

Willingness to Pay

An Examination of Nonuse Values

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Abstract

This paper examines the willingness to pay for the removal of roads in the North Rim of the Grand Canyon. I begin by giving a brief description of the good in question, with a normative and positive examination for funding this project attached in Appendix A. I then turn to the discussion of how to estimate willingness to pay for this project given that willingness to pay was measured by use of a dichotomous choice question. To estimate willingness to pay I will specify a likelihood function and maximize this function in terms of its parameters with the assistance of a program called Mathematica. This function, as will be seen, is based on the assumption that the error term is normally distributed with a mean of 0, and a variance of σ^2 . After completing these estimations I will make general inferences based on the signs of the estimated parameters. In the end, I hope to better understand how to use maximum likelihood estimation in Mathematica, and the methods/steps behind parameter estimation in terms of dichotomous choice data.

I. Introduction

In 1993 Patricia Champ constructed a contingent valuation survey to measure Wisconsin resident's willingness to pay for the removal of old, unpaved roads along the North Rim of the Grand Canyon. Completion of this environmental project was necessary before the North Rim could be officially designated as a Wilderness Area. Thus, the survey is measuring individual's willingness to pay for a seldom-visited public good (i.e. it is measuring nonuse values¹). Although the examined population lived in Wisconsin at the time they were not exempt from visiting the park. This does not mean that the data is a poor measurement of nonuse values. My reasoning is twofold. First, "The North Rim is 215 miles by road from the popular and often-visited South Rim. As a result, relatively few visitors to the Grand Canyon visit the North Rim (Champ et al., 1993)." Second, it seems as though Wisconsin residents will not directly benefit from this project since the probability that a Wisconsin resident will actually visit the North Rim of the Grand Canyon National Park is small (based on the previous statement made by Champ). Thus, it seems as though this data is a valid source for measuring nonuse benefits.

For the purposes of this paper I will leave the discussion of the normative and positive prescriptions for funding this project to Appendix A. My exploration through the data provided by Patricia Champ will proceed as follows: the simple form for estimating willingness to pay, an analysis of the data set, summary statistics, model

¹ Nonuse values are the benefits individuals receive from knowing that a certain state of the world exists. They are not benefits related to actual use of the good/item in question. For example, a nonuse benefit to me might be whale preservation. Although I will never see these whales I still obtain benefits from knowing that they exist.

specification, parameter estimation, significance testing, and concluding remarks (Sections II, III, IV, V, VI, VII, and VIII respectively). Before continuing note that my main interest in sections VI and VII is to use and better understand maximum likelihood estimation in Mathematica, in order to estimate the parameters of a function of willingness to pay, so I can draw some general inferences from the signs of the parameter estimates.

II. Estimating Willingness to Pay (The Simple Form)

As noted above, the data I have obtained is survey based. Four sets of surveys were mailed. The first two sets employed a dichotomous choice² question asking the respondents willingness to pay. Half of these surveys were hypothetical³, while the other half asked the individual to actually donate the money. The second two sets were also split in a manner where half of the surveys were hypothetical and the other half were actual payment. The only difference is that the second two sets employed an open-ended question in asking willingness to pay.

For the purposes of this assignment I will use the hypothetical treatment and actual payment treatment cases of the dichotomous choice question. A detailed description of the particular dichotomous choice question measuring willingness to pay will be presented below. For now, let's assume that an estimation of willingness to pay from these two sets will take the simple form:

$$WTP_i = \alpha + \beta X_i + v_i \tag{1}$$

² Dichotomous Choice: A dichotomous choice question is one to which the respondent can answer either yes or no. Thus, the willingness to pay question presented some dollar value that the respondent could either accept or decline.

³ Hypothetical: Throughout the paper I will refer to the surveys that hypothetically asked the respondents willingness to pay as the "hypothetical treatment."

where WTP_i is the willingness to pay of individual i , α is some constant, X_i is a vector of demographic variables, and ν_i is a normally distributed random term with a mean of zero and a variance = σ^2 (i.e. $\nu_i \sim N(0, \sigma^2)$). The parameters of this regression, α and β , will then be estimated using the maximum likelihood method. The particulars of the estimation are discussed in greater detail below.

III. The Data Set

Before presenting the summary statistics a brief discussion of the particular dichotomous choice question measuring willingness to pay, the specific variables to be used to estimate willingness to pay, and data cleanup is a must. Willingness to pay was measured in the following way. Each respondent was presented with an offer. These offers, which varied across surveys, were \$1, 5, 8, 12, 15, and 50. If the respondent would pay this amount he/she circled yes, and if not he/she circled no. The remainder of the survey asked a number of preference and demographic questions (in my estimation of willingness to pay I include only five explanatory variables). Note that since I am not going to make any concrete conclusions from these parameter estimates I feel that there is no need to worry about any omitted variable biases.

