Let me give you the quick and dirty introduction to discrete-choice random utility models

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Discrete-choice random utility models in a nutshell

- Assume individual i faces one or more choice occasions.
- On each choice occasion, c, he is presented with J alternatives. He must pick one and only one alternative on each choie occasion.
- The individual has some choice-occasion budget constraint, y_{ci}.
- Money not spent on the chosen alternative, p_{cji}, is spent on a numeraire. That is, $(y_{ci} - p_{cki})$ is spent on the numeraire if the individual chooses alternative k.
- The utility individual gets on choice occasion c if he chooses alternative k, $k = 1, 2, \ldots J$. is $u_{cki} = v_{cki} + \varepsilon_{cki}$ where v_{cki} is deterministic from both the researcher's and individuals perspective. ε_{cki} is know to the individual but a random variable from the investigator's perspective with some known distribution.
- $v_{cki} = f((y_{ci} - p_{cki}), \mathbf{x}_{ck}, \mathbf{s}_i)$, where $\mathbf{x}_{ck}$ are the characteristics of alternative k on choice occasion c, and $\mathbf{s}_i$ are the relevant characteristics of the individual.
- On each choice occasion, the individual chooses the alternative that maximizes his or her utility.

Let denote $f(\boldsymbol{\varepsilon})$ the joint density function of the random components. This can be as simple or as complicated as one desires. For now make the simplifying assuming that each ε_{cji} is an independent draw from the univariate density function $f(e)$

Determine the probability that individual i will choose alternative 1 on choice occasion c

$$\begin{aligned}
\Pr(ci1) &= \Pr(u_{c1i} > u_{cji} \ \forall j \neq 1) \\
&= \Pr(v_{c1i} + \varepsilon_{c1i} > v_{cji} + \varepsilon_{cji} \ \forall j \neq 1) \\
&= \Pr(\varepsilon_{c1i} - \varepsilon_{cji} > v_{cji} - v_{c1i} \ \forall j \neq 1) \\
&= \Pr((\varepsilon_{cji} - \varepsilon_{c1i}) < (v_{c1i} - v_{cji}) \ \forall j \neq 1)
\end{aligned}$$

Note that $(\varepsilon_{cji} - \varepsilon_{cki})$ is the difference between two random variables, so is a random variable. Depending on what on assumes about $f(\varepsilon)$, the density function for the difference might take a simple form.

To simplify the notation, let $\gamma_{cji} \equiv (\varepsilon_{cji} - \varepsilon_{cki})$. So

$$\Pr(ci1) = \Pr(\gamma_{cji} < (v_{c1i} - v_{cji})\ \forall j \neq 1)$$

Consider the simple case where $J = 2$. In which case the probability of choosing alternative 1 is

$$\Pr(ci1) = \Pr(\gamma_{cji} < (v_{c1i} - v_{c2i}))$$

Denote $f(\gamma)$ the density of γ and $F(\gamma)$ as its joint density. Therefore

$$\begin{aligned}\Pr(ci1) &= \Pr(\gamma_{cji} < (v_{c1i} - v_{c2i})) \\ &= F(v_{c1i} - v_{c2i})\end{aligned}$$

That is it, in a nutshell. The form of $F(\gamma)$ depends on the form of $f(\varepsilon)$. It can be simple or a mess.

If one assume $f(\varepsilon) = e^{-x}\exp[-e^{-x}]$, a simple extreme value distribution, $f(\gamma)$ has a logistic distribution and, in the two alternative case,

$$P(ci1) = \frac{e^{v_{c1i}}}{e^{v_{c1i}} + e^{v_{c2i}}}$$