

Exact Consumer Surplus Measures and Utility Theoretic Demand Systems: A Historical Review

The intent of this note is to review the historical development of empirical demand theory as it relates to the estimation of consumer surplus measures.

Simply put, the consumer surplus, CS , associated with some change in the price and/or characteristics of a market commodity (or commodities), and/or a change in the level of nonmarket commodities ¹, is a money measure of the utility change that results from the change.

CS is therefore a money metric of the utility change

formal definition to follow

The note has six parts

1. The Ad hoc Historical Approach
2. Utility Theoretic Demand Systems: The Direct Approach
3. Duality Theory: Demand Estimation and Consumer Surplus
4. Examples
5. Complications: Estimation and Random Utility
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¹ Individuals face parametric prices for market commodities and endogenously choose the consumption quantities for these commodities subject to commodity prices and the individual's other constraints. In contrast, nonmarket commodities are not purchased by rather exist in levels exogenous to the individual. Coke and Pepsi are examples of market commodities; the weather, miles of roads, and the level of national defense are examples of nonmarket commodities. Characteristics of Coke and Pepsi are color, calories, taste, etc. Note that, like nonmarket commodities, the characteristics of market commodities are exogenous to the individual.

The Ad hoc Historical Approach

Objective is to estimate the CS associated with the change in the price of some good

Estimate the demand function for that good and then measure the CS as the area under that curve between the initial and new price levels.²

E.g. assume

$$(1) \quad y_1^* = \alpha + \beta_1 p_1 + \beta_2 p_2 + \beta_3 p_3 + \beta_4 I$$

where

y_1^* is the quantity demanded of good 1 (food)

p_1 = price of food

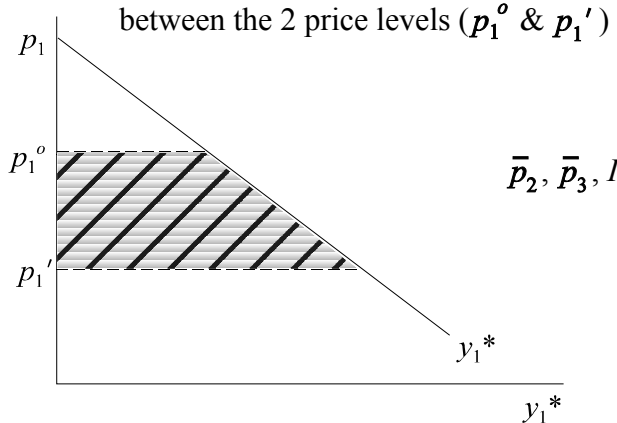
p_2 = price of clothing

p_3 = price of housing

I = income

Note that food, clothing and housing are all market commodities.

Estimate the parameters of (1) then measure CS as the area under the inverse demand curve



This area is called the Marshallian CS measure because its the area under a Marshallian Demand Function

²As we will come to realize, estimating CS for a single price change is a silly activity.

Problems with this approach

(1) It is maybe okay if one just wants to estimate demand, but it's not okay if one's intent is to derive *CS* measures. Measuring *CS* is just a disguised attempt to measure utility, so one will not be able to estimate *CS* from a demand function unless that demand function is consistent with an underlying utility function.

The properties that a system of demand equations must have to be consistent with an underlying utility function are called regularity or integrability conditions.

Integrability conditions because they are the conditions that are required to be able to integrate back to the *U* function.

e.g. the Marshallian *D* function should be homog of degree zero in prices and income - no money illusion. Equation (1) is obviously not.

If the system of “demand functions” don't fulfill these conditions, they are not demand functions.

(2) Even if Equation (1) was the true Marshallian *D* function, which it is not, the area under the Marshallian *D* curve between the two price levels is not, in general, a money metric of the utility change. Two money metrics of the utility change are the *compensating variation* and the *equivalent variations*. The *MCS* is only an approximation to the compensating and equivalent variation (*CV & EV*). The *CV* and *EV* are what we are trying to measure. The *CV & EV*, in contrast to the Marshallian measure, are often called exact *CS* measures.

For a price Δ from p_1^o to p_1'

The CV is the amount of money that would make the consumer indifferent between facing

$(p_1^o, p_2^o, p_3^o, I^o)$ and $(p_1', p_2^o, p_3^o, I^o - CV)$

If $p_1' < p_1^o$, the $CV > 0$ and is the maximum amount the consumer would pay to bring about the price change.³

The EV is the amount of money that would make the consumer indifferent between facing

$(p_1', p_2^o, p_3^o, I^o)$ and $(p_1^o, p_2^o, p_3^o, I^o + EV)$

If $p_1' < p_1^o$, the $EV > 0$ and is the min amount of money you would have to pay the consumer to get him to voluntarily forego the price Δ .⁴

The second problem with the ad hoc approach is that the Marshallian CS measure is only an approx to the CV and EV .