The five variables of choice include treatment, children, age, gender, and income. Treatment refers to the survey specification (i.e. was the survey hypothetical in nature or actual). Age is measured in its numerical equivalent, i.e. if respondent 6 had an age value of 33 this person is 33 years old. On the other hand, the children variable simply specifies whether or not the family has children. Gender, obviously, specifies whether or

not the respondent is male or female. Income is measured in brackets as shown in Table

1. For example, an individual reporting a 4 falls in the income

Table 1

1	Less than \$10,000	6	\$50,000 to \$59,999
2	\$10,000 to \$19,999	7	\$60,000 to \$69,000
3	\$20,000 to \$29,999	8	\$70,000 to \$79,999
4	\$30,000 to \$39,999	9	\$80,000 to \$99,999
5	\$40,000 to \$49,000	10	\$100,000 or more

range of \$30,000 to \$39,999 per year. Thus, income, children, gender, and treatment will be measured through the use of dummy variables.

So, returning to the estimation of WTP_i , when I include all five variables Equation

(1) can be rewritten as:

$$WTP_i = \alpha + \beta_1 D_{1i} + \beta_2 D_{2i} + \dots + \beta_9 D_{9i} + \beta_{10} D_{10i} + \beta_{11} D_{11i} + \beta_{12} D_{12i} + \beta_{13} X_i + v_i \quad (2)$$

where $D_{1i} = 1$ if \$10,000 to \$19,999

= 0 otherwise,

$D_{2i} = 1$ if \$20,000 to 29,999

= 0 otherwise ...

$D_{8i} = 1$ if \$80,000 to \$99,999

= 0 otherwise

$D_{9i} = 1$ if greater than \$100,000

= 0 otherwise

$D_{10i} = 1$ if the respondent has children

= 0 if no children

$D_{11i} = 1$ if male

= 0 if female

$D_{12i} = 1$ if the survey actually asked the respondent to pay

= 0 if the survey hypothetically asked the respondent to pay, and

X_i = age of individual i .

The data sets I have obtained are fairly comprehensive; unfortunately, some of the individuals did not report all of the relevant information, so in order to simplify

calculations I cut from the sets any individual that failed to report any of these values. Thus, for the hypothetical treatment I cut 50 respondents leaving 342 observations, and for the actual payment treatment I cut 65 respondents leaving 276. With a decent idea of the data I am going to use and how it is measured in mind I shall now turn to the summary statistics.

IV. Summary Statistics

Tables 2 and 3 show the summary statistics for the dichotomous choice hypothetical treatment case and actual payment treatment case, respectively. It is interesting to note that the mean, median, and mode for the age variable in both the hypothetical and actual case are relatively close to one another. The mean, median, and mode for age in the hypothetical and actual cases are 49, 47, 42, and 50, 47, 38, respectively. The reason this is interesting to note is due to the fact that a variable exhibiting the property of the mean, median, and mode equaling each other is said to be normally distributed. Although they are not exactly equal here one might expect that as we increase the sample size these figures will become strikingly similar.

I have also calculated, in Tables 2 and 3, the gender percentage of the population, the percentage of the population with children, the percentage of males and females that responded yes to their offer, the total population income distribution, the income distribution across each offer, and the percentage of individuals that responded either yes or no to each offer (1, 5, 8, 12, 15, 50). The hypothetical (actual) population is approximately 74% (76%) male, and 26% (24%) female. Of these males, 46% (20%) responded yes to any given offer, whereas 55% (24%) of the females responded yes to any given offer. Approximately 76% (63%) of the population had children at the time.

It is most interesting to note that the percentage of respondents answering yes to a given offer dramatically decreases in the event that they have to actually pay. Should one be surprised by this fact? No, for, as noted in Appendix A, this is exactly what theory would have us believe. It is also interesting to note that in the actual case the percentage of individuals responding yes to the \$5 offer decreases to about 16% (from 30% at the \$1 level), then increases to about 30% for the \$8 offer and slightly decreases to 22% for the \$12 offer before dropping off dramatically to 15% and 5% for the \$15 and \$50 offers, respectively. My first thought was that the distribution of income for the \$8 and \$12 group was skewed more towards the higher income brackets as compared to the \$5 group. However, as one can see, this is not the case; instead, they are somewhat similar. Perhaps this phenomenon can simply be explained by the fact that the \$8 individuals have greater nonuse benefits than those given the \$5 offer, or that those given the \$5 offer have tendencies to free ride more often. There is also the possibility that since each offer group had relatively few respondents (ranging from a low of 27 to a high of 69) it would only take a few outliers to skew the results of the group.

Summary statistics are important in giving one an idea of their data set; however, they can only go so far, and if one is looking to estimate willingness to pay he/she will need to turn to parameter estimation. Although I will draw some general inferences based on the signs of the estimated parameters, I am not interested in estimating willingness to pay in such an exact manner to which one can make concrete conclusions. Instead, as noted above, I want to use maximum likelihood estimation to estimate the parameters in Equation (2) in an attempt to better understand maximum likelihood

estimation in Mathematica and the methods/steps involved in estimating willingness to pay given a survey that employs a dichotomous choice question.

