The CDF for having sex

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7818 Take-home quiz
7818Qz5aF08_SexCDF

groups of three

Assume that the number of times one has sex in a year is a random variable, S, that is not restricted to a countable number of values.

Further assume that one cannot have sex a negative number of times or more than 400 time a year. Further assume that some percent of the population, $0 < \gamma < 1$ never has sex, and that $0 < \nu < 1$ percent of the population has sex 400 times a year.

Specify two or three CDFs for this random variable. Then graph the CDF for each of your examples.

For fun, and only fun, you could derive the corresponding density functions.

0.1 I will start by specifying a CDF, $G(S)$ with the following properties: $G(s) = 0$ if $s \leq 0$ and $G(s) < 1$ for some $s > 400$.

0.2

With this CDF, no one has sex a negative amount of times, and a positive proportion of the population has sex more than 400. $G(S)$ does not have all of the required properties. Before modifying it to conform to the required properties, consider some possible examples of $G(S)$.

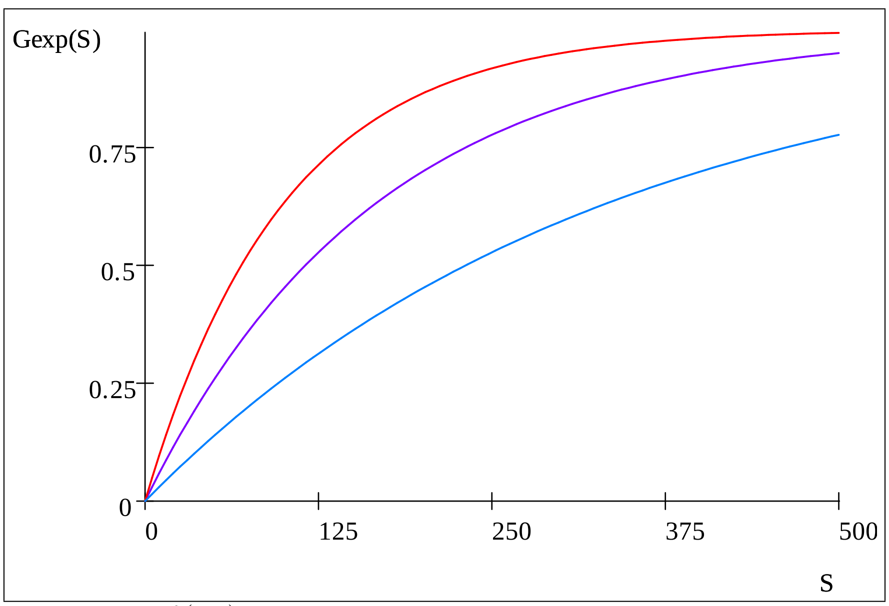
0.2.1 Consider the exponential

$$G_{\exp}(s) \begin{cases} 1 - e^{-\lambda(s)} & if \quad s \geq 0 \\ 0 & otherwise \end{cases}$$

$\lambda > 0$.

Note that one can have sex 17.374 times.

Graphing $1 - e^{-\lambda(s)}$ for some different values of λ. I chose values for λ so their is a positive probability of having sex more than 400 times.



$1 - e^{-\lambda(wtp)}$. purple $\lambda = .006$, red$\lambda = .01$ and blue $\lambda = .003$

This distributions does not fit the bill for three reasons: $G(0) = 0$ **and** $G(s > 400) > 0$**, and the probability of having sex exactly 400 times is zero.**

