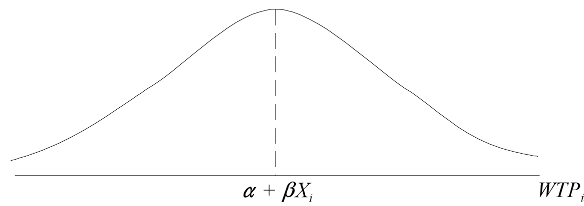
Estimating *WTP* with Referendum *CV* Data

Assume $WPT_i = E(WPT_i) + \epsilon_i$ where $\epsilon_i \sim N(o, \sigma^2)$

and

$$E(WPT_i) = \alpha + \beta X_i$$



One has referendum data; that is, individual i is asked if they will pay payment p_i and individual i either says yes $(y_i = 1)$ or no $(y_i = 0)$.

We also observe $X_i \ \forall_i$, and p_i which varies across individuals,

How will you estimate $\alpha, \beta, \& \sigma$?

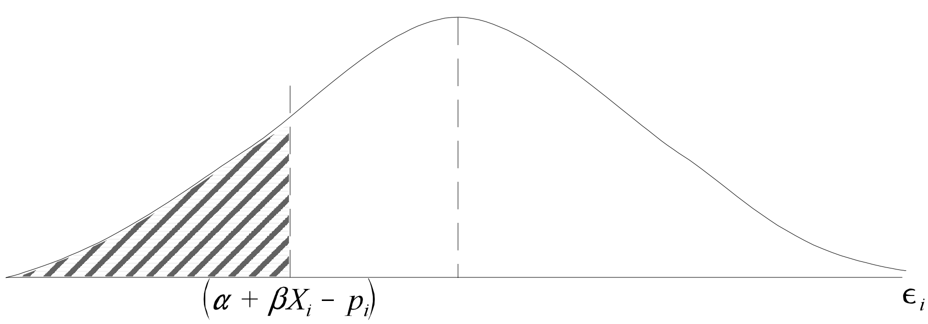
1st: Determine condition prob that individual i will say yes to p_i

Prob $\left(WTP_i \geq p_i : E(WTP_i) = \alpha + \beta X_i ; \sigma\right)$

$= \text{Prob}\left(\alpha + \beta X_i + \epsilon_i \geq p_i\right)$

$= \text{Prob}\left(\epsilon_i \geq p_i - \alpha - \beta X_i\right)$

$= \text{Prob}\left(\epsilon_i \leq \alpha + \beta X_i - p_i\right)$



Shaded area $= \text{Prob}\left(\epsilon_i \leq \alpha + \beta X_i - p_i\right)$

So, summarizing to this point,

$$\text{Prob}\ \left(WTP_i \geq p_i : \alpha + \beta X_i \text{ and } \sigma\right)$$

$$= \text{Prob}\ \left(\epsilon_i \leq \alpha + \beta X_i - p_i\right)$$

dividing through by σ

$$= \text{Prob}\ \left(\frac{\epsilon_i}{\sigma} \leq \frac{\alpha + \beta X_i - p_i}{\sigma}\right)$$

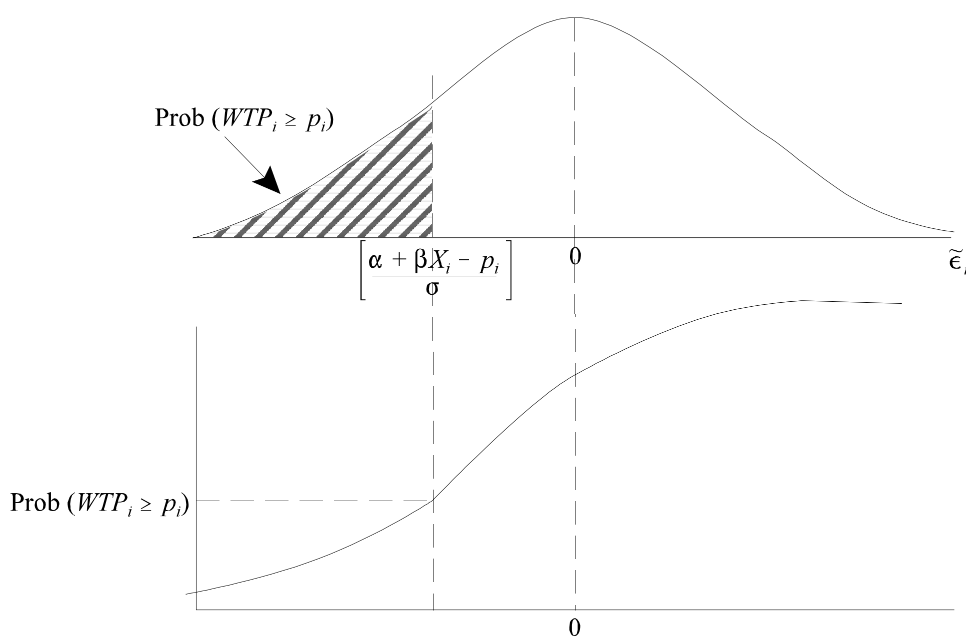
$$= \text{Prob}\ \left(\tilde{\epsilon}_i \leq \frac{\alpha + \beta X_i - p_i}{\sigma}\right)$$

where $\tilde{\epsilon}_i = \dfrac{\epsilon_i}{\sigma} \sim N(0,\ 1)$

$$= \Phi\left(\frac{\alpha + \beta X_i - p_i}{\sigma}\right)$$

where Φ is the *CDF* of the standard normal.

Graphically,



So, summarizing to here,

$$\text{Prob} \quad \left(WTP_i \geq p_i : \alpha + \beta X_i \text{ and } \sigma \right)$$

$$= \Phi \left(\frac{\alpha + \beta X_i - p_i}{\sigma} \right)$$

2nd: Therefore log of the likelihood function for individual i is

$$\ln L_i = y_i \ln \left[\Phi \left(\frac{\alpha + \beta X_i - p_i}{\sigma} \right) \right] + (1 - y_i) \ln \left[1 - \Phi \left(\frac{\alpha + \beta X_i - p_i}{\sigma} \right) \right]$$

And, for a sample if N individual

$$\ln h = \sum_{i=1}^{N} y_i \ln \left[\Phi \left(\frac{\alpha + \beta X_i - p_i}{\sigma} \right) \right] + (1 - y_i) \ln \left[1 - \Phi \left(\frac{\alpha + \beta X_i - p_i}{\sigma} \right) \right]$$

Choose α, β and σ to max $\ln L$
given Y, X and P