V. Model Specification

As noted in Equation (2) willingness to pay will take the form:

$$WTP_i = \alpha + \beta_1 D_{1i} + \beta_2 D_{2i} + \dots + \beta_9 D_{9i} + \beta_{10} D_{10i} + \beta_{11} D_{11i} + \beta_{12} D_{12i} + \beta_{13} X_i + v_i \quad .$$

So, how do we can estimate these parameters given that willingness to pay was measured from a yes/no type question? First, for simplicity let's begin with one explanatory variable. Thus, I will simply use Equation (1), from above:

$$WTP_i = \alpha + \beta X_i + v_i$$

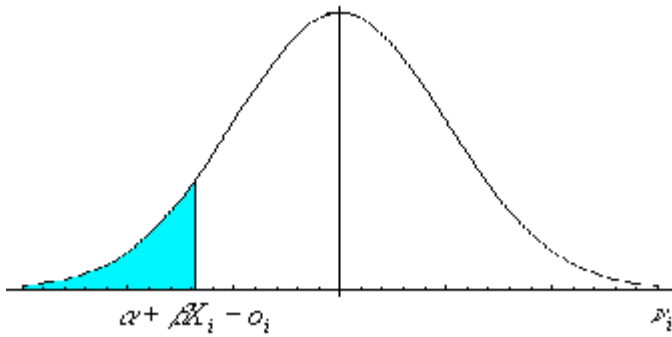
Next, let's specify what we know. Individual i is asked if sh/e will pay offer o_i , which varies across individuals. This individual's response is either yes ($p_i = 1$) or no ($p_i = 0$). Recall that I assumed $v_i \sim N(0, \sigma^2)$, and that we want to estimate α , β , and σ^2 . The first step in our estimation is to determine the probability that individual i will circle yes to o_i .

That is:

$$\begin{aligned} & \text{Prob}(WTP_i \geq o_i) . \text{ By substituting (1) in for } WTP_i \text{ we get:} \\ & = \text{Pr ob}(\alpha + \beta X_i + v_i \geq o_i) \\ & = \text{Pr ob}(v_i \geq o_i - \alpha - \beta X_i) \\ & \text{Pr ob}(WTP_i \geq o_i) = \text{Pr ob}(v_i \leq \alpha + \beta X_i - o_i) \end{aligned} \quad (3)$$

Graphically, (3) is represented by the shaded area in Figure 1.

Figure 1



Now, divide (3) through by σ :

$$\begin{aligned}
 &= \text{Pr ob} \left(\frac{v_i}{\sigma} \leq \frac{\alpha + \beta X_i - o_i}{\sigma} \right). \text{ Let } \hat{v}_i = \frac{v_i}{\sigma} \sim N(0,1) \\
 &= \text{Pr ob} \left(\hat{v}_i \leq \frac{\alpha + \beta X_i - o_i}{\sigma} \right).
 \end{aligned}$$

By dividing through by σ we now have a standard normal distribution. Therefore,

$$\text{Pr ob}(WTP_i \geq o_i) = \Phi \left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right) \quad (4)$$

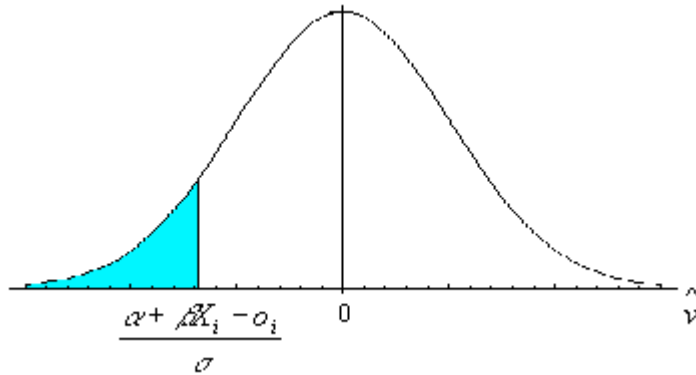
where Φ is the cumulative density function (CDF) of the standard normal distribution.

Why is this true? Graphically, the $\text{Pr ob}(WTP_i \geq o_i)$ is the area under the probability density function (PDF) of the standard normal distribution from negative infinity to

$\left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right)$ (see Figure 2), and this is simply the area under the CDF up to

$\left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right)$.

Figure 2



Finally, we can define the likelihood function⁴, and the log of the likelihood function for my sample of 618.

$$L = \prod_{i=1}^{618} \left[\Phi \left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right)^{p_i} \left[1 - \Phi \left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right) \right]^{(1-p_i)} \right] \quad (5)$$

$$\ln L = \sum_{i=1}^{618} p_i \ln \left[\Phi \left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right) \right] + (1 - p_i) \ln \left[1 - \Phi \left(\frac{\alpha + \beta X_i - o_i}{\sigma} \right) \right] \quad (6)$$

I can now use Mathematica to find α , σ , and the β 's that maximize $\ln L$, (6).

Before continuing to the parameter estimation section note that in order to define Equation (5) one must have a random sample. It can be argued that the sample my data originated from is not random in the sense that it targeted Wisconsin residents. Although it targeted this state the surveys were randomly mailed throughout the state.

So, to the extent that this data came from a randomly sampled Wisconsin population I can safely define Equations (5) and (6).

VI. Parameter Estimation Through The Use of Mathematica

The next step in this project is to import both the hypothetical and actual data sets into Mathematica and use its FindMinimum command to find the values of my parameters that maximize Equation (6).