³Note that we have defined the CV without any notion of a utility function.

⁴ If $p_1' > p_1^o$, the $CV < 0$ and, in absolute terms, is the WTA the higher price, and the $EV < 0$ and, in absolute terms, is the WTP to avoid the higher price. What are the relationships between CV , EV , WTA (willingness to accept) and WTP (willingness to pay) for $p_1' < p_1^o$? For $p_1' > p_1^o$?

To solve the first of these two problems people began to estimate demand equations that were consistent with an underlying utility function.

Utility-Theoretic Demand Systems: The Direct Approach

Make sure that the estimated demand functions are consistent with an underlying utility function.

There are two direct ways to guarantee consistency:

- (1) Specify the functional form of the direct utility function and then derive the system of demand equation from it, or
- (2) Directly specify the functional form(s) of the demand equation(s), but make sure that the functional forms chosen are consistent with the required regularity conditions.

See Morey (1984a) for a discussion of the relative merits of these two approaches.

First, consider the first of these two approaches.

Problem is to max $U(y_1, y_2, y_3)$

wrt y_1, y_2, y_3

$$\text{s.t.} \quad I = \sum_{k=1}^3 p_k y_k$$

Solve for the Marshallian demand equations by assuming that the direct utility function $U(y_1, y_2, y_3)$ has some explicit functional form.

For example, one might assume a CES utility function

$$(2) \quad U = \sum_{j=1}^3 y_j^\beta h_j$$

where $1 > \beta \neq 0$ and h_j is of the same sign \forall_j (these are the regularity conditions). Note that h_j can be interpreted as an index of the "quality" of commodity j .

If one max (2) s.t. $I = \sum_{k=1}^3 p_k y_k$ one obtains the following system of 3 demand equations.

$$(3) \quad y_j^* = \left(\frac{p_j}{h_j} \right)^{-\sigma} I / \left[\sum_{k=1}^3 p_k \left(\frac{p_k}{h_k} \right)^{-\sigma} \right] \quad j = 1, 2, 3$$

where $\sigma \equiv -1/(\beta-1) = 1/(1-\beta)$ is the constant elasticity of substitution.

To get the *CS* associated with a Δ from p_1^o to p_1' one needs to estimate either the single demand function for y_1 or the whole system of demand equations; estimating the whole system will obviously lead to more efficient estimates of h_1, h_2, h_3 and σ .

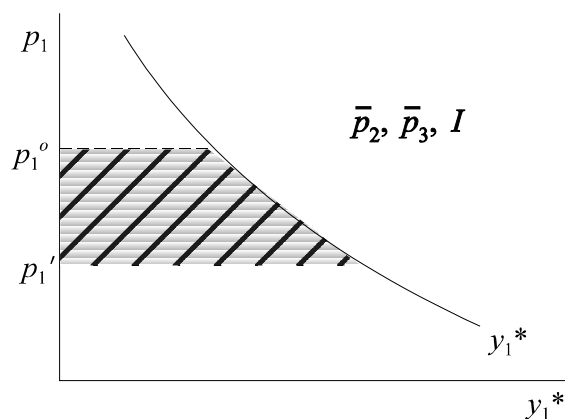
The question is: How do we use our parameter estimates $(\hat{h}_1, \hat{h}_2, \hat{h}_3, \hat{\sigma})$ to derive the *CS* measure associate with a Δ from p_1^o to p_1' .

We have a system of estimated Marshallian demand equations and we want to use those to get a money measure of the Utility Δ associated with a price Δ from p_1^o to p_1' .

What we want to do in theory is to integrate back from the Marshallian D functions to get a money metric of the ΔU .

Integrating back to the utility function is what we approximate when we define the *CS* associated with the Δ as the area under the Marshallian demand function between the two price levels.

In this case the Marshallian CS measure is



The Marshallian CS measure is the integral under the Marshallian D function evaluated between p_1^o and $p_1^'$.

This approach to estimating CS gets around the first problem associated with the ad hoc historical approach (here, the estimated D functions are consistent with some underlying utility function) but it doesn't get around the second problem (the Marshallian CS measure is only an approximation) to an exact CS measure).

This is seen by expressing the CV and EV in terms of the Hicksian demand functions.

The Hicksian demand function for good j is the quantity demanded of good j as a function of prices and the utility level.

$$(4) \quad y_j^h = m_j(U, p_1, p_2, p_3)$$

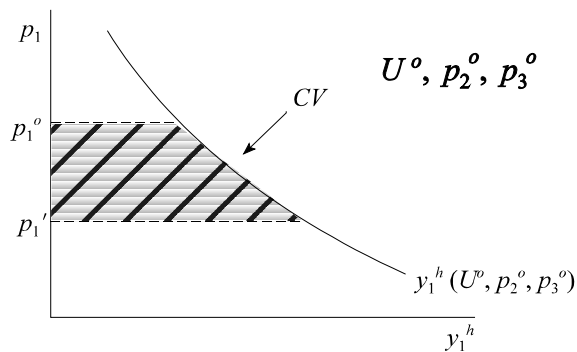
Both the CV and EV for a price Δ from p_1^o to p_1' can be defined as areas under the Hicksian demand function for good 1. This will be proven later.