0.3 Consider the modified CDF

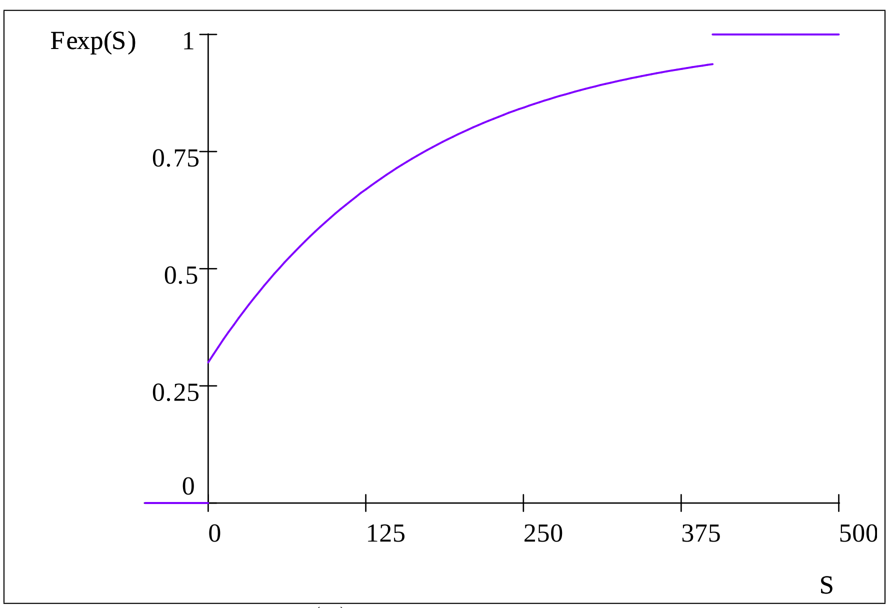
$$F(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \gamma + (1-\gamma)G(s) & if \quad 0 < s \le 400 \\ 1 & if \quad s > 400 \end{cases}$$

This function should do the trick.

0.3.1 If $G(S) = G_{\exp}(S) \equiv (1 - e^{-\lambda(s)})$

$$F_{\exp}(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \gamma + (1-\gamma)(1 - e^{-\lambda(s)}) & if \quad 0 < s \le 400 \\ 1 & if \quad s > 400 \end{cases}$$

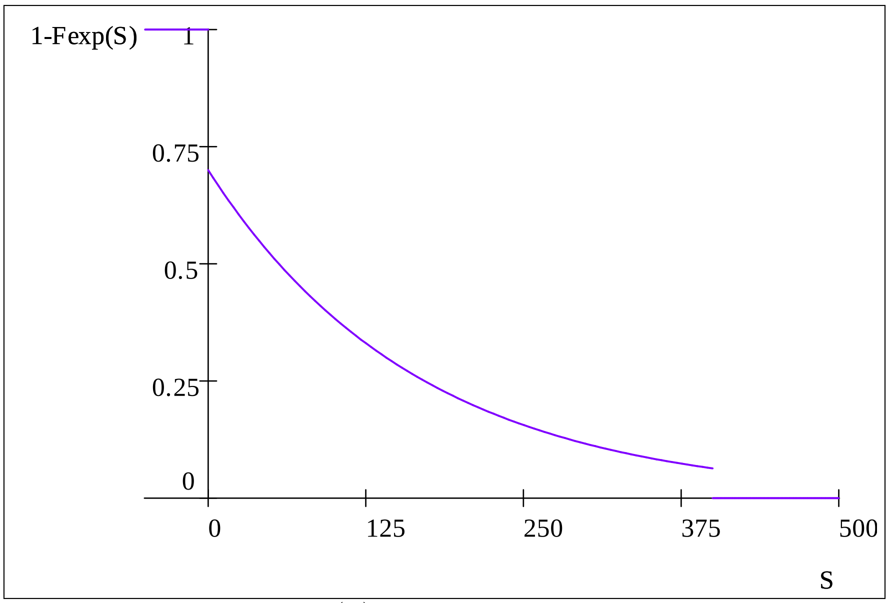
Graphing this for $\gamma = .3$ and $\lambda = .006$



$F_{\exp}(S)$ for $\gamma = .3$ and $\lambda = .006$

In this example, 30% of the population does not have sex, and approximately 10% have it 400 times. No one has it a negative number of times and no one has it more than 400 times.