Let's begin with a brief discussion of the main problem I encountered in Mathematica. In my first attempt at estimating the parameters I created a variable for

$\Phi\left(\frac{\alpha + \beta X_i - o_i}{\sigma}\right)$. That is, I set the former equal to K . However, when I plugged K into

Equation (6) Mathematica did not realize that I wanted it to also sum for all of the X_i 's and o_i 's. Since it did not sum for all of the X_i 's and o_i 's the ln L did not have a maximum. Thus, when I ran the FindMinimum command for the negative of Equation (6), with respect to its parameters, Mathematica produced an output that consisted of errors. To solve this problem I had to plug the form of K directly into Equation (6), which is what you will see directly below.

In my first parameter estimation trial I only included the constant, α . With only this one parameter the log of the likelihood looks like the following in Mathematica:

⁴ For a discussion of likelihood functions and maximum likelihood estimation see Mood, Graybill, and Boes pages 276 to 286.

$$L = \sum_{i=1}^{618} ((p_i * \text{Log}[\text{CDF}[\text{ndist}, ((\alpha - o_i) / (\sigma))]]) + ((1 - p_i) * \text{Log}[1 - \text{CDF}[\text{ndist}, ((\alpha - o_i) / (\sigma))]]))$$

Where *ndist* is the normal distribution with a mean of 0 and a variance of 1 (i.e. it is the standard normal). Recall, this is what was shown up to Equation (4) and carried into Equations (5) and (6). After Mathematica ran this I asked it to find the values of α , and σ that maximized L. That is, I asked Mathematica to:

FindMinimum[-L, { α , -19.8}, { σ , 10}] .

From this it gave me an output of {398.789, { $\alpha \rightarrow -19.8035$, $\sigma \rightarrow 9.59858$ }} (AB.1 in Appendix B). (To run the program yourself and obtain these results please see the attached diskette.) Notice above that I am finding the minimum of $\sim L$, which is the same as finding the maximum of L. Also, notice that the parameter values I told Mathematica to search for the maximum around are strikingly similar to the actual values that maximize L. When I began I did not know where the maximum occurred. So, in order to obtain these starting values I first ran the command searching around arbitrarily chosen values for all of the parameters. Then, I took the output values, made them my new starting points, and ran the process again. I repeated this step once more before accepting the values.

Interpreting the output is fairly straightforward. In this simple case $\alpha = -19.8$, and $\sigma = 9.60$, and the value of -L at these points is 398.789 (the value of the function will be of greater importance when testing for significance). So, $\hat{WTP}_i = -19.8035$, which implies that everyone's expected willingness to pay is equal to -19.8035.

In my second trial I added age to the equation. Then I added the treatment variable (actual or hypothetical), followed by children, then gender, and finally I added the dummy variables for income. All of the results are shown in Appendix B. Notice in Appendix B that all of the parameter labels correspond to those listed on page 4. The only difference is that I had to use B1, B2, and B3 in place of β_1 , β_2 , and β_3 since Mathematica considers, for example, β_2 and β_{12} the same. When I included all of the explanatory variables the output became:

{343.871, { $\alpha \rightarrow 31.153$, $\beta_{13} \rightarrow -0.500905$, $\beta_{12} \rightarrow -49.0918$,
 $\beta_{10} \rightarrow -8.13489$, $\beta_{11} \rightarrow -21.9528$, B1 $\rightarrow 14.328$, B2 $\rightarrow 12.888$, B3 $\rightarrow 14.2973$, $\beta_4 \rightarrow 25.2027$,
 $\beta_5 \rightarrow 42.3251$, $\beta_6 \rightarrow 45.9897$, $\beta_7 \rightarrow 59.3985$, $\beta_8 \rightarrow 39.2471$, $\beta_9 \rightarrow 98.7834$, $\sigma \rightarrow 7.84336$ }}

In terms of Equation (2) \hat{WTP}_i becomes:

$$\hat{WTP}_i = 31.15 + 14.33D_{1i} + 12.88D_{2i} + 14.30D_{3i} + 25.20D_{4i} + 42.33D_{5i} + 45.99D_{6i} + 59.40D_{7i} + 39.25D_{8i} + 98.78D_{9i} - 8.13D_{10i} - 21.95D_{11i} - 49.09D_{12i} - 0.50X_i$$

The income, treatment, and children signs are what we would expect. All of the income signs are positive and become larger as income increases (with the exception of D_{2i} , and D_{8i}). It is possible that these decreases can be explained by the fact that, as noted earlier, since each offer group had relatively few respondents (ranging from a low of 27 to a high of 69) it would only take a few outliers to skew the results of the group. In addition to the increasing nature of the income coefficients, notice the relative closeness in the values of D_{1i} , D_{2i} , D_{3i} and D_{5i} , D_{6i} , D_{7i} . This suggests that one could group the income variable into three brackets (poverty, middle class, and upper class), as opposed to the ten brackets I am currently using.

The sign on D_{12i} is negative, indicating that those who are asked to actually pay gave less, on average. The sign on children is also negative, implying that families with children tended to pay less. Although it is possible to make general inferences based on the signs of the coefficients one cannot make any concrete conclusions based on the values of the coefficients. This is due to the fact that, as noted earlier, by selectively choosing a handful of all the possible explanatory variables I was bound to have omitted some of those that might have some importance in estimating willingness to pay.

VII. Significance Testing

It is of particular interest to be able to examine the parameters and discuss their meaning. However, examining an insignificant parameter is of little value to us. Thus, it is only fitting that we test the significance of the explanatory variables used. In testing for significance I am going to use the likelihood ratio test.

