial S} \frac{\partial S}{\partial B} \frac{\partial B}{\partial L_B} - 1 = 0$$

(42) $$\frac{\partial \pi_{B+F}}{\partial L_F} = P_F \frac{\partial F}{\partial L_F} - 1 = 0$$

and

(43) $$\frac{\partial \pi_{B+F}}{\partial L_S} = P_F \frac{\partial F}{\partial S} \frac{\partial S}{\partial L_S} - 1 = 0$$

This merger does not effect the wheat industry so the necessary condition for profit maximization in the wheat industry remains:

$$P_W \cdot \frac{\partial W}{\partial L_W} - 1 = 0 \tag{24}$$

Equations (41) - (43) and (24) implies the efficiency conditions (16), (17) and (18); that is, i.e., **the competitive equilibrium with the mergers will be efficient.**

A demonstration of this result proceeds as follows. From (42) and (24), one obtains the result that in equilibrium

(44) $$\frac{P_W}{P_F} = \frac{\partial F}{\partial L_F} \Big/ \frac{\partial W}{\partial L_W} \equiv MRT_{WF}$$

Combining this with the result that in competitive equilibrium

$$MRS_{WF} = \frac{\partial U}{\partial W} \Big/ \frac{\partial U}{\partial F} = \frac{P_W}{P_F} \tag{21}$$

one obtains efficiency condition (18). Combining (42) and (43), one obtains

(45) $$P_F \frac{\partial F}{\partial L_F} - 1 = P_F \frac{\partial F}{\partial S} \frac{\partial S}{\partial L_S} - 1$$

Rearranging (45), one obtains efficiency condition (16).

Solving (41) for $\frac{P_B}{P_F}$ one obtains

(46) $$\frac{P_B}{P_F} = \frac{1}{P_F\left(\frac{\partial B}{\partial L_B}\right)} - \frac{\partial F}{\partial S}\frac{\partial S}{\partial B}$$

From (42), $P_F = 1 \Big/ \frac{\partial F}{\partial L_F}$, substituting this into (46) one obtains the result that in equilibrium

(47) $$\frac{P_B}{P_F} = \left[\frac{\partial F}{\partial L_F} \Big/ \frac{\partial B}{\partial L_B}\right] - \frac{\partial F}{\partial S}\frac{\partial S}{\partial B}$$

Combining this with the result that in equilibrium

$$MRS_{BF} \equiv \frac{\partial U}{\partial B} \Big/ \frac{\partial U}{\partial F} = \frac{P_B}{P_F} \qquad (20)$$

one obtains efficiency condition (17).

Summarizing, the equilibrium with the mergers implies (16) - (18), the conditions for efficiency.

The second issue is whether the firms have an incentive to merge. I leave it to the reader to prove that **in a competitive equilibrium the firms have an incentive to merge**. That is, demonstrate

$$\pi_{B+F} \geq \pi_B + \pi_F$$

Mergers (or buyouts) have undoubtably occurred for this reason. The likelihood of the inefficiency being eliminated by merger (or buyout) will increase as the number of impacted firms decreases, as the damage increases, and as the cost of negotiating decreases. Cases where the smoke from a single beer factory affects many firms will probably not be corrected by merger. Even with a small number of recipient firms, negotiation costs could be high enough to preclude a merger. For example, if the firms produce dissimilar products, neither party will know much about the others technology, and the costs of negotiating a merger will be high.

VI. Pigouvian Tax

Assume $S = S(B, L_S)$. Assume competitive price-taking behavior. Assume that even though a pareto improvement is possible through bribery or merger that this potential has not been realized; i.e. there is no bribery or merger and there is no expectation of either. Such a state is likely if negotiation costs are high.

In this situation, the inefficiency will persist unless an outside agent can intervene and eliminate it. Consider now whether the government can eliminate the inefficiency by imposing a tax on the smoke. Assume that the beer industry pays a tax, t, on every unit of smoke produced.

(48) $T = t \cdot S(B(L_B), L_S)$

where

$T \equiv$ total taxes collected from the beer industry, and

$t \equiv$ per unit tax on smoke.