The CV for the Δ from p_1^o to p_1' is

$$(5) \quad CV = \int_{p_1'}^{p_1^o} m_1(U^o, p_1, p_2^o, p_3^o) dp_1$$

where U^o is the max utility associated with $(p_1^o, p_2^o, p_3^o, I^o)$

Graphically, assuming $p_1' < p_1^o$

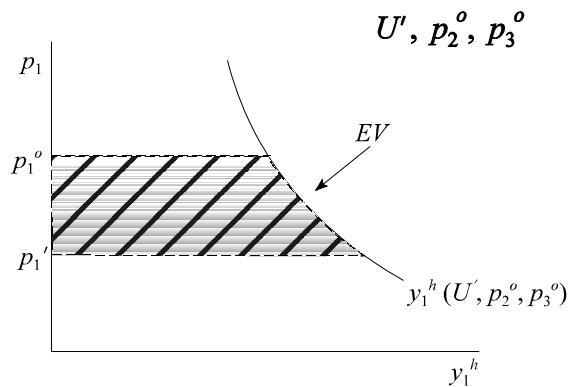


The EV for the Δ from p_1^o to p_1' is

$$(6) \quad EV = \int_{p_1'}^{p_1^o} m_1(U', p_1, p_2^o, p_3^o) dp_1$$

where U' is the max utility associated with (p_1', p_2^o, p_3^o, I)

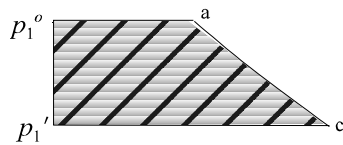
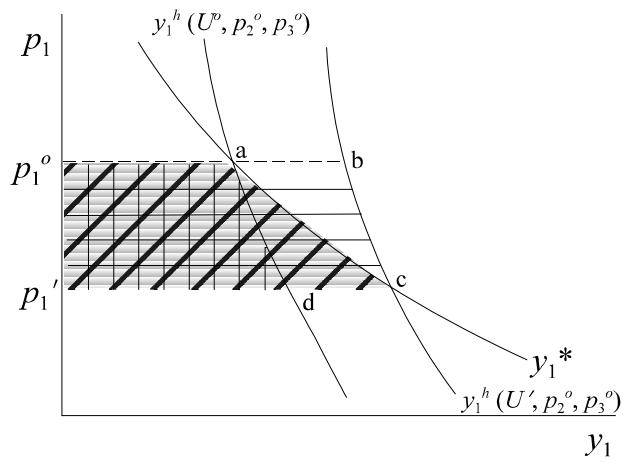
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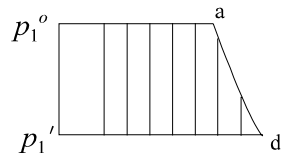
The CV is evaluated at the original utility level, U^o , and the EV is evaluated at the new utility level, U' .

For the Δ , p_1^o to p_1' , the CV , EV and Marshallian CS measure can all be put on the same graph.

Assuming $p_1' < p_1^o$



is the Marshallian CS measure



is the CV

and



is the EV

The Marshallian CS measure is an approx to the CV and EV.

Note that

$$EV \geq MCS \geq CV \quad (\text{this is true for } p_1^o > p_1' \text{ and } p_1^o < p_1')^5$$

An important issue is how much can the CV and EV differ.

In a very famous article, "Consumer's Surplus Without Apology," Robert Willig (1976) showed that for a single price Δ , MCS will be a very good approximation to both the CV and EV if certain conditions are fulfilled.

These conditions are basically that expenditures on the good are not a large proportion of total income and the price Δ is not too large.

The Willig result has been used to justify the use of the MCS whenever one wants to measure CS .

⁵Note that if $p_1^o < p_1'$, EV , CV and MCS are all negative, so $CV \leq MCS \leq EV$, the same relationship as when $p_1' < p_1^o$

However, there are a number of reasons why the Willig result cannot always be used to justify the *MCS* as a good approximation to the *CV* and *EV*:

- (1) Even for a single price Δ , the Willig necessary conditions are not always fulfilled.
- (2) The Willig result doesn't carry over to the multiple prices changes
e.g., a Δ from (p_1^o, p_2^o, p_3^o) to (p_1', p_2', p_3') .
- (3) Often we are trying to estimate the *CS* associated with a Δ in the prices and characteristics of some good or goods and/or a change in the level of nonmarket commodities, but the Willig result doesn't carry over to characteristics/nonmarket space (see Hanemann, 1991 and Shogrun et. al., 1994).
- (4) *MSC* is not well defined fro multiple price changes and/or multiple characteristics/nonmarket changes.
- (5) There is no need to approximate. We can get the exact *CS* measures.

