Econ 6818: Econometric Methods and Applications

Quiz 1

1. (3 points) We have studied sample spaces, outcomes, events, probability and probability functions. Explain to the reader what is meant by the “probability of an event”.

Simply put, the *probability of an event* is the number associated with that event by the *probability function*. A *probability function* is a function with certain properties that associates a number with every possible event. This is a sufficient explanation but not that enlightening.

Explaining further: The probability function imposes certain properties on the number it attaches to an event. The number must be between zero and one, inclusive. In addition, if one “summed” the numbers associated with all possible events, the sum would equal one. The number associated with no event occurring is zero, and the number associated with events A or B occurring cannot be less than the number associated with either A or B alone.

What does this *number* indicate. Considering its properties, it indicates the “likelihood” of an event, in that the “probability” of nothing happening is zero (zero indicates the event cannot happen), the “probability” of something happening is one (one indicates the event is certain), and the “probability” of events A and B happening can’t be less the the “probability” of either alone.

One might define the probability of an event in the following way. If an experiment were repeated an infinite number of time, the probability of the event would equal the proportion of times the event occurred. This would be a particular specification of the probability function. One could then estimate the probability of an event by the proportion of times it occurs in n experiments.

The classical definition of the probability of an event and the frequency definition of the probability of an event imply two specific probability functions.

Note that:

> A giraffe is a giraffe and a probability is a probability but neither phrase defines the term.

2. (3 points) What is a random variable? and what role do they play in statistics?

A rv can be defined in two ways. I find both definitions add insights.

A rv is a variable that has some distribution. That is, it has some density function or probability density function.

A rv is a function that associates a real number with each event in event space. Because it associates different numbers with events, it varies.

Simply put, rvs characterize events in terms of the value of a real number. They are the link between probability theory and distribution theory. Since events are uncertain, the specific value a rv will take is also uncertain, but it will have some distribution, which can be described by a density function or probability density function. Associating numbers with events makes it much easier to study events and their probabilities.

3. (4 points) Consider two events, A and B. In general, what is $P(A \cup B)$ and how does it relate to $P(A)$ and $P(B)$? Why? What if A and B were independent?

$P(A \cup B)$ means the probability of either events A or B or both occurring.
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where $P(A \cap B)$ is the probability that events A and B simultaneously occur. If one did not subtract $P(A \cap B)$ from $P(A) + P(B)$, one would be double counting in terms of the likelihood of A or B.

If A and B are mutually exclusive then $(A \cap B) = f \Rightarrow P(A \cap B) = 0$. In which case, $P(A \cup B) = P(A) + P(B)$.

However, mutually exclusive and independent are, in general, not compatible. If A and B are independent, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ because independence implies $P(A \cap B) = P(A)P(B)$.

Note that if $P(A) > 0$, $P(B) > 0$ and A and B are independent, A and B cannot be mutually exclusive.

If A and B are independent, $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ simplifies to $P(A \cup B) = P(A) + P(B)$ only if if $P(A) = 0$ or $P(B) = 0$.