Given this tax function, profits in the beer industry are

(49) $\pi_B = P_B \cdot B(L_B) - L_B - L_S - t \cdot S(B(L_B), L_S)$

and the necessary conditions for profit maximization are

(50) $$\frac{\partial \pi_B}{\partial L_B} = P_B \cdot \frac{\partial B}{\partial L_B} - 1 - t\,\frac{\partial S}{\partial B}\,\frac{\partial B}{\partial L_B} = 0$$

and

(51) $$\frac{\partial \pi_B}{\partial L_S} = -1 - t \frac{\partial S}{\partial L_S} = 0$$

Compare (49) - (51) with (31) - (33), the comparable equations for the beer industry in the bribery case where TB = M − β · S(B (L_B), L_S). The magnitude of M does not influence the input decision. So, from the perspective of the beer industry, β is effectively a per unit tax on smoke. The beer industry responds to t in the same way they would respond to β.

The only difference between the tax on smoke and the bribery case is that t is not a choice variable in the flower industry and that there is no presumption that the tax revenues will be passed along to the flower industry; that is, in the tax case, profits in the flower industry are

(52) $\pi_F = P_F \cdot F(L_F, S) - L_F$ rather than (28),

and the necessary condition for π maximization is just

$$\frac{\partial \pi_F}{\partial L_F} = P_F \frac{\partial F}{\partial L_F} - 1 = 0 \qquad (29)$$

Referring back to conditions (17), (20), (29) and (35), fulfillment of efficiency condition (17) requires that

$$P_B^* = 1 \Bigg/ \frac{\partial B}{\partial L_B} - \left[\frac{\partial F}{\partial S} \Bigg/ \frac{\partial F}{\partial L_F}\right] \frac{\partial S}{\partial B} \qquad (35)$$

One can solve for the t that will achieve this by substituting (35) into (50) and solving for t* one obtains

$$(53) \qquad t^* = -\left(\frac{\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial S}\right) \Big/ \left(\frac{\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial L_F}\right) = -\, MRTS^F_{SL_F}(L_B^*, L_S^*, L_F^*) > 0$$

where (L_B^*, L_F^*, L_W^*, L_S^*) is the optimal allocation of labor.

Plugging t* into (51) demonstrates that t* also fulfills efficiency condition (16). Since t* does not directly influence the input decisions in either the flower or wheat industry, efficiency condition (18) will also be fulfilled. In words, **the attainment of efficiency at the optimal allocation L_B^*, L_F^*, L_W^*, L_S^* requires that the per unit tax, t, is set equal to minus the marginal rate of technical substitution between smoke and labor in the production of flowers evaluated at the optimal allocation of labor. Note that t* = β*.**

Summarizing, **a Pigouvian tax on smoke can be used to eliminate the inefficiency associated with the smoke externality.**

Could a tax on the output (B) or the input (L_B) work just as well? This is an important policy question because it is often much easier to monitor the output or input(s) level than to monitor the amount of pollution produced.

Will a tax on beer production, B, work if $S = S(B(L_B), L_S)$? Intuition suggests that it won't because the efficiency requires that $L_S > 0$ and a tax on B will not induce the beer industry to allocate labor to smoke abatement. The math confirms our intuition.

Given a per unit tax, t_B, on beer production, profits in the beer industry are

$$(54) \qquad \pi_B = P_B \cdot B(L_B) - L_B - L_S - t_B B(L_B)$$

and the first order conditions for profit maximization are

$$(55) \qquad \frac{\partial \pi_B}{\partial L_B} = P_B \cdot \frac{\partial B}{\partial L_B} - 1 - t_B \cdot \frac{\partial B}{\partial L_B} = 0$$

and

$$(56) \qquad \frac{\partial \pi_B}{\partial L_S} = -1 \neq 0 \Rightarrow L_S = 0$$

Plugging (35) into (55) and solving for t_B^* one obtains the result that efficiency condition (17) will be fulfilled if

$$(57) \qquad t_B^* = \frac{\left(\dfrac{\partial F(L_F^*,\ S(B(L_B^*),\ L_S^*))}{\partial S} \right)}{\left(\dfrac{\partial F(L_F^*,\ S(B(L_B^*),\ L_S^*))}{\partial L_F} \right)} \cdot \frac{\partial S\,(B(L_B^*),\ L_S)}{\partial B}$$

But will t_B^* also fulfill efficiency conditions (16) and (18)? Efficiency condition (18) will be fulfilled, but t_B^* will not achieve efficiency condition (16); i.e., t_B^* does not substitute into (56) to achieve (16).

