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Introduction to the Theory of Statistics

THIRD EDITION

To HARRIET A.M.M.
To my GRANDCHILDREN F.A.G.
To JOAN, LISA, and KARIN D.C.B.

INTRODUCTION TO THE THEORY OF STATISTICS
INTERNATIONAL EDITION 1974

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PREFACE TO THE THIRD EDITION

The purpose of the third edition of this book is to give a sound and self-contained (in the sense that the necessary probability theory is included) introduction to classical or mainstream statistical theory. It is not a statistical-methods-cookbook, nor a compendium of statistical theories, nor is it a mathematics book. The book is intended to be a textbook, aimed for use in the traditional full-year upper-division undergraduate course in probability and statistics, or for use as a text in a course designed for first-year graduate students. The latter course is often a "service course," offered to a variety of disciplines.

No previous course in probability or statistics is needed in order to study the book. The mathematical preparation required is the conventional full-year calculus course which includes series expansion, multiple integration, and partial differentiation. Linear algebra is not required. An attempt has been made to talk to the reader. Also, we have retained the approach of presenting the theory with some connection to practical problems. The book is not mathematically rigorous. Proofs, and even exact statements of results, are often not given. Instead, we have tried to impart a "feel" for the theory.

The book is designed to be used in either the quarter system or the semester system. In a quarter system, Chaps. I through V could be covered in the first quarter, Chaps. VI through part of VIII the second quarter, and the rest of the book the third quarter. In a semester system, Chaps. I through VI could be covered the first semester and the remaining chapters the second semester. Chapter VI is a "bridging" chapter; it can be considered to be a part of "probability" or a part of "statistics." Several sections or subsections can be omitted without disrupting the continuity of presentation. For example, any of the following could be omitted: Subsec. 4.5 of Chap. II; Subsecs., 2.6, 3.5, 4.2, and 4.3 of Chap. III; Subsec. 5.3 of Chap. VI; Subsecs. 2.3, 3.4, 4.3 and Secs. 6 through 9 of Chap. VII; Secs. 5 and 6 of Chap. VIII; Secs. 6 and 7 of Chap. IX; and all or part of Chaps. X and XI. Subsection 5.3 of Chap VI on extreme-value theory is somewhat more difficult than the rest of that chapter. In Chap. VII, Subsec. 7.1 on Bayes estimation can be taught without Subsec. 3.4 on loss and risk functions but Subsec. 7.2 cannot. Parts of Sec. 8 of Chap. VII utilize matrix notation. The many problems are intended to be essential for learning the material in the book. Some of the more difficult problems have been starred.

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EXCERPTS FROM THE FIRST AND SECOND EDITION PREFACES

This book developed from a set of notes which I prepared in 1945. At that time there was no modern text available specifically designed for beginning students of mathematical statistics. Since then the situation has been relieved considerably, and had I known in advance what books were in the making it is likely that I should not have embarked on this volume. However, it seemed sufficiently different from other presentations to give prospective teachers and students a useful alternative choice.

The aforementioned notes were used as text material for three years at Iowa State College in a course offered to senior and first-year graduate students. The only prerequisite for the course was one year of calculus, and this requirement indicates the level of the book. (The calculus class at Iowa State met four hours per week and included good coverage of Taylor series, partial differentiation, and multiple integration.) No previous knowledge of statistics is assumed.

This is a statistics book, not a mathematics book, as any mathematician will readily see. Little mathematical rigor is to be found in the derivations simply because it would be boring and largely a waste of time at this level. Of course rigorous thinking is quite essential to good statistics, and I have been at some pains to make a show of rigor and to instill an appreciation for rigor by pointing out various pitfalls of loose arguments.

While this text is primarily concerned with the theory of statistics, full cognizance has been taken of those students who fear that a moment may be wasted in mathematical frivolity. All new subjects are supplied with a little scenery from practical affairs, and, more important, a serious effort has been made in the problems to illustrate the variety of ways in which the theory may be applied.

The problems are an essential part of the book. They range from simple numerical examples to theorems needed in subsequent chapters. They include important subjects which could easily take precedence over material in the text; the relegation of subjects to problems was based rather on the feasibility of such a procedure than on the priority of the subject. For example, the matter of correlation is dealt with almost entirely in the problems. It seemed to me inefficient to cover multivariate situations twice in detail, i.e., with the regression model and with the correlation model. The emphasis in the text proper is on the more general regression model.

The author of a textbook is indebted to practically everyone who has touched the field, and I here bow to all statisticians. However, in giving credit to contributors one must draw the line somewhere, and I have simplified matters by drawing it very high; only the most eminent contributors are mentioned in the book.

I am indebted to Catherine Thompson and Maxine Merrington, and to E. S. Pearson, editor of *Biometrika*, for permission to include Tables III and V, which are abridged versions of tables published in *Biometrika*. I am also indebted to Professors R. A. Fisher and Frank Yates, and to Messrs. Oliver and Boyd, Ltd., Edinburgh, for permission to reprint Table IV from their book "Statistical Tables for Use in Biological, Agricultural and Medical Research."

Since the first edition of this book was published in 1950 many new statistical techniques have been made available and many techniques that were only in the domain of the mathematical statistician are now useful and demanded by the applied statistician. To include some of this material we have had to eliminate other material, else the book would have come to resemble a compendium. The general approach of presenting the theory with some connection to practical problems apparently contributed significantly to the success of the first edition and we have tried to maintain that feature in the present edition.