This is most easily seen by appealing to duality theory.

Duality Theory: Demand Estimation and Consumer's Surplus

There are a number of ways to specify a preference ordering. The historical way is to use a direct utility function.

$$(7) \quad U = U(y_1, y_2, y_3)$$

Generalizing this to include the characteristics of the goods one obtains in matrix notation⁶.

$$(8) \quad U = U(Y, A)$$

where

$Y \equiv [y_j]$, where y_j is the quantity consumed of good j

and

$A = [a_{kj}]$, where a_{kj} is the quantity of characteristic k associated with good j .

However, a preference ordering can also be specified in terms of either the indirect utility function or the expenditure function.

⁶ Generalizing this to include the levels of nonmarket commodities.

$$(8a) \quad U = U(Y, A, C)$$

where

$C \equiv [c_m]$, where c_m is the level of nonmarket commodity m

For now, suppress C . We will bring it to the forefront later.

The indirect utility function identifies maximum utility as a function of prices, characteristics and income

$$(9) \quad U = V(P, A, I) \quad j=1,2,\dots,J$$

where

$P \equiv [p_j]$, where p_j is the price of good j .⁷

The expenditure function identifies the minimum expenditures as a function of prices, characteristics and the utility level.

$$(10) \quad E = E(U, P, A)$$

where

E is the minimum level of expenditures required to achieve U given P and A .

The direct, the indirect and the expenditure function are just different (dual) ways to represent a preference ordering.

The expenditure function can be obtained from the indirect by solving the indirect for I .

One can derive the system of demand equations from the indirect U function much more easily than from the direct utility function. So use of the indirect greatly simplifies empirical demand estimation.

⁷Looking ahead, if there are nonmarket commodities $U = V(P, A, C, I)$

If one starts with the direct utility function, $U(Y, A)$ the demand equations are derived by solving the constrained optimization problem.

$\max U(Y, A) \text{ s.t. } P'Y = I$ (This can get very messy.)

But, if one starts with the indirect utility function $U = V(P, A, I)$,

the system of demand functions can be easily obtained using Roy's Identity:

$$(11) \quad y_j^* = \frac{-\partial V(P, A, I) / \partial p_j}{\partial V(P, A, I) / \partial I}$$

The demand equations are therefore easily obtained by differentiation of the indirect utility function.

For example, the indirect form for the *CES* preference ordering that we examined earlier is, with two characteristics incorporated,

$$(12) \quad U = V(P, A, I) = I / \left[\sum_{j=1}^3 h_j(a_{1j}, a_{2j})^{-1/(\beta-1)} p_j^{\beta/(\beta-1)} \right]^{(\beta-1)/\beta}$$

where, for example

$$(13) \quad h_j(a_{1j}, a_{2j}) = \alpha_j + \alpha_1 a_{1j} + \alpha_2 (a_{1j} a_{2j})^{1/2} + \alpha_3 a_{2j} \\ + \alpha_4 a_{1j}^{1/2} + \alpha_5 a_{2j}^{1/2}$$

If we applied Roy's identify to (12), we would obtain the system of Marshallian demand equations, (3), with h_k replaced by $h_k(a_{1k}, a_{2k})$.

Other examples of indirect utility functions are the Flexible Functional Forms developed by Diewert, Jorgensen, Lau, McFadden and others.

Using the indirect utility function to specify the preference ordering makes it very easy to derive the exact *CS* measures for changes in the prices or characteristics of market commodities.⁸

Since the parameters in the system of demand equations are also the parameters in the direct utility function, the indirect *U*, function and the expenditure function, estimation of the demand system means we have estimated the direct utility function, or at least the relevant chunk of it.

⁸This statement is really too loose and too dirty, but is ok for now. It is correct, as stated, if there are no nonuse values associated with the existence of the characteristics of the market commodities. The statement in the text does not hold for nonmarket commodities. More on this later.

For example, for the *CES*, the expenditure function is

$$(14) \quad E = E(U, P, A) = U \left[\sum_{j=1}^3 h_j (a_{1j}, a_{2j})^{-1/(\beta-1)} p_j^{\beta/(\beta-1)} \right]^{(\beta-1)/\beta}$$

(Equation (14) is just equation (12) solved for I .)

The estimated expenditure function can be used to obtain the *CV* and *EV* associated with changes in prices and characteristics.

Lemma, the *CV* and *EV* for a change from (P^o, A^o) to (P', A') are

$$(15) \quad CV = E(U^o, P^o, A^o) - E(U^o, P', A')$$

where $U^o = V(P^o, A^o, I^o)$

$$(16) \quad EV = E(U', P^o, A^o) - E(U', P', A')$$

where $U' = V(P', A', I^o)$

Proof: The CV for a change from (P^o, A^o) to (P', A') is defined as the amount that must be subtracted from, or added to, one's income in the new state to make one indifferent between the original state, I^o, P^o, A^o and the new state with the adjustment to income, $I^o - CV, P', A'$.