Summarizing, if $S = S(B(L_B), L_S)$ then the inefficiency associated with the smoke externality cannot be eliminating by taxing the output of the polluting industry.

However, if there is no abatement technology ($S = S(B(L_B))$) then efficiency condition (16) disappears and t_B^* can be used to eliminate the inefficiency associated with the smoke externality.

Could a per unit tax, t_{L_B}, on the labor used in the beer industry, rather than a tax on B or S, be used to eliminate the inefficiency associated with the smoke externality? No if there is abatement technology. Yes if there is no abatement technology[1]. This result is demonstrated as follows.

With a per unit tax on the labor used in the beer industry, profits are

(58) $$\pi_B = P_B \cdot B(L_B) - L_B - L_S - t_{L_B} \cdot L_B$$

and the first order conditions for profit maximization are

(59) $$\frac{\partial \pi_B}{\partial L_B} = P_B \cdot \frac{\partial B}{\partial L_B} - 1 - t_{L_B} = 0$$

[1]Note that this result would not, in general, hold if beer production required multiple inputs.

and

(60) $$\frac{\partial \pi_B}{\partial L_S} = -1 \neq 0$$ a corner solution: again no L_S will be used.

Plugging (35) into (59) and solving for $t^*_{L_B}$ one obtains the result that efficiency condition (17) will be fulfilled if

(61) $$t^*_{L_B} = \left[\frac{\left(\frac{\partial F(L^*_F, S(B(L^*_B), L^*_S))}{\partial S} \right)}{\left(\frac{\partial F(L^*_F, S(B(L^*_B, L^*_S))}{\partial L_F} \right)} \right] \cdot \frac{\partial S(B(L^*_B, L^*_S))}{\partial B} \frac{\partial B(L^*_B)}{\partial L_B}$$

Efficiency condition (18) will also be fulfilled but, as with the tax on output, $t^*_{L_B}$ will not achieve efficiency condition (16).

In summary, if there is abatement technology in the beer industry, the inefficiency associated with the smoke externality cannot be eliminated by taxing the sole input of the polluting industry. [How about by subsidizing L_S?]

However, as with the tax on output, a tax on L_B can be used to eliminate the inefficiency if there is no abatement technology in the beer industry ($S = S(B(L_B))$). (See the qualification in footnote 1.)

Summarizing this section, **the inefficiency associated with the smoke externality can always be eliminated with the appropriate tax on the smoke, but a tax on B or L_B can only eliminate the inefficiency if there is no abatement technology in the beer industry.** When there is no abatement technology, there is a one to one relationship between smoke and beer and a one to one relationship between smoke and the labor input. In which case, a tax on beer (or L_B) is equivalent to a tax on smoke.

Two asides are in order before we proceed to consider subsidies to the flower industry.

1. While beyond the scope of our model, it is appropriate to wonder if the inefficiency associated with the smoke externality can ever be eliminated by taxing one of the inputs when there are multiple inputs. The answer is yes, but only if there is no abatement technology in the beer industry and, in addition, there is a fixed relationship between the output of beer and the taxed input.

2. It is sometimes argued in the literature that a tax on smoke is not consistent with efficiency because even though the tax by itself, will achieve efficiency, there will still be an incentive even after the tax is imposed, for the flower industry to bribe the beer industry. If the bribe takes place, smoke production will fall below its efficient level. This is considered possible in a small number case i.e., only one or few polluters (beer firms in our story) and only one or few victims (flower firms). See "The Theory of Environmental Policy" by William J. Baumol and Wallace E. Oates, 2nd edition, 1993, page 32-35.

While formally correct, this point does not negate the fact that the inefficiency can be eliminated with a tax on the smoke because such a tax will only be imposed if there is no potential for a bribe to take place. If a bribe was going to take place, it would have already happened. There is still an incentive to bribe after the tax is imposed, but the potential gains from the bribe are less than were in the absence of the tax.

VII. Subsidy

Can the inefficiency associated with the smoke externality be eliminated by subsidizing the affected firms?

Assume competitive price-taking behavior. Assume that even though a pareto improvement is possible through bribery or merger that this potential has not been realized and that there is no expectation that it will be. Further assume that the beer industry is immune from government taxation or control (they generously contribute to political campaigns). In this situation, can the inefficiency associated with the smoke externality be eliminated by subsidizing the flower industry? The answer is no if there is abatement technology in the beer industry ($S = S(B, L_S)$). If $S = S(B, L_S)$, efficiency requires that $L_S > 0$. If the beer industry is immune from control they will not allocate labor to smoke abatement, independent of whether some other industry is subsidized because of the smoke produced.

