WTP, CDF. and probability of voting yes

Edward Morey

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In class quiz, groups of three

Consider a proposal in Boulder to remove mining waste (pollution) from Boulder Creek. Assume willingness-to-pay, WTP, for this project is a random variable with density $f(WTP)$ and CDF $F(WTP)$. Denote individual $i's$ WTP, wtp_i. Let c_i be the cost of the project to individual i and assume that if $wtp_i \geq c_i$ individual i will vote for the project, and if $wtp_i < c_i$ the individual will vote against the project.

1 Specify, in general terms the probability that individual i will vote for the project.

answer: note that $F(c_i)$ is the probability that individual $i's$ wtp is less than c_i, so the probability that individual i will vote no. So, $1 - F(c_i)$ is the probability of a yes vote. That is

$$\begin{aligned} \Pr(yes_i \,| c_i\,) &= \Pr(wtp_i \geq c_i) \\ &= 1 - F(c_i) \end{aligned}$$

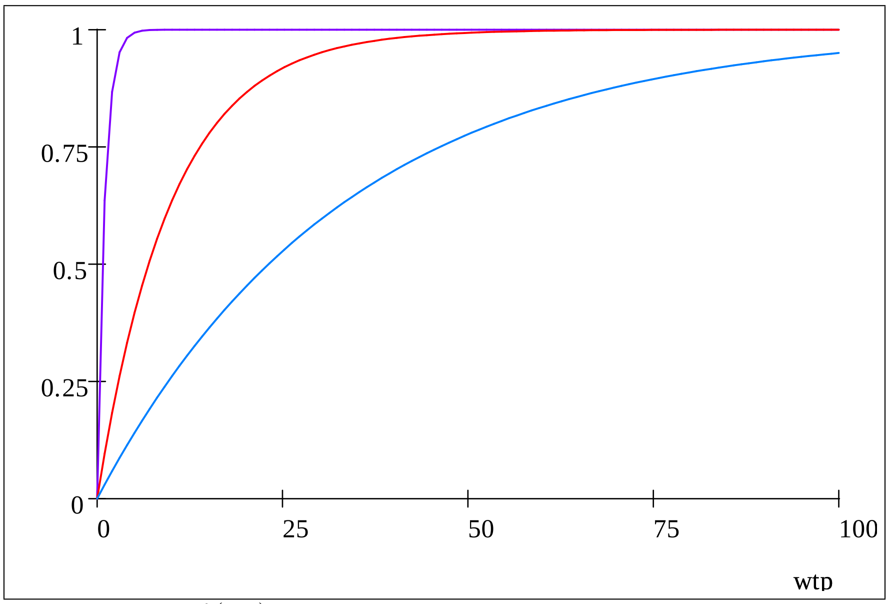
2 Now specify a specific distribution for WTP making it a function of individual $i's$ income, y_i, and derive the specifc probability of a yes as a function of c_i and y_i. Keep is simple.

answer: I am going to assume WTP is distributed

$$F(wtp) \begin{cases} 1 - e^{-\lambda(wtp)} & if \quad wtp \geq 0 \\ 0 & otherwise \end{cases}$$

$\lambda > 0$. This is the first continuous rv CDF presented in MGB. It is the expondential distribution.

