The University of Colorado at Boulder
Deparment of Economics
ECON 6808-1998

Quiz Four--September 29, 1998

SCENARIO: Wilbur maximizes the linear utility function $u = a s + b b$ subject to a linear money constraint $m = p_s s + p_b b$ and a linear time constraint $t = t_s s + t_b b$, where s is the number of ski trips taken, b is the number of games bowled, p_s is the of a ski trip, p_b is the cost to bowl a game, m is total income, t_s is how many hours are required for each ski trip, t_b is how hours are required to bowl a game, and t is the total time available.

1. Sketch out three possible cases of Wilbur's constraints: one where the money constraint plays no role, one where the time constraint plays no role, and one where both constraints can play a role.

Wilbur solves the following problem:

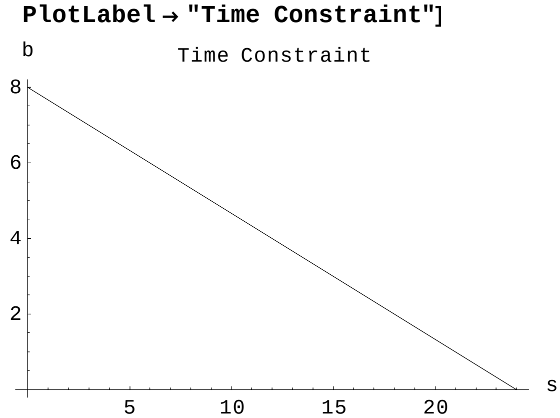
Let's assume that $p_s=30$, $p_b=20$, $m=100$, $t_s=1$, $t_b=3$ and $t=24$.

With the above values, the two constraints can be graphically represented as follows:

```
timeconstraint = Line[{{0, 24 / 3}, {24, 0}}]

Line[{{0, 8}, {24, 0}}]

timegraph = Show[Graphics[timeconstraint], Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "Time Constraint"]
```

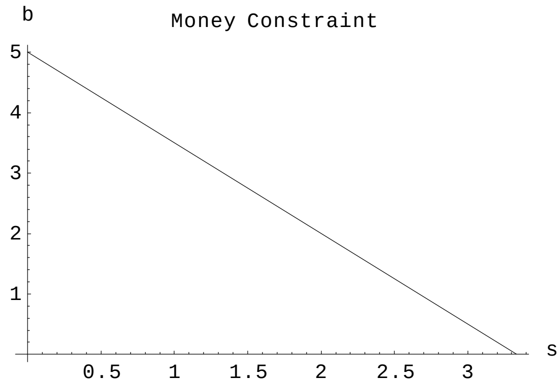



```
Graphics
```

```
moneyconstraint = Line[{{0, 100 / 20}, {100 / 30, 0}}]
```

Line[{{0, 5}, {10/3, 0}}]

```
moneygraph = Show[Graphics[moneyconstraint], Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "Money Constraint"]
```

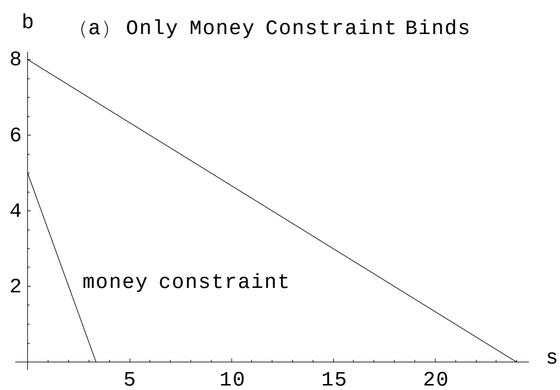


Graphics

Lets sketch three possible cases of Wilbur's constraints: (a) the time constraint plays no role; (b) the money constraint plays no role; and (c) both constraints can play a role.

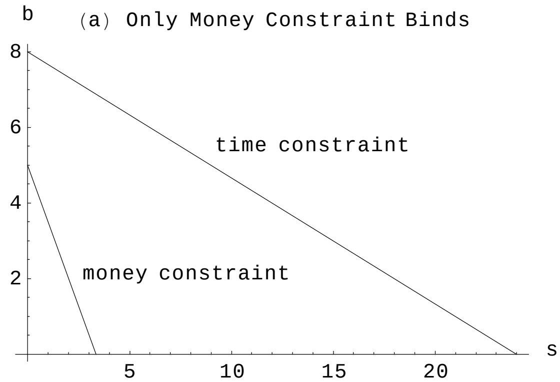
For case (a), the following parameter values are assumed: p_s=30, p_b=20, m=100, t_s=1, t_b=3, t=24. Only money constraint binds. The bundles of ski trips and bowling games feasible with the money endowment are also feasible with the time endowment, but not visa versa.

```
notime =
 Show[Graphics[Text["money constraint", {2.5, 2}, {-1, 0}]], moneygraph, timegraph,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(a) Only Money Constraint Binds"]
```



