1 Some notes on the uniform distribution

How does one write the density function for the uniform distribution on $[0, 1]$?

$$f_X(x) = \begin{cases} 1 \text{ if } 0 \leq x \leq 1 \\ 0 \text{ otherwise} \end{cases}$$

It can be shown (MGB page 106) that

$$E[X] = \frac{1}{2}$$

and

$$var(X) = \frac{1}{12}$$

How would you write the density function for the uniform distribution if $f(x) > 0 \forall a \leq x \leq b$?

$$f_X(x) = \begin{cases} \frac{1}{b-a} \text{ if } a \leq x \leq b \\ 0 \text{ otherwise} \end{cases}$$

Further assume $b = a + 3$, so we can write the density function

$$f_X(x) = \begin{cases} \frac{1}{3} \text{ if } a \leq x \leq a + 3 \\ 0 \text{ otherwise} \end{cases}$$

Assume you draw a random sample of one observation from this distribution

and the result is 5. What is your best estimate of a? A good estimate of a is 3.5. Note that if $a = 3.5$, then the range on our uniform distribution is $[3.5, 6.5]$, and 5 is the midpoint of this range. Note that it can be shown that 3.5 is the maximum likelihood estimate of a. What is $E[x]$ if $a = 3.5$?

$$E[x : a = 3.5] = \int_{3.5}^{6.5} x \frac{1}{3} dx = \frac{1}{6} x^2 \Big|_{3.5}^{6.5} = 5$$

This is a special case of the proof on page 106 of MGB.