That is

$$(I^o, P^o, A^o) \sim (I^o - CV, P', A')$$

But, if the individual is indifferent between (I^o, P^o, A^o) and $(I^o - CV, P', A')$, it must be the case that

$$(17) \quad U^o = V(I^o, P^o, A^o) = V(I^o - CV, P', A')$$

This is the definition of the compensating variation in terms of the indirect utility function.

The CV in terms of the expenditure function, $E(U, P, A)$, is easily derived from equation (17).

Equation (17) can be rewritten as two separate equations

$$(17a) \quad U^o = V(I^o, P^o, A^o)$$

and

$$(17b) \quad U^o = V(I^o - CV, P', A')$$

Solve equation (17a) for I^o to obtain the expenditure function evaluated at (U^o, P^o, A^o) ; that is

$$(18) \quad I^o = E(U^o, P^o, A^o)$$

and solve 17b for $(I^o - CV)$ to obtain the expenditure function evaluated at (U^o, P', A') ; that is,

$$(19) \quad I^o - CV = E(U^o, P', A')$$

Rearranging equation (19), one obtains

$$(20) \quad CV = I^o - E(U^o, P', A')$$

Plugging equation (18) into equation (20) one obtains equation (15)

$$(21) \quad CV = E(U^o, P^o, A^o) - E(U^o, P', A')$$

which is the definition of the compensating variation in terms of the expenditure function.

qed.

The equivalent variation is defined as

$$(I^o + EV, P^o, A^o) \sim (I^o, P', A')$$

\Rightarrow

$$V(I^o + EV, P^o, A^o) = V(I^o, P', A') \equiv U'$$

\Rightarrow

$$(16) \quad EV = E(U', P^o, A^o) - E(U', P', A')$$

where

$$U' = V(P', A', I)$$

qed.

Earlier (equation 5), I defined the *CV* for a single price change as an area under the Hicksian demand function for the good whose price has changed.

We now see that this area is a special case of equation (15); that is,

the *CV* for the Δp_1^o to p_1' is

$$CV = \int_{p_1'}^{p_1^o} m_1(U^o, p_1, p_2^o, p_3^o) dp_1$$

$$= \int_{p_1'}^{p_1^o} \left(\frac{\partial E(U^o, p_1, p_2^o, p_3^o)}{\partial p_1} \right) dp_1$$

by Shephard's Lemma which states that

$$m_j = \partial E(U, P) / \partial p_j$$

$$= E(U^o, p_1^o, p_2^o, p_3^o) - E(U^o, p_1', p_2^o, p_3^o) \text{ by integration}$$

Equation (6) can be derived from equation (16) in an analogous manner.

In the case of the *CES*, the specific formulas for the *CV* and *EV* for a change from (P^o, A^o) to (P', A') are obtained by substituting (14) into (15) and (16) to obtain

$$(22) \quad CV = U^o \left\{ \left[\sum_{j=1}^3 h_j (a_{1j}^o, a_{2j}^o)^{-1/(\beta-1)} p_j^{o\beta/(\beta-1)} \right]^{\beta/(\beta-1)} - \left[\sum_{j=1}^3 h_j (a'_{1j}, a'_{2j})^{-1/(\beta-1)} p_j'^{\beta/(\beta-1)} \right]^{\beta/(\beta-1)} \right\}$$

where

$$(23) \quad U^o = I^o / \left[\sum_{j=1}^3 h_j (a_{1j}^o, a_{2j}^o)^{-1/(\beta-1)} p_j^{o\beta/(\beta-1)} \right]^{\beta/(\beta-1)} \quad \text{from equation (12)}$$

and

$$(24) \quad EV = U' \left\{ \left[\sum_{j=1}^3 h_j (a_{1j}^o, a_{2j}^o)^{-1/(\beta-1)} p_j^{o\beta/(\beta-1)} \right]^{\beta/(\beta-1)} - \left[\sum_{j=1}^3 h_j (a'_{1j}, a'_{2j})^{-1/(\beta-1)} p_j'^{\beta/(\beta-1)} \right]^{\beta/(\beta-1)} \right\}$$

where

$$U' = I^o / \left[\sum_{j=1}^3 h_j (a'_{1j}, a'_{2j})^{-1/(\beta-1)} p_j^{\beta/(\beta-1)} \right]^{(\beta-1)\beta} \quad \text{from equation (12)}$$

So, subject to our earlier caveat about nonuse values, if the parameters of the demand system are estimated, we can use them to derive the *CV* and *EV* for any change in prices and characteristics.

Keep in mind that these results are general and that the *CES* was just presented as an example.

Examples

I've used this approach to estimate the exact *CS* measures associated with changes in the prices and characteristics of recreational sites such as ski areas and parks.