Graphics

```
notime2 = Show[Graphics[Text["time constraint", {9, 5.4}, {-1, 0}]], notime,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(a) Only Money Constraint Binds"]
```



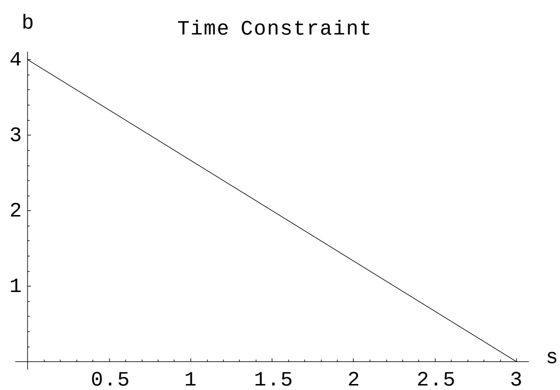

```
Graphics
```

For case (b), the following parameter values are assumed: p_s=30, p_b=20, m=100, t_s=8, t_b=6, t=24. Only time constraint binds. The bundles of ski trips and bowling games feasible with the time endowment are also feasible with the money endowment, but not visa versa.

```
timeconstraint = Line[{{0, 24 / 6}, {24 / 8, 0}}]
```

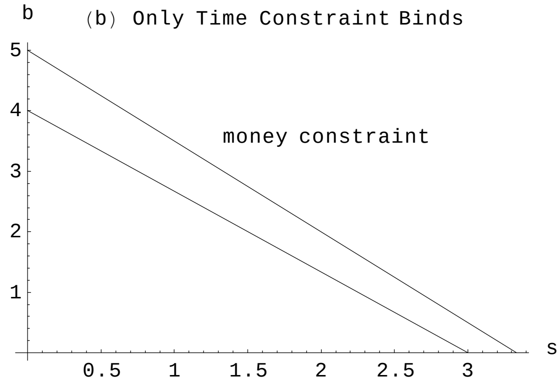
```
Line[{{0, 4}, {3, 0}}]
```

```
timegraph = Show[Graphics[timeconstraint], Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "Time Constraint"]
```



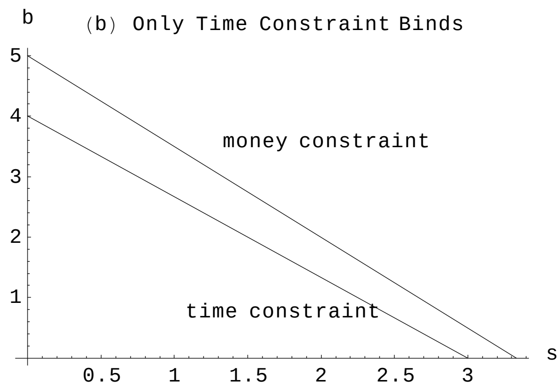

```
Graphics
```

```
nomoney =
 Show[Graphics[Text["money constraint", {1.3, 3.5}, {-1, 0}]], moneygraph, timegraph,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(b) Only Time Constraint Binds"]
```




```
Graphics
```

```
nomoney2 = Show[Graphics[Text["time constraint", {2.4, 0.7}, {1, 0}]], nomoney,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(b) Only Time Constraint Binds"]
```



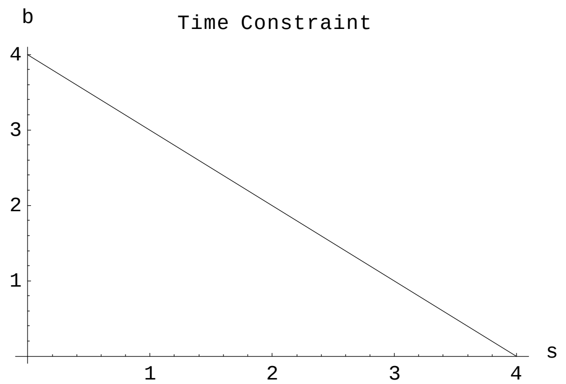

```
Graphics
```

For case (c), the following parameter values are assumed: p_s=30, p_b=20, m=100, t_s=6, t_b=6, t=24. Both constraints can play a role. For bundles of zero to two ski trips, the time constraint can bind; for bundles of two to 3.33 ski trips, the money constraint can bind.

```
timeconstraint = Line[{{0, 24 / 6}, {24 / 6, 0}}]
```

