1 Odds, odd ratios, probabilities, and the *logit*

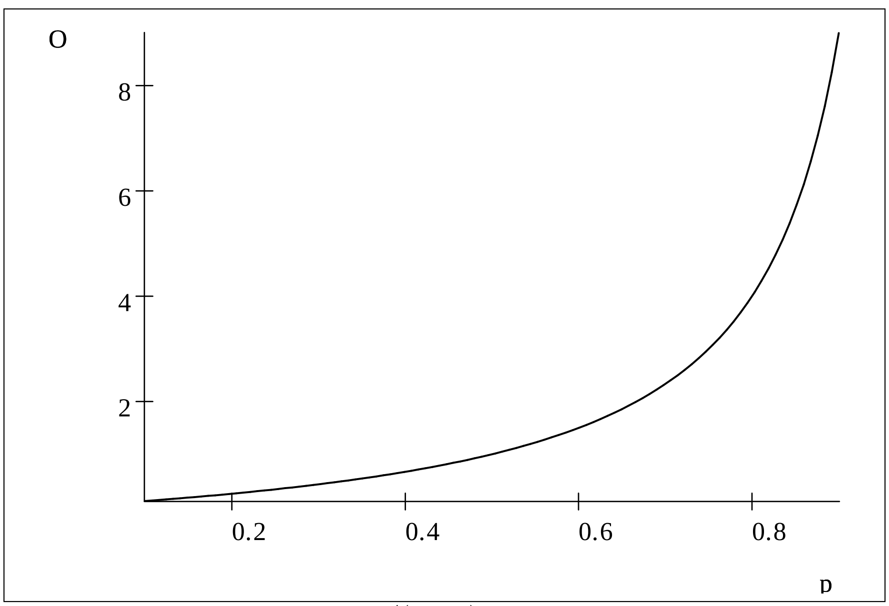
draft September 27, 2009

If you start reading statistics books rather than econometrics books, or statsitics books written by economists, you will need to how statisticans don't always talk like economists

1.1 odds

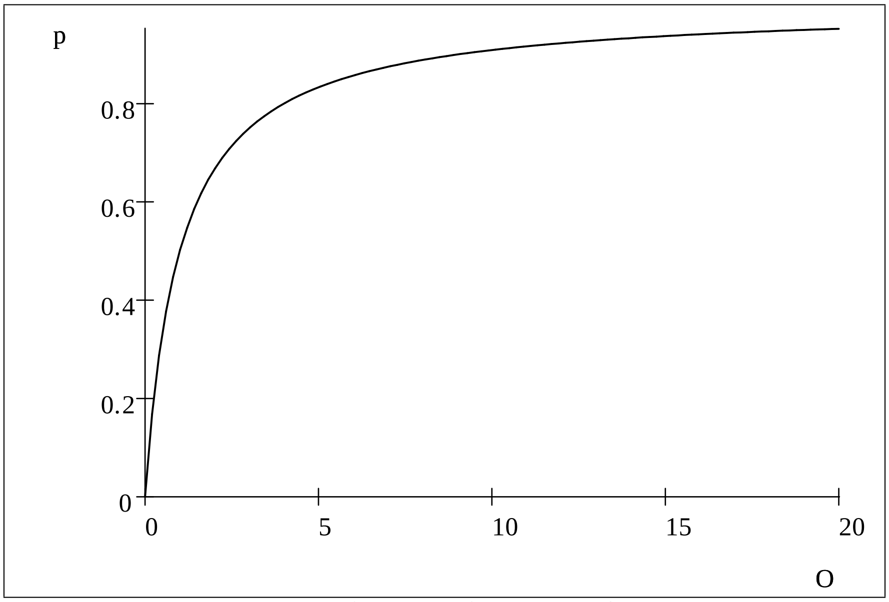
When there are two alternatives, $x = 1$ and $x = 0$, with $\Pr[x = 1] = p$ (a Bernoulli), it is common to interpret $x = 1$ as a success and $x = 0$ as a failure. In such common circumstances, statisticians and people who go to the track, usually not economists, like to talk about the odds of a success rather than the probability of a success, where the odds, O, are

$$O = \frac{p}{1-p}$$



The odds, $O = p/(1-p)$, as a function of p

Note that $0 \leq O$, and O is undefined for $p = 1$. Solving $O = \frac{p}{1-p}$ for p, $p = \frac{O}{(O+1)}$