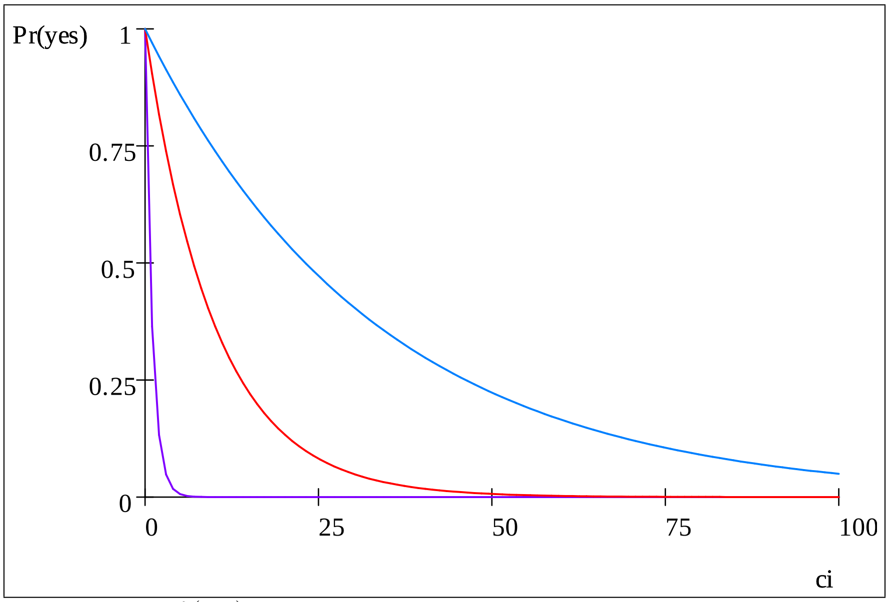
Graphing $1 - e^{-\lambda(wtp)}$ for different values of λ.



$1 - e^{-\lambda(wtp)}$. purple $\lambda = 1$, red$\lambda = .1$ and blue $\lambda = .03$

Notice it starts at zero.
So, in this case,

$$\begin{aligned}
\Pr(yes_i \,|c_i) &= \Pr(wtp_i \geq c_i) \\
&= 1 - F(c_i) \\
&= 1 - (1 - e^{-\lambda(c_i)}) \\
&= e^{-\lambda(c_i)}
\end{aligned}$$



$e^{-\lambda(wtp)}$. purple $\lambda = 1$, red$\lambda = .1$ and blue $\lambda = .03$

Notice that it starts at one.

2.1 Now I will make the one parameter in the CDF a function of y_i.

Specifically, assume $\lambda = \lambda(y_i) = \alpha + \beta y_i$ in which case

$$\begin{aligned} \Pr(yes_i \,|c_i, y_i\,) &= \Pr(wtp_i \geq c_i) \\ &= e^{-\lambda(y_i)(c_i)} \\ &= e^{-(\alpha+\beta y_i)(c_i)} \end{aligned}$$

There are two unknown parameters in this probability α and β.

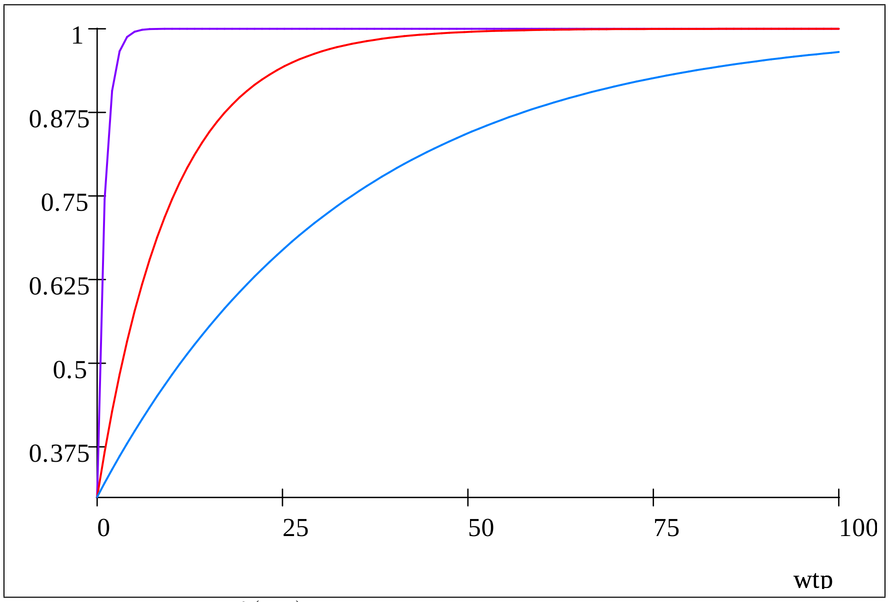
3 Just for fun

Imagine that some proportion of the population has a zero WTP, that is $\Pr(wtp = 0) > 0$. The above CDF is not consistent with this possibility. So, following MGB, let's modify it so that it is possible. Assume

$$F(wtp) \begin{cases} \gamma + (1-\gamma)(1 - e^{-\lambda(wtp)}) & if \quad wtp \geq 0 \\ 0 & otherwise \end{cases}$$

γ is the proportion of the population with $wtp = 0$.