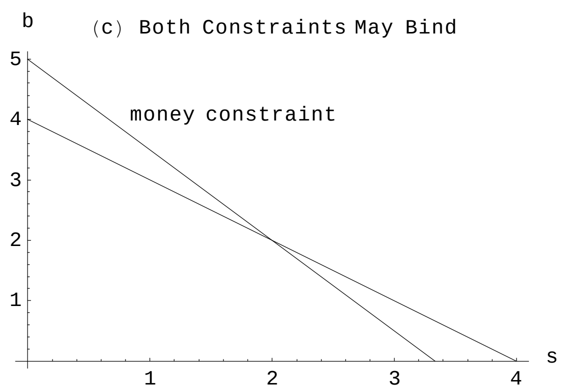
```
Line[{{0, 4}, {4, 0}}]
```

```
timegraph = Show[Graphics[timeconstraint], Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "Time Constraint"]
```



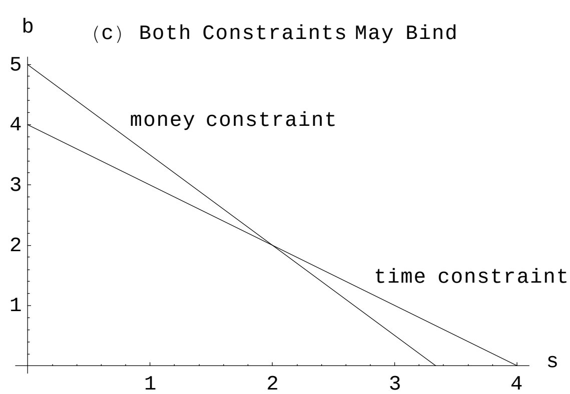

```
Graphics
```

```
both =
 Show[Graphics[Text["money constraint", {0.8, 4}, {-1, 0}]], moneygraph, timegraph,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(c) Both Constraints May Bind"]
```




```
Graphics
```

```
both2 = Show[Graphics[Text["time constraint", {2.8, 1.4}, {-1, 0}]], both,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(c) Both Constraints May Bind"]
```




```
Graphics
```

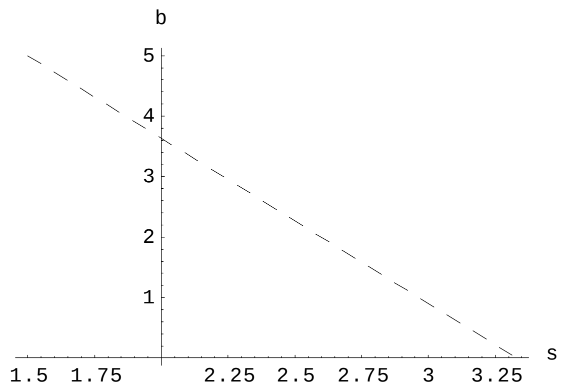
2. Think about what the solution might look like. Consider whether Wilbur will spend all of his money, whether Wilbur will utilize all his time, and whether Wilbur will both bowl and ski.

Let's consider three possible solutions: (i) Wilbur spends all his money and a fraction of his time; (ii) Wilbur spends all his time and a fraction of his money;and (iii) Wilbur spends all his money and time.

For all three cases the following parameter values are assumed: p_s=30, p_b=20, m=100, t_s=6, t_b=6, t=24.

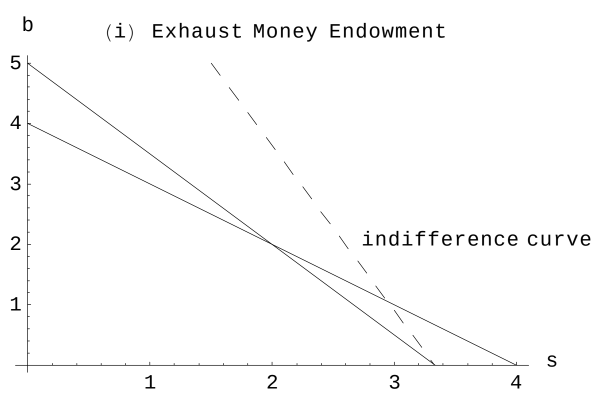
In case (i), Wilbur spends all his money (m=100) and some of his time (t=19.98) to consume 3.33 ski trips and zero bowling game.

```
utilitybline = Show[Graphics[{Dashing[{0.03, 0.03}], Line[{{1.5, 5}, {3.33, 0}}]}],
  Axes → True, AxesLabel → {"s", "b"}]
```