The logic behind the likelihood ratio test is as follows. Assume that the log-likelihood function takes the form:

$$\ln L = \sum_{i=1}^{618} p_i \ln \left[\Phi \left(\frac{\alpha + \beta_{13} X_i - o_i}{\sigma} \right) \right] + (1 - p_i) \ln \left[1 - \Phi \left(\frac{\alpha + \beta_{13} X_i - o_i}{\sigma} \right) \right] \quad (7).$$

Now, suppose that our null hypothesis (H_0) is that β_{13} , the coefficient of the age variable (X_i), is equal to zero. In this case the log-likelihood function will become:

$$\ln L = \sum_{i=1}^{618} p_i \ln \left[\Phi \left(\frac{\alpha - o_i}{\sigma} \right) \right] + (1 - p_i) \ln \left[1 - \Phi \left(\frac{\alpha - o_i}{\sigma} \right) \right] \quad (8).$$

"Equation (8) is known as the restricted log-likelihood function (RLLF) because it is estimated with the restriction that, a priori, β is zero, whereas Equation (7) is known as

the unrestricted log-likelihood function (ULLF) (Gujarati, 280-281)." To test H_0 , the likelihood ratio test obtains the following test statistic:

$$\lambda = -2(RLLF - ULLF).$$

If the sample size is large, this test statistic is chi-square distributed with degrees of freedom equal to the number of restrictions imposed. Thus, for one degree of freedom, at the 5% level of significance, one would reject the null hypothesis if $\lambda > 5.02$ or if $\lambda < 0.000982$.

This test seems so simple, and normally it is; however, in my first attempt at testing for significance I experienced difficulties. Normally, one would expect the functional value for the unrestricted log-likelihood function to be greater than the value for the restricted log-likelihood function. That is, $\ln L_{UR} > \ln L_R$. Why is this?

Maximum likelihood estimation maximizes the log-likelihood function, and dropping variables generally leads to a smaller, or at least no larger, log-likelihood functional value. However, notice, in Appendix B, that as I add explanatory variables the functional value of the log-likelihood decreases. So, all of my test statistics are negative, which implies that, no matter the number of degrees of freedom or the significance level, I will always reject the null hypothesis, and all of my explanatory variables will be significant. Unfortunately, this seems problematic, for the chi-square distribution is never negative.

So, where did my problem lie? Recall from above that the value of the negative log-likelihood function for the case where I only include the constant term and standard deviation is 398.789. Thus, the functional value for the positive log-likelihood function is -398.789. With this problem solved we are now able to perform the likelihood ratio test.

To begin I will test the null hypothesis that $\beta_{13} = 0$. To do this I will need to take the negative of the functional value of the restricted log-likelihood function (AB.1), subtract it from the unrestricted log-likelihood function (AB.2), and multiply by -2.

$$\lambda = -2(-398.789 - -394.403) = 8.722$$

Notice, AB.1 only includes α , making it the RLLF, whereas AB.2 includes both α and the age variable, making it the ULLF.

Recall that λ is chi-square distributed with degrees of freedom equal to the number of restrictions. In this case we have 1 degree of freedom. So, we reject H_0 if $\lambda < 0.000982$ or if $\lambda > 5.02$. Since 8.772 is greater than 5.02 we can reject H_0 . Thus, age is a significant variable. Although age is a significant variable we cannot place much importance on it in the case where I include all of the explanatory variables. This is due to the fact that the coefficient for age is $\bar{.5}$, which is not significantly different from zero.

Appendix C shows the results for the remainder of the variables. It is interesting to note that children and gender, when tested individually, are significant; furthermore, when testing the group of variables we find that, as a group, they are significant (as compared to AB.1).

VIII. Concluding Remarks

At times Mathematica may seem difficult and confusing in its functionality. However, as can be seen, its power and usefulness provides us with a tool that can accomplish large and seemingly impossible tasks. In this paper not only should the reader better understand willingness to pay as an economic topic, but also how to use

maximum likelihood estimation in Mathematica to estimate willingness to pay given survey data that employs a dichotomous choice question. In addition, the reader should feel confident in using the likelihood ratio test to see if the particular parameters used in the estimation are significant. In the end, I feel confident of my understanding of maximum likelihood estimation, its use in estimating willingness to pay given referendum data, and significance testing with the likelihood ratio test.

Appendix A

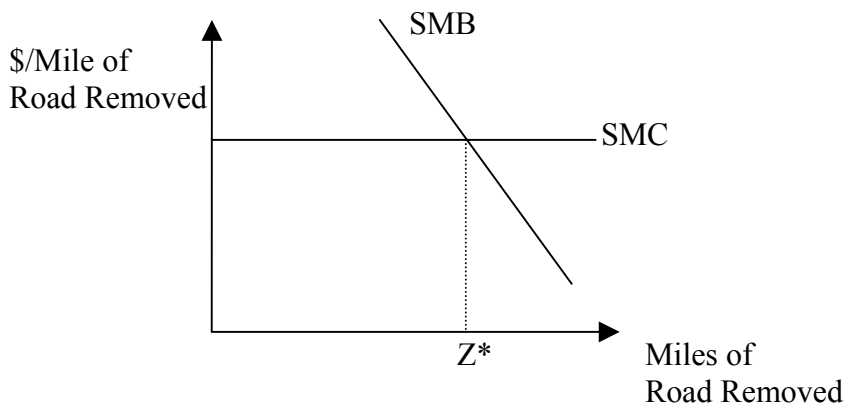
AI. Introduction

Throughout this appendix I will refer to the removal of these roads as public in nature. My reasoning for this reference is as follows. When considering the efficient level of a public good one often thinks of both the quality and quantity of the good. For example, when I think of visiting a local park I consider its size and cleanliness. If Scott Carpenter Park were dirty with garbage then clean up would increase the quality of the park. Therefore, since the quality of the North Rim increases with the removal of the roads, in the sense that it will have the prestigious status of a Wilderness Area, I will consider this project as public in nature.