Graphing this assuming $\gamma = .3$

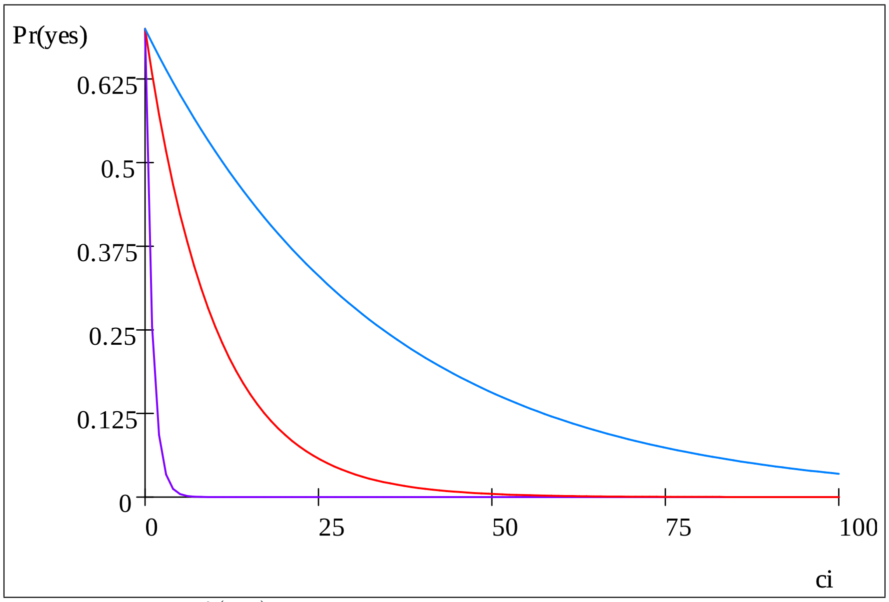


$.3 + .7(1 - e^{-\lambda(wtp)})$. purple $\lambda = 1$, red$\lambda = .1$ and blue $\lambda = .03$

Notice that it starts at .3.

In this case, the probability of a yes is

$$\begin{aligned} \Pr(yes_i \,|c_i\,) &= \Pr(wtp_i \geq c_i) \\ &= 1 - F(c_i) \\ &= 1 - (.3 + .7(1 - e^{-\lambda(c_i)})) \\ &= .7e^{-\lambda(c_i)} \end{aligned}$$



$1 = e^{-\lambda(wtp)}$. purple $\lambda = 1$, red$\lambda = .1$ and blue $\lambda = .03$

Notice that it starts at .7.

4 If you answered in terms of $f(WTP)$ rather than $F(WTP)$

The probability of a no vote is $\Pr(no\,|c_i) = \Pr(wtp_i < c_i) = \int_{-\infty}^{c_i} f(wtp)d(wtp)$.

So the probability of a yes vote is

$$\Pr(yes\,|c_i) = \Pr(wtp_i \geq c_i) = 1 - \int_{-\infty}^{c_i} f(wtp)d(wtp)$$

Second part: For my expodential density function $f(wtp) = \lambda e^{-\lambda(wtp)}$, so

$$\begin{aligned} \Pr(yes\,|c_i) &= \Pr(wtp_i \geq c_i) = 1 - \int_{-\infty}^{c_i} \lambda e^{-\lambda(wtp)} d(wtp) \\ &= e^{-\lambda(c_i)} \end{aligned}$$