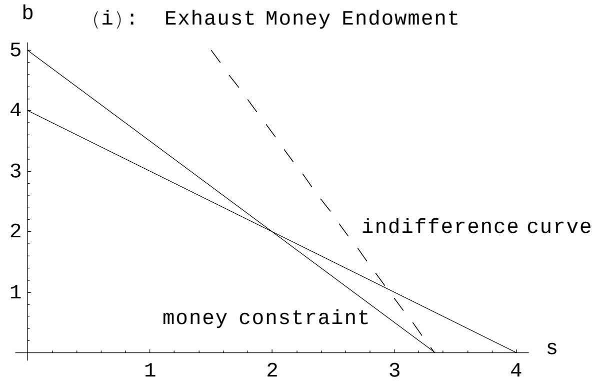

```
Graphics
```

```
moneybinds = Show[Graphics[Text["indifference curve", {2.7, 2}, {-1, 0}]],
  moneygraph, timegraph, utilitybline, Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "(i) Exhaust Money Endowment"]
```



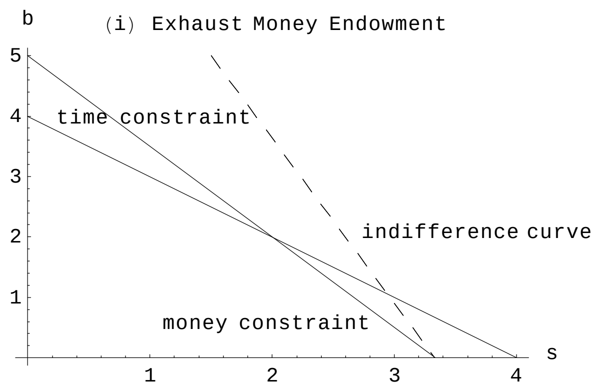

```
Graphics
```

```
moneybinds2 = Show[Graphics[Text["money constraint", {2.8, 0.5}, {1, 0}]],
  moneybinds, utilitybline, Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "(i):  Exhaust Money Endowment"]
```



Graphics

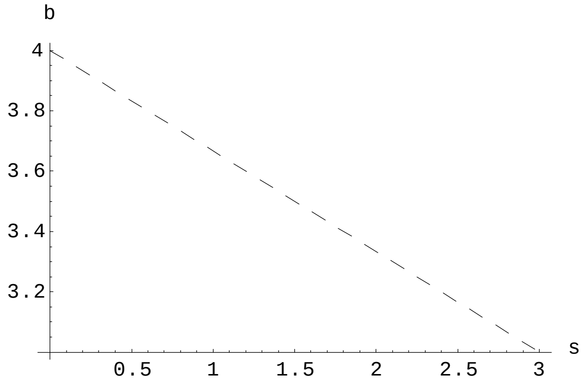
```
moneybinds3 = Show[Graphics[Text["time constraint", {0.2, 3.9}, {-1, 0}]],
  moneybinds2, utilitybline, Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "(i) Exhaust Money Endowment"]
```



Graphics

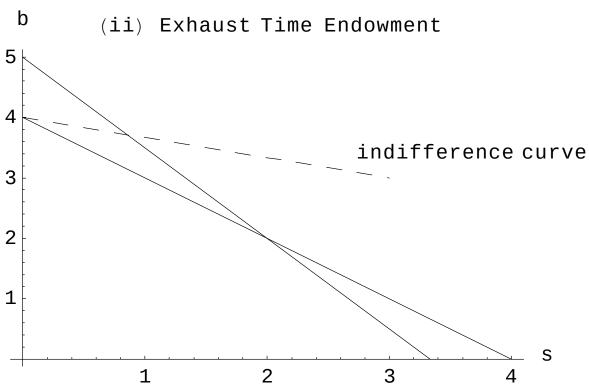
In case (ii), Wilbur spends all his time (t=24) and some of his money (m=80) to consume 4 bowling games and zero ski trip.

```
utilitybline = Show[Graphics[{Dashing[{0.03, 0.03}], Line[{{0, 4}, {3, 3}}]}],
  Axes → True, AxesLabel → {"s", "b"}]
```



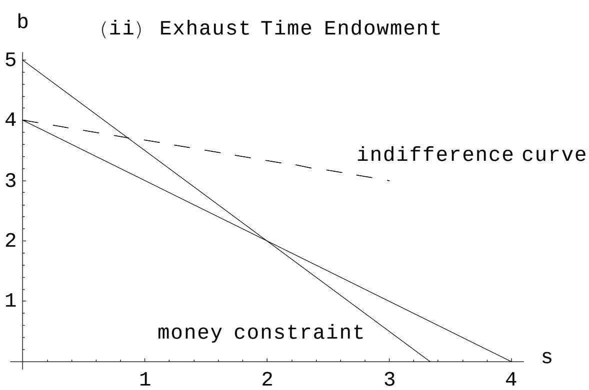
Graphics

```
timebinds = Show[Graphics[Text["indifference curve", {2.7, 3.35}, {-1, 0}]],
  moneygraph, timegraph, utilitybline, Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "(ii) Exhaust Time Endowment"]
```