For example, the *CVs* and *EVs* that different individuals would associate with the introduction of a new recreational site as a function of the individual's characteristics and the site's characteristics.

Some references:

"The Demand for Site-Specific Recreational Activities: A Characteristics Approach,"

JEEM 1981;

"The Choice of Ski Areas: Estimation of a Generalized Preference Ordering with

Characteristics," *REStat*, 1984; and

"Characteristics Consumer Surplus and New Activities," *Journal of Public Economics*, 1985.

Complications: Estimation and Random Utility

Up to this point, I've ignored two important components of the problem, but in my defense they are things many have ignored, most until recently.

They are

(1) parameter estimates will vary across samples, so are random variables. Since the CV and EV are functions of these random variables they are also random variables. So far, we have not considered this.

and

(2) In addition to the fact that the estimated parameters are random variables, the utility function might contain components (random components) that vary across individuals in ways unobservable to the investigator, so even if God told the investigator the true parameters values, the CV and EV would still vary across individuals in ways the investigator cannot observe. This means that even if we know the true parameter values, we could calculate only the individual's expected CV , $E(CV)$, not his or her CV .

With respect to point (1): the estimated CV will vary across samples; that is, it has a sampling distribution. Two issues arise: how to best estimate the CV , and what is the confidence interval on the CV .

If the utility function contains random parameters that we do not observe, (1) and (2) together imply that we cannot estimate an individual's CV rather only his $E(CV)$, and it will vary across samples. So one has to determine how best to estimate $E(CV)$ and its confidence interval.

Ideally we want to take both of these important factors into account.

Consider first point (1), ignoring the issue of random components in the utility function.

The positive variance on the parameter estimates implies that the estimated CV s and EV are also statistics with positive variance.

It also means that our best estimate of the $CV(EV)$ is not obtained by plugging in the parameter estimates into the deterministic formula for the $CV(EV)$. That is, the expected value of a nonlinear function of a random variable is not the nonlinear function evaluated at the expected values of the parameters.

In spite of this, this is how most of us estimate the CV and EV .⁹

⁹And elasticities and other nonlinear functions of the parameters.

It is better to estimate the CV with simulations. This will also produce an estimate of the confidence interval on the CV.¹⁰

The simulation approach proceeds as follows: One takes the estimated parameter vector and the estimated var/cov matrix of the parameters, and uses them to randomly draw parameter vectors from a normal distribution with these means and variances. For each draw, one calculates the CV associated with that parameter vector. One takes hundreds or even thousands of these draws. This generates a simulated distribution of the CV. The mean of this distribution is our best estimate of the CV and the variance of this distribution is the estimated variance of the sampling distribution.

Note that the mean of this simulated distribution, our best estimate of the CV, will typically be “close” in value to the estimate obtained by plugging the estimated parameters into the CV formula, so doing the latter will often not result in a big mistake.

One uses the simulated distribution of the CV to get the confidence interval on the CV. For example, the 95% confidence interval is obtained by lopping off the top and bottom 2.5% of the distribution.

¹⁰If one does not simulate the sampling distribution of the CV, one can, in theory, analytically determine its distribution. This amounts to determining the distribution of a random variable that is a nonlinear function of a vector of random variables given the distribution of that vector of random variables. This is often very difficult.

However, there are often problems associated with the simulation approach. More needs to be added to the discussion.

In addition to simulating it, it is also possible to analytically approximate the variance on the sampling distribution of the CV.

Theil and others show that if $y = f(x_1, \dots, x_n)$

where Ω is the var cov matrix of the x 's.

then an approximation to the variance of y is

$$\text{var } y \approx \begin{bmatrix} \frac{\partial f(\cdot)}{\partial x_1} & \frac{\partial f(\cdot)}{\partial x_2} & \dots & \frac{\partial f(\cdot)}{\partial x_n} \end{bmatrix} \Omega \begin{bmatrix} \frac{\partial f(\cdot)}{\partial x_1} \\ \frac{\partial f(\cdot)}{\partial x_2} \\ \frac{\partial f(\cdot)}{\partial x_n} \end{bmatrix}$$

This approximation formula for the variance can be used to calculate the standard deviation, and confidence interval, on the estimated CV given an estimate of the var/cov matrix of the parameter estimates.

For an example, see

Morey and Shaw, "An Economic Model to Assess the Impact of Acid Rain."

When welfare measure are estimated for litigation, confidence intervals are very important.

Historically, many applied welfare economists have tended to forget that the estimated CV is a random variable, and discuss it as if it's deterministic.

My results with Douglass Shaw indicate that knowledge of the standard deviations is also important. In some cases we will not be able to reject the null that the true CV is zero even though its estimated value is a large positive number. The standard error give us an indication of the accuracy of our welfare measures.

Another issue with estimated demand functions

One specifies a deterministic preference ordering, derives a system of deterministic demand functions, and then adds error terms onto those demand equations so the parameters in the demand functions can be estimated.