AII. Normative Analysis

In a perfect world how would one like to fund this project? First, let's assume that the efficient level of quality in the North Rim is associated with complete road removal. Complete road removal is shown in Figure 1 as Z^* . Notice in Figure 1 that the social marginal cost is the same for each mile of road removed. This is a reasonable assumption, for the removal of the first mile of road is likely to cost the same as the removal of the last mile.

Figure 1



So, in our perfect world, the social marginal benefit curve would be calculated from the vertical summation of all the private marginal benefit (PMB) curves. With knowledge of every household's PMB one would want to fund this project by making each household donate an amount proportional to the benefits each household receives. Therefore, household h will pay an amount equal to its PMB multiplied by Z^* (i.e., household h pays = $PMB \times Z^*$)⁵.

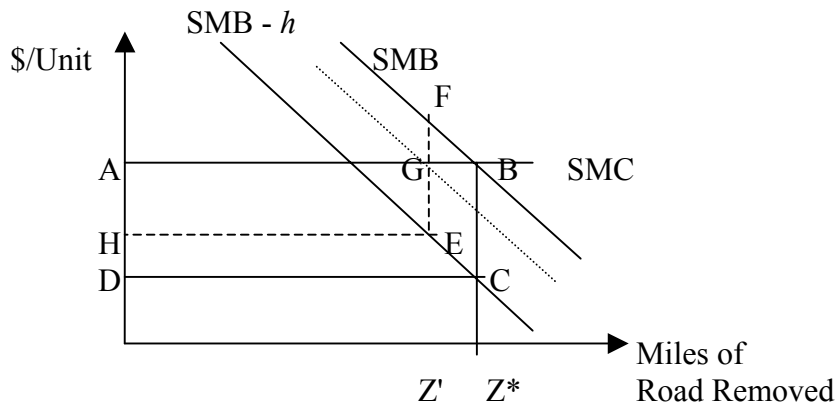
Unfortunately, this is not how the real world works. For example, let's say I am working for the Grand Canyon National Park, and they wanted to know if some group of people, Wisconsin residents in this case, would be willing to fund Z^* miles of road removal. To answer their question I would go out and hypothetically sample the population. After sampling the population I would have some figure indicating the potential amount of road removal. Assume, for the moment, that from the hypothetical prescription I conclude that it is possible to fund the project. However, when I collect donations the figure from above will, more than likely, be different from the hypothetical amount, which will be different from the optimal road removal amount. Based on economic theory one would expect the actual amount of road removal to be less than the hypothetical and optimal amount of road removal. My reasoning is threefold.

First, assume I actually collect funds during my first sampling of the population, instead of hypothetically. Optimally, I would like to make everyone pay based on the benefit principle. So, I let the sample know that after they tell me their PMB I am going to make them pay based on the benefit principle. That is, their cost share will equal their PMB share at the efficient level of road removal (Z^*). The problem encountered through

⁵ This is known as the Benefit Principle. Under this principle costs are shared in proportion to benefits received. For proof that household h pays an amount equal to $PMB \times Z^*$ see the end of this appendix.

the use of this method is that of free riding. To help explain why individuals would free ride let's examine Figure 2.

Figure 2

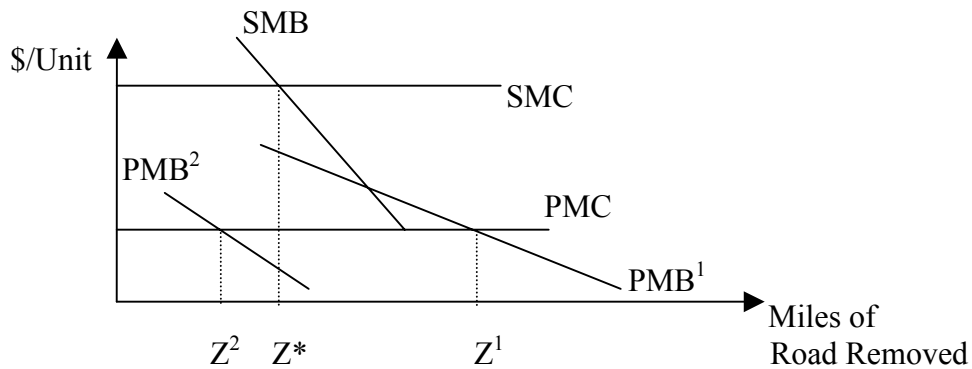


Note, in Figure 2, $SMB - h$ is the social marginal benefit excluding household h . So, after asking household h for its PMB it would think, "If everybody truthfully reports, including myself, then all of the road will be removed together and we will have to pay a total of ABCD dollars. On the other hand, if everybody truthfully reports and I under-declare (represented by the dotted line), then I will lose benefits equal to BCEF and save BCDHEG. Since the amount I save is greater than my loss I will under-declare." So, it is obvious to see that there are gains to be made from free riding. To the extent that people free ride, which seems rational since there are gains to be made, funding will fall short of complete road removal (Z^*).