Now let's address the subsidy question in the case where there is no abatement technology in the beer industry ($S = S(B)$). In this case, can the efficiency associated with the smoke externality be eliminated by subsidizing the flower industry? The answer is a qualified yes.

Note that when $S = S(B)$, the conditions for efficiency are (14), (17) and (18) but not (16). The first issue that needs to be addressed is the form of the subsidy. If a subsidy scheme is going to have any potential to eliminate the inefficiency, it must influence behavior in the flower industry. For example, an output subsidy for flower production will influence its behavior, but subsidizing the flower industry for the amount of smoke produced will not.

That is, an output subsidy might work, a subsidy for the amount of smoke produced will definitely not work. Pursuing the possibility of a subsidy for flower production, assume that the government subsidizes the flower industry γ for every unit of flowers produced.

In which case, profits in the flower industry are

(62) $\pi_F = P_F \cdot F(L_F, S) - L_F + \gamma \cdot F(L_F, S)$

and the first order condition for profit maximization is

(63) $$\frac{\partial \pi_F}{\partial L_F} = P_F \cdot \frac{\partial F}{\partial L_F} - 1 + \gamma \frac{\partial F}{\partial L_F} = 0$$

Compare (63) with (23) - the condition for profit maximization when $S_F = 0$. Referring back to page 11, fulfillment of efficiency condition (17) requires that

$$\frac{P_B}{P_F} = \left[\frac{\partial F}{\partial L_F} \bigg/ \frac{\partial B}{\partial L_B}\right] - \frac{\partial F}{\partial S}\frac{\partial S}{\partial B} \qquad (34)$$

Since, in this case, the beer industry is immune from control, profits in the beer industry will be maximized when

$$P_B = 1 \bigg/ \frac{\partial B}{\partial L_B} \qquad (22)$$

Substituting (22) into (34) one obtains the result that fulfillment of efficiency condition (17) requires that

$$(64) \qquad P_F^* = 1 \Bigg/ \left[\frac{\partial F}{\partial L_F} - \frac{\partial F}{\partial S} \frac{\partial S}{\partial B} \frac{\partial B}{\partial L_B} \right]$$

One can solve for the γ^* that will achieve this by substituting (64) into (63) and solving for S_F^*. One obtains

$$(65) \quad \gamma^* = \frac{1}{\left(\dfrac{\partial F(L_F^*, S(B(L_B^*), L_S))}{\partial L_F} \right)} - \frac{1}{\left[\dfrac{\partial F(L_F^*, S(B(L_B^*), L_S))}{\partial L_F} - \dfrac{\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial S} \dfrac{\partial S(B(L_B^*), L_S)}{\partial B} \dfrac{\partial B(L_B^*)}{\partial L_B} \right]}$$

where (L_B^*, L_F^*, L_W^*, L_S^*) is the optimal allocation of labor.

Will this subsidy, by itself, also fulfill efficiency condition (18)?

$$MRS_{WF} \equiv \frac{\partial U}{\partial W} \Bigg/ \frac{\partial U}{\partial F} = \frac{\partial F}{\partial L_F} \Bigg/ \frac{\partial W}{\partial L_W} \equiv MRT_{WF} \tag{18}$$

Given that consumers will adjust such that in equilibrium

$$MRS_{WF} \equiv \frac{P_W}{P_F} \tag{21}$$

Fulfillment of efficiency condition (18) requires that

(66) $$\frac{P_W}{P_F} = \frac{\partial F}{\partial L_F} \Big/ \frac{\partial W}{\partial L_W}$$

Given (64), (66) will be fulfilled only if

(67) $$P_W^* = \frac{\frac{\partial F}{\partial L_F}}{\frac{\partial W}{\partial L_W}\left[\frac{\partial F}{\partial L_F} - \frac{\partial F}{\partial S}\frac{\partial S}{\partial B}\frac{\partial B}{\partial L_B}\right]} \qquad i.e. \quad \frac{P_W^*}{P_F^*} = \frac{\frac{\partial F}{\partial L_F}}{\frac{\partial W}{\partial L_W}}$$