The approach is ad hoc in that we don't have a theory that explains why the demand equations have a stochastic component. Is it measurement error, that utility has a random component from our perspective, or something else?

The random utility approach

Rather than add error terms onto demand functions in an ad hoc fashion, it is more theoretically appealing to assume that there is a random component in the utility function.

That is, one assumes a

Random Utility Function

where one specifies the distribution of the random components in the U function.

The most basic examples are Logit, Probit and Tobit analysis of consumer demand.

McFadden has been a pioneer in this area.

The random components in the U function imply the distribution of the demand functions.

In this context, the CV is a random variable from the investigator's perspective, and as noted earlier the best we can do is estimate its expectation.¹¹

¹¹Remember the distinction between the CV and the estimated CV . Even if the CV is not a random variable, the estimated CV is a random variable.

Pioneering work with random utility models has been done by

McFadden, "Economic Models of Probabilistic Choice," 1981

Small and Rosen, "Applied Welfare Economics with Discrete Choice Models,"

Econometrica, 1981

and

Hanemann, "Welfare Analysis with Discrete Choice Models," *Econometrica* 1985.

For a review of some of this literature, see the survey article on my web page, "Two Rums Uncloaked". It lists many applications.

An Example of a Simple Random Utility Model

Consider the choice between two fishing sites, site 1 and site 2

where p_j is the cost of a trip to site j and c_j is the expected catch rate for site j

Consider the following model

$$V = \max (U_1, U_2)$$

where

$$U_j = V_j + \epsilon_j \quad j = 1, 2.$$

where V_j and ϵ_j are deterministic from the individual's perspective

but ϵ_j is a random variable from the investigator's perspective such that ϵ_j is randomly drawn from some distribution

Therefore

$$\begin{aligned} V &= \max (U_1, U_2) \\ &= \max (V_1 + \epsilon_1, V_2 + \epsilon_2) \end{aligned}$$

So, $V(\cdot)$ is a random utility function from the investigator's perspective.

Given that the individual chooses to fish, what is the probability that she will choose site 1?

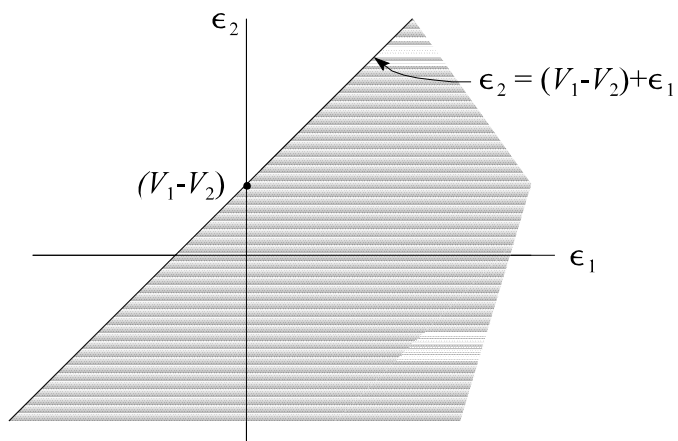
$$\begin{aligned} \text{Prob}_1 &= \text{Prob} [U_1 > U_2] = \text{Prob} [V_1 + \epsilon_1 > V_2 + \epsilon_2] \\ &= \text{Prob} [V_2 + \epsilon_2 < V_1 + \epsilon_1] \\ &= \text{Prob} [\epsilon_2 < (V_1 - V_2) + \epsilon_1] \end{aligned}$$

Let $f(\epsilon_1, \epsilon_2)$ and $F(\epsilon_1, \epsilon_2)$ denote the density function and CDF for ϵ_1 and ϵ_2

In which case

$$\begin{aligned} \text{Prob}_1 &= \text{Prob} [\epsilon_2 < (V_1 - V_2) + \epsilon_1] \\ &= \int_{\epsilon_1 = -\infty}^{+\infty} \int_{\epsilon_2 = -\infty}^{(V_1 - V_2) + \epsilon_1} f(\epsilon_1, \epsilon_2) d\epsilon_2 d\epsilon_1 \end{aligned}$$

which is the area under $f(\epsilon_1, \epsilon_2)$ above the shaded area



It is common to assume $E(\epsilon_j) = 0 \forall_j$

If ϵ_1 and ϵ_2 are each independently drawn from a normal distribution, the model is a simple bivariate probit model.

If ϵ_1 and ϵ_2 are each independently drawn from an extreme value distribution the model is a simple bivariate logit model.

For example, if each ϵ_j is independently drawn from the Extreme value distribution

$$F(\epsilon) = \exp(-e^{-\epsilon}) \quad \Leftrightarrow \quad f(\epsilon) = e^{-\epsilon} \exp[-e^{-\epsilon}]$$

the probability of choosing site j is

$$\text{Prob } j = \frac{e^{V_j}}{e^{V_1} + e^{V_2}}$$

The model is completed by assuming something like

$$\begin{aligned} V_j &= V(c_j, (FB - p_j)) \\ &= \alpha c_j + \beta (FB - p_j) \end{aligned}$$

where

FB is the fishing budget for the choice occasion.