Since it is known that the above method leads to free riding I will change the way in which I make each household pay. In this second method I will inform the sample that their cost share is going to be independent of the stated PMB (i.e. everyone will pay the same amount, and face the same PMC curve). Again, households would probably not give their true PMB. Why is this? Take, for example, the PMB of household 1, PMB^1 in

Figure 3. This person is better off knowing that all of the roads have been removed. In other words, household 1 strongly supports Wilderness Areas. So, to make sure the project is completed, this household will over-declare. Similarly, a household, household 2 in Figure 3, may have reasons to under-declare. Thus, the sample will tend to give non-sensical messages.

Figure 3



Finally, as noted earlier, people tend to over-declare in hypothetical situations. That is, when costs are free, households tend to state more than if they were asked to actually give. Through examination of the data it is obvious that households tend to under-declare. I further explore this idea in the summary statistics and parameter estimation sections in the main text.

Thus far I have shown that, in a perfect world with perfect information, one would like to collect each household's PMB and divide the costs by the benefit principle. Unfortunately, to the extent that information is imperfect, we have to ask each household for its PMB, and this, as we have seen, tends to give the incorrect/inefficient amount of the public good in question.

AIII. Positive Analysis

Since we will never be able to accurately obtain household h 's PMB we must use a different mechanism to collect the funds for such a project. Under normal circumstances it seems logical that the Grand Canyon National Park would simply allocate part of its budget towards the project⁶. However, at the time, the park did not have the funding to pay for food and supplies for the volunteers who were to remove the roads. This is where Patricia Champ and her survey come into play. Her survey asked people to make voluntary contributions for the project. This form of fundraising, as the reader may already be aware, is problematic. To examine the problems encountered let's take a numerical approach.

Let's assume that the household with the highest PMB has contributed enough money for the removal of ten miles of road, and that the optimal amount of road removal is 50 miles. Further, assume there are five other households in the population with the same PMB (equal to \$640), and that the removal of one mile of road costs \$640. So, if my family were to contribute enough money for the removal of another mile of road they would gain a PMB of \$640 and lose a cost of \$640. Thus, their net benefit equals zero. Since their net benefit is zero they are indifferent as to whether or not they should contribute. Imagine a case where their PMB equaled \$630; in such a case they would refrain from contributing. Therefore, when deciding whether or not to contribute, each household only considers its individual PMB and PMC. Since each household ignores the benefit that could be gained by other households from its contribution "too little" is provided. However, if all five families were made to contribute for the removal of one mile of road, my family would gain a PMB equal to \$3,200 ($5 \times \640), and lose PMC

equal to \$640. Thus, my family's net benefit equals \$2,560. Notice, in this example, that every family has the same PMB schedule. For such a program, where someone forces everyone to contribute, to be fair every household must have relatively close PMB curves. If, for example, a household did not care about the program at all then they are obviously made worse off.

In the end, theory tells us that unless people are forced to pay they will contribute "too little," and this is exactly what happened to Patricia Champ. She notes that the money raised through her survey was insufficient to cover the project in full. However, since the area has been designated as a Wilderness Area additional funding must have come from somewhere. It is possible that the National Park eventually integrated this project into its budget. Under such a situation those who benefit the most from use of the park, i.e. those who visit the most, are the people that helped fund the project. This type of funding is beneficial, for not all households get equal benefit from knowing that the North Rim is a Wilderness Area, and since those households who benefit more, presumably those who go to the park more often, pay more, this benefit principle is considered more fair. Note, there are a couple of ideas dealing with entrance fees that, for the purposes of this paper, the reader needs only to be aware of. First, funding through the use of entrance fees moves one away from measuring non-use values to the measurement of direct-use values. Second, there is a deadweight loss associated with the entrance fees (higher entrance fees correspond to a higher deadweight loss).

⁶ A majority of the parks revenues comes from entrance fees.

Benefit Principle

Under the Benefit Principle the price household h pays at equilibrium is equal to:

$$\begin{aligned} & \left(\frac{PMB^h}{SMB} \right) \times (TotalCost) \quad \text{where } TC = SMC \times Z^* \\ & = \left(\frac{PMB^h}{SMB} \right) \times (SMC \times Z^*) \quad \text{at equilibrium } SMB = SMC \\ & = PMB^h \times Z^* \end{aligned}$$

Appendix B

Note : All of the parameter labels correspond to that which

is listed on page 10 of the text . The only difference is that I had to use B1, B2, and B3 in place of β_1 , β_2 , and β_3 since Mathematica considers, for example, β_2 and β_{12} the same .

```
FindMinimum[-L, { $\alpha$ , -19.8}, { $\sigma$ , 10}]
```

```
{398.789, { $\alpha$   $\rightarrow$  -19.8035,  $\sigma$   $\rightarrow$  9.59858}}
```