However, if the wheat industry is left to itself, this will not happen. If not subsidized, the wheat industry will maximize profits by setting

$$P_W = 1 \Big/ \frac{\partial W}{\partial L_W} \qquad (24)$$

Therefore, if S = S(B) and flower output is subsidized but not wheat output, efficiency will not be achieved. While "correcting" the relationships in beer and flowers, S_F^* has distorted the allocation of labor between flower and wheat production. **To achieve efficiency through subsidy, the government needs to subsidize both flower and wheat production**. If the government subsidizes wheat production at the rate ζ, profits in the wheat industry are

(68) $\pi_W = P_W \cdot W(L_W) - L_W + \zeta \cdot W(L_W)$

And the necessary condition for profit maximization is

$$(69) \qquad \frac{\partial \pi_W}{\partial L_W} = P_W \cdot \frac{\partial W}{\partial L_W} - 1 + \zeta \cdot \frac{\partial W}{\partial L_W} = 0$$

One can solve for the S_W that, when combined with S_F, will achieve efficiency condition (18) by substituting (67) into (69) and solving for ζ. The result is

$$(70) \qquad \zeta^* = \frac{1}{\frac{\partial W(L_W^*)}{\partial L_W}} - \frac{\left(\frac{\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial L_F}\right)}{\frac{\partial W(L_W^*)}{\partial L_W}\left[\frac{\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial L_F} - \frac{\partial F(L_F^*, S(B(L_B^*), L_S^*))}{\partial S}\frac{\partial S(B(L_B^*), L_S^*)}{\partial B}\frac{\partial B(L_B^*)}{\partial L_B}\right]}$$

Summarizing, the attainment of efficiency at L_B^*, L_F^*, L_W^* using output requires that flower output be subsidized at the rate S_F^* and that, in addition, wheat be subsidized at the rate ζ^* **flower production cannot achieve efficiency if there is abatement technology in the beer industry**.

The subsidy case points out the importance of including a third industry in the model. If a third

industry exists, subsidizing just the affected industry will not achieve efficiency. This point would have been missed if a third industry had not been included in the model.

VIII. Transferable Emission Permits

Can the inefficiency caused by the externality be eliminated by a transferable emission permit system? The answer is yes, if all the polluting firms are price takers in the emission permit market. Under this system, all polluters are required to have permits to emit. In our example, this would mean that all the beer firms must have the necessary number of permits to emit smoke. The permits are freely transferable. **The pollution control authority determines the efficient level of emission and issues exactly the necessary number of permits to achieve that.** The method of issuance of the permits by the control authority and its initial allocation among polluters are immaterial. A competitive system achieves efficiency because the producers of the negative externality are forced to pay for the external effects by requiring them to buy additional permits for any incremental emission beyond the number of permits they hold.

For simplicity, we adopted the fiction in our model that each industry's output is produced by just one firm and this firm is forced to behave as a price taker in both output and input markets. We now further assume that the beer firm which is the only polluter in this fiction is a price taker in the pollution permit market also. The permits issued by the control authority are traded in a competitive market and there is a price prevailing in the market. Attainment of efficiency in a competitive system with transferable permits can be demonstrated as follows.

Profits in the beer industry are:

(71) $\pi_B = P_B \,.\, B(L_B) - L_B - L_S - P_S \,.\, S(B(L_B),L_S)$

where P_S is the price of each of S number of emission (smoke) permits which the beer firm procures to emit S level of smoke.

Equation (71) looks exactly like the equation (49) for the case of Pigovian tax (page 16) except that t is replaced by P_S. Similar to the case of Pigovian tax, we can show that **the competitive equilibrium with**

transferable emission permits achieves efficiency. I leave this exercise to the reader. **The price of the permits in the competitive market will be equal to t^* as expressed in equation (53) in page 16.**

All polluting firms are price takers in the permit market is a crucial assumption for efficiency. If one of the firms is large in this market, efficiency may not be achieved unless the large firm obtains initially from the control authority exactly the same number of permits which is consistent with the efficient outcome. See "Market Power and Transferable Property Rights" by Robert W. Hahn in The Quarterly Journal of Economics, November 1984. According to Hahn, if the large firm obtains more permits initially, it exercises monopoly power, and if it obtains less initially, it exercises monopsony power in the permit market introducing inefficiency.