For this simple logit model, it can be shown that

$$E(V) = \ln [e^{V_1} + e^{V_2}] = .57721 \dots$$

and the expected value of the compensating variation associated with a change from

$$(p_1^o, c_1^o, p_2^o, c_2^o) \text{ to } (p_1', c_1', p_2', c_2')$$

is

$$E(CV) = E(EV) = \left(\frac{1}{\beta} \right) [\ln D' - \ln D^o]$$

change in expected
utility

where

$$D = e^{V_1} + e^{V_2}$$

Demand Functions Don't Tell All

Consider the more general utility function:

$$U = U(Y, A, C) \text{ or in terms of the indirect } U = V(I, P, A, C)$$

where

Y is the vector of market commodities.

$A \equiv [a_{kj}]$, where a_{kj} is the quantity of characteristic to be associated with market commodity j .

$C = [c_m]$, where c_m is the level of nonmarket commodity m .

The question is

Can one always estimate all the parameters in $U(\cdot)$ by estimating the complete demand system for market commodities?

The question and answer relate to the issue of whether use and nonuse values can be estimated solely on the basis of observed behavior.

Start by considering a world without nonmarket commodities.

$$U = U(Y, A)$$

or

$$U = V(P, A, I)$$

Will all of the parameters in the utility function end up in the system of demand functions for the market commodities, or is it possible for important components of the preference ordering to not have an impact on the demand for market commodities?

In all of our examples to this point (remember my earlier caveat), all the parameters in the utility function appear in the system of market demand functions, but this need not be the case.

Consider the following modified CES utility function

$$U = \sum_{j=1}^3 h(a_{1j}, a_{2j}) y_j^{\beta} + g(A)$$

This utility function generates the same demand system as $\sum_{j=1}^3 h(a_{1j}, a_{2j}) y_j^{\beta}$, so the parameters in $g(A)$, while being important determinants of utility, do not influence observed behavior.

What is going on?

The characteristics of market commodities can influence utility directly and indirectly: indirectly by influencing the experience associated with each unit of each market commodity consumed—the $h(a_{1j}, a_{2j})$ effect; and directly, independently of the vector of market commodities consumed—the $g(A)$ effect.

For example, let y_j denote the number of fishing trips to river j and a_{1j} the stock of trout in river j . The trout in river j influence me in two ways: their presence increases the enjoyment I get fishing river j , and I get pleasure from just knowing trout prosper and multiply in Colorado's rivers.

If one calculates a CV for a change in a_{1j} from a_{1j}^o to a_{1j}' solely on the basis of that portion of the utility function that can be recovered from the demand functions, one will get an incorrect answer if preferences have significant nonuse components—like $g(A)$.

Value has two components – use value and nonuse value. From demand functions one can recover use values but not nonuse values.

It is tempting to assume use value is a lower bound on total value, but this is not necessarily the case. For an individual who loves catching fish but, otherwise hates the fact that fish live in water (what did W.C. Field's say about what fish do in water?)

$$\frac{\partial h(a_{1j}, a_{2j})}{\partial a_{1j}} > 0 \quad \text{but} \quad \frac{\partial g(A)}{\partial a_{1j}} < 0$$

Often, applied welfare economists make assumptions sufficient to imply there are no nonuse values associated with the characteristics of market commodities.

One such assumption is Måler's weak complementarity assumption with respect to a market commodity and their characteristics

a_{kj} is said to be weakly complimentary with y_j if

$$\frac{\partial U(y_1, y_2, \dots, y_{j-1}, 0, y_{j+1}, \dots, y_j; A)}{\partial a_{kj}} = 0$$

In words, if this condition holds, one is only influenced by a change in a_{kj} if one is consuming a positive amount of y_j . That is, a_{kj} only influences an individual through the consumption of commodity j .

If Måler's WCC holds $\forall k$ and j , there are no nonuse values associated with the characteristics of market commodities and one can recover the CV or EV associated with a change in A from the complete system of market demand functions.

Note that Måler's WCC is a sufficient condition for recovering complete preferences from market behavior, not a necessary condition.

The issue of recovering preferences from observed market behavior becomes even more problematic in a world with nonmarket commodities.

$$U = U(Y, A, C) \text{ or } U = V(P, A, I, C)$$

It is possible that individuals care about C in ways that do not influence market behavior.

In which case, one cannot infer everything one wants to know about the preferences for nonmarket commodities from the demand functions for market commodities.

When one can, and cannot, recover preferences from observed behavior is an ongoing area of research.

If one can't infer value from observed behavior, the only alternative is to ask the individual to state her preferences.

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