The corresponding proportion who are having sex at least s times (the survival function) is



$1 - F_{\exp}(S)$ for $\gamma = .3$ and $\lambda = .006$

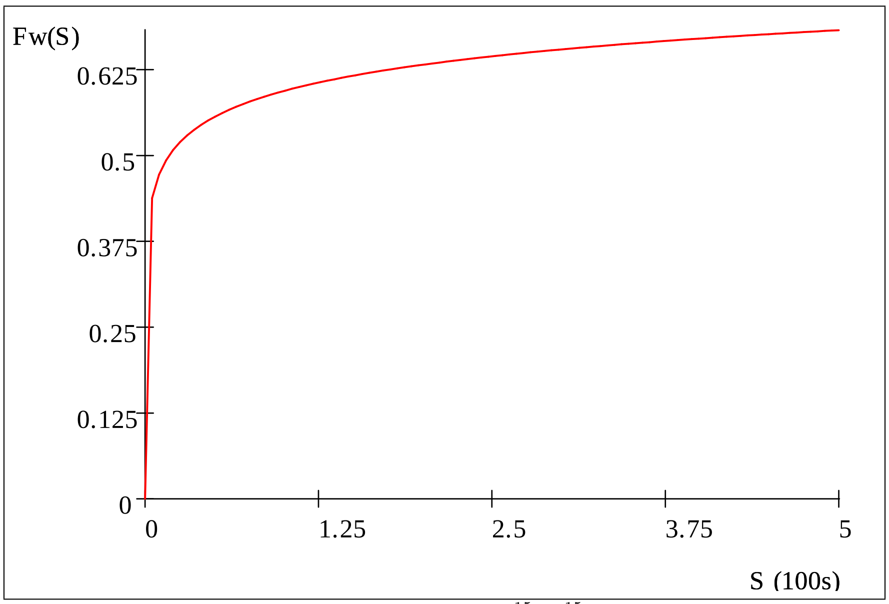
Notice that at no sex the proportion is .7; the proportion then continuously drops as $s \longrightarrow 400$ and then past 400 it drops to zero.

0.4 Consider a second example, the Weibull

$$G_W(s) \begin{cases} 1 - e^{-x^a b^{-a}} & if \quad s \geq 0 \\ 0 & otherwise \end{cases}$$

where $a, b > 0$

Graphing this with $a = .15$, $b = 2$ and S measured in hundreds



$$G_W(S) = 1 - e^{-s^{.15} 2^{-.15}}$$

This distributions does not fit the bill for the same three reasons: $G(0) = 0$ **and** $G(s > 4) > 0$**, and the probability of having sex exactly 400 times is zero.**

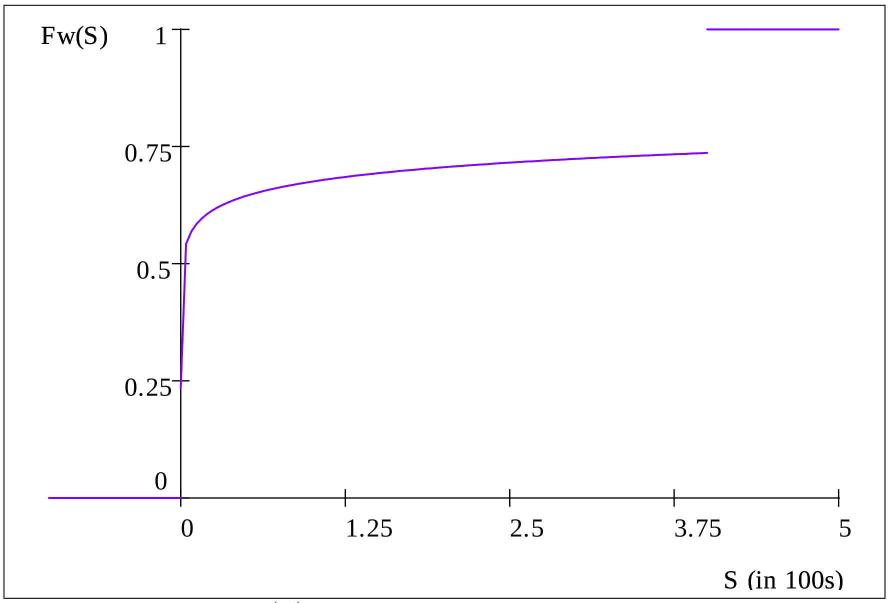
0.4.1 Consider again the modified CDF

$$F(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \gamma + (1-\gamma)G(s) & if \quad 0 < s \leq 4 \\ 1 & if \quad s > 4 \end{cases}$$