Here I only estimated the constant and standard deviation . (AB .1)

```
FindMinimum[-L, { $\alpha$ , 26.75}, { $\beta_{13}$ , -.9597}, { $\sigma$ , 9.66475}]
```

```
{394.403, { $\alpha$   $\rightarrow$  26.7679,  $\beta_{13}$   $\rightarrow$  -0.96375,  $\sigma$   $\rightarrow$  9.68905}}
```

Here I estimated the constant term, the parameter on age, and the standard deviation . (AB .2)

```
FindMinimum[-L, { $\alpha$ , 47.78}, { $\beta_{13}$ , -.76}, { $\beta_{12}$ , -57.99}, { $\sigma$ , 8.58}]
```

```
{367.769, { $\alpha$   $\rightarrow$  49.1656,  $\beta_{13}$   $\rightarrow$  -0.787625,  $\beta_{12}$   $\rightarrow$  -60.7435,  $\sigma$   $\rightarrow$  8.74518}}
```

Here I estimated the constant term, the paramaters on age and treatment, and the standard deviation . (AB .3)

```
FindMinimum[-L, { $\alpha$ , 47.93}, { $\beta_{13}$ , -.73}, { $\beta_{12}$ , -58.74}, { $\beta_{10}$ , -1.98}, { $\sigma$ , 8.62}]
```

```
{367.747, { $\alpha$   $\rightarrow$  49.8724,  $\beta_{13}$   $\rightarrow$  -0.774827,  $\beta_{12}$   $\rightarrow$  -59.1373,  $\beta_{10}$   $\rightarrow$  -1.74288,  $\sigma$   $\rightarrow$  8.64951}}
```

Here I estimated the constant term, the paramaters on age, treatment, and children, and the standard deviation . (AB .4)

```
FindMinimum[-L, { $\alpha$ , 56.2}, { $\beta_{13}$ , -.727}, { $\beta_{12}$ , -58.04}, { $\beta_{10}$ , -.88}, { $\beta_{11}$ , -12.95}, { $\sigma$ , 8.558}]
```

```
{366.734,  
{ $\alpha$   $\rightarrow$  57.626,  $\beta_{13}$   $\rightarrow$  -0.764496,  $\beta_{12}$   $\rightarrow$  -58.6192,  $\beta_{10}$   $\rightarrow$  -0.313117,  $\beta_{11}$   $\rightarrow$  -12.4519,  $\sigma$   $\rightarrow$  8.59158}}
```

Here I estimated the constant term, the paramaters on age, treatment, children, and gender and the standard deviation . (AB .5)

```
FindMinimum[-L, { $\alpha$ , 30.3321}, { $\beta_{13}$ , -.447946}, { $\beta_{12}$ , -51.9329}, { $\beta_{10}$ , -9.58529},  
{ $\beta_{11}$ , -22.9526}, {B1, 15.0199}, {B2, 13.7692}, {B3, 14.2882}, { $\beta_4$ , 24.6092},  
{ $\beta_5$ , 40.851}, { $\beta_6$ , 43.9601}, { $\beta_7$ , 59.2655}, { $\beta_8$ , 38.4094}, { $\beta_9$ , 98.1715}, { $\sigma$ , 8.558}]
```

```
{343.871, { $\alpha$   $\rightarrow$  31.153,  $\beta_{13}$   $\rightarrow$  -0.500905,  $\beta_{12}$   $\rightarrow$  -49.0918,  $\beta_{10}$   $\rightarrow$  -8.13489,  $\beta_{11}$   $\rightarrow$  -21.9528,  
B1  $\rightarrow$  14.328, B2  $\rightarrow$  12.888, B3  $\rightarrow$  14.2973,  $\beta_4$   $\rightarrow$  25.2027,  $\beta_5$   $\rightarrow$  42.3251,  $\beta_6$   $\rightarrow$  45.9897,  
 $\beta_7$   $\rightarrow$  59.3985,  $\beta_8$   $\rightarrow$  39.2471,  $\beta_9$   $\rightarrow$  98.7834,  $\sigma$   $\rightarrow$  7.84336}}
```

Here I added in all of the income variables . Thus, this includes all of the explanatory variabels . (AB .6)

Appendix C: Likelihood Ratio Testing at the 5% Level of Significance

Null Hypothesis	λ	df	RLLF Value / ULLF Value	Reject If	Reject Null?
$\beta_{13} = 0$	8.772	1	AB.1 / AB.2	$\lambda < 0.000982$ or $\lambda > 5.02$	Yes
$\beta_{12} = 0$	53.268	1	AB.2 / AB.3	$\lambda < 0.000982$ or $\lambda > 5.02$	Yes
$\beta_{10} = 0$	0.044	1	AB.3 / AB.4	$\lambda < 0.000982$ or $\lambda > 5.02$	No
$\beta_{11} = 0$	2.026	1	AB.4 / AB.5	$\lambda < 0.000982$ or $\lambda > 5.02$	No
β_1 through $\beta_9 = 0$	45.726	9	AB.5 / AB.6	$\lambda < 2.70$ or $\lambda > 19.0$	Yes
β_1 through $\beta_{13} = 0$	109.836	13	AB.1 / AB.6	$\lambda < 5.01$ or $\lambda > 24.7$	Yes