p as a function of the odds, O

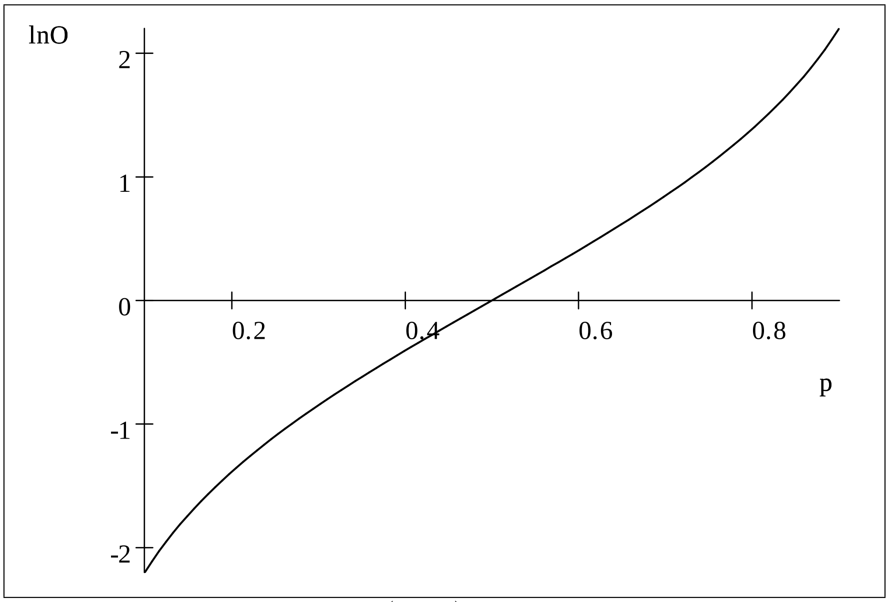
So, talking in terms of the odds and talking in terms of p are two different ways of saying the same thing ($p \Longleftrightarrow O$). Note the the odds of a success is 1 if $p = .5$, less than one if $p < .5$ and greater than 1 if $p > .5$. Think about the odds of your horse winning the race (a success). To win you want the odds to be greater than 1. In table form

p	.1	.2	.3	.4	.5	.6	.7	.8	.9
O	1/9	1/4	3/7	2/3	1	3/2	7/3	4	9

1.1.1 $\ln O$, the *logit*

Sometimes it is fun, and informative, to work with the log of the odds

$$\begin{aligned}\ln O &= \ln(\frac{p}{1-p}) \\ &= \ln p - \ln(1-p)\end{aligned}$$



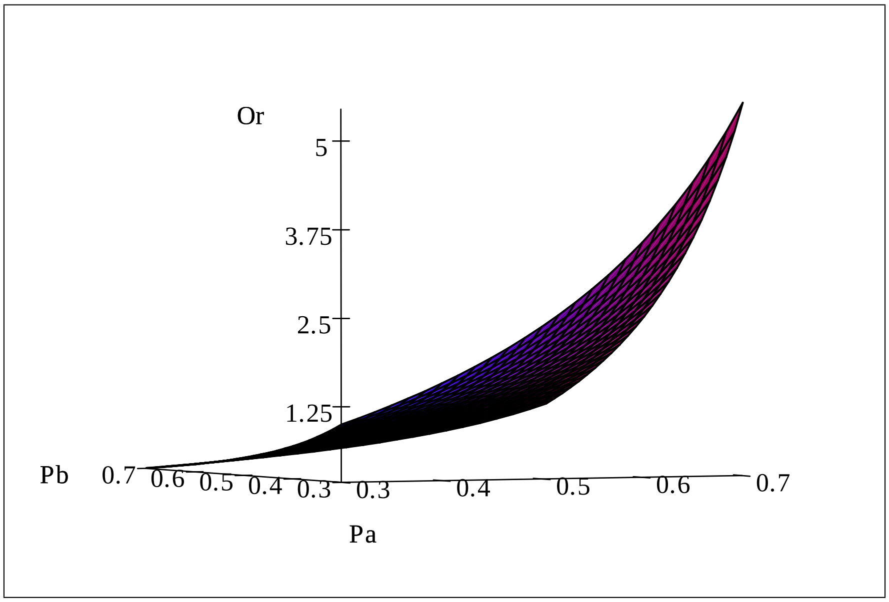
$\ln O = \ln p - \ln(1-p)$, as a function of p

Note that $\ln O$ is zero when $p = .5$ (even odds)

1.2 odd ratios

Now imagine two groups: A and B, maybe women and men, such that the probability of a success for A, p_A, is different from the probability of success for B, p_B. In which case consider the odds ratio

$$O_r = \frac{\frac{p_A}{1-p_A}}{\frac{p_B}{1-p_B}}$$



For example, if $p_A = .75$ and $p_B = .5$, then

$$O_r = \frac{\frac{p_A}{1-p_A}}{\frac{p_B}{1-p_B}} = \frac{\frac{.75}{1-.75}}{\frac{.5}{1-.5}} = 3$$

How would you intuite, in words, this result.