If $G(S) = G_W(S)$

$$F_W(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \gamma + (1-\gamma)(1 - e^{-s^a b^{-a}}) & if \quad 0 < s \leq 4 \\ 1 & if \quad s > 4 \end{cases}$$

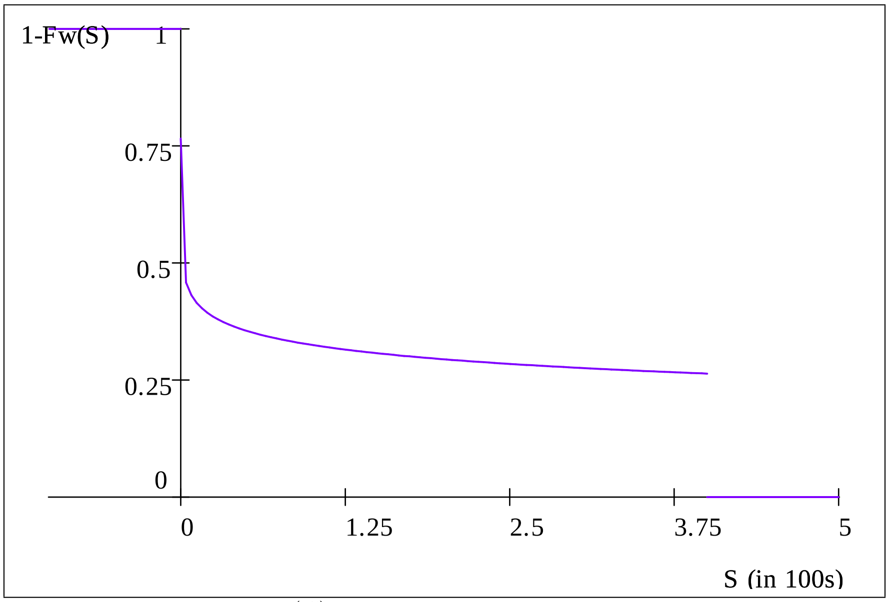
Graphing this for $a = .15$, $b = 2$ and $\gamma = .2$



$F_W(S)$ with $a = .15$, $b = 2$ and $\gamma = .2$

Note the jump at zero and again at 4.

For the Weibull, the corresponding proportion who are having sex at least s times (the survival function) is



$1 - F_W(S)$ with $a = .15$, $b = 2$ and $\gamma = .2$

0.5 For fun, and only fun:

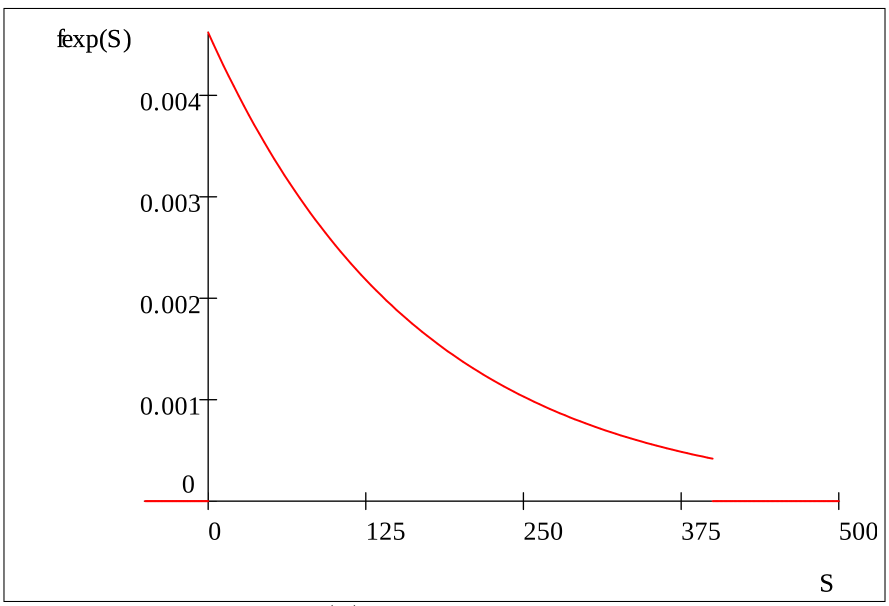
0.5.1 The density function corresponding to

$$F_{\exp}(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \gamma + (1-\gamma)(1 - e^{-\lambda(s)}) & if \quad 0 < s \leq 400 \\ 1 & if \quad s > 400 \end{cases}$$

Is

$$f_{\exp}(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \frac{(1-\gamma)\lambda e^{-x\lambda}}{\int_0^{400} \lambda e^{-\lambda t} dt} & if \quad 0 < s \leq 400 \\ 0 & if \quad s > 400 \end{cases}$$

Graphing this for $\gamma = .3$ and $\lambda = .006$. The density function is



$f_{\exp}(S)$: $\gamma = .3$ and $\lambda = .006$

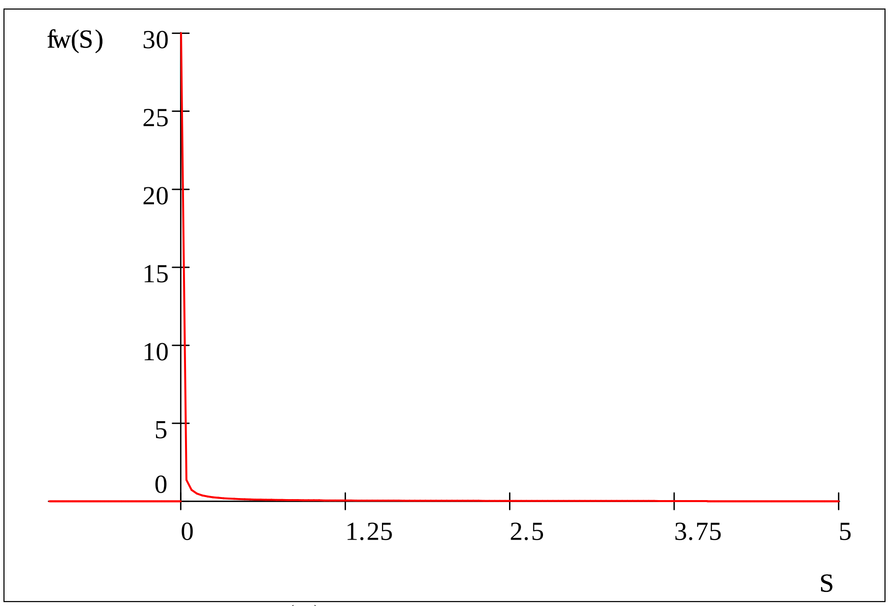
0.5.2 The density function corresponding to

$$F_W(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \gamma + (1-\gamma)(1 - e^{-s^a b^{-a}}) & if \quad 0 < s \le 4 \\ 1 & if \quad s > 4 \end{cases}$$

Is (I hope)

$$f_W(s) = \begin{cases} 0 & if \quad s < 0 \\ \gamma & if \quad s = 0 \\ \frac{(1-\gamma)ab^{-a}s^{a-1}e^{-s^a b^{-a}}}{\int_0^4 ab^{-a}t^{a-1}e^{-t^a b^{-a}}\,dt} & if \quad 0 < s \le 4 \\ 0 & if \quad s > 4 \end{cases}$$

Graphing this for $a = .15$, $b = 2$ and $\gamma = .2$. The density function, I hope, looks like



$f_W(S)$: $a = .15$, $b = 2$ and $\gamma = .2$

There is a jump from zero to .2 at zero, then the function jumps above .2 thereafter declining to 4, at which point in drops to zero. This last drop is too small to see. In this Weibull example, most people do not have sex very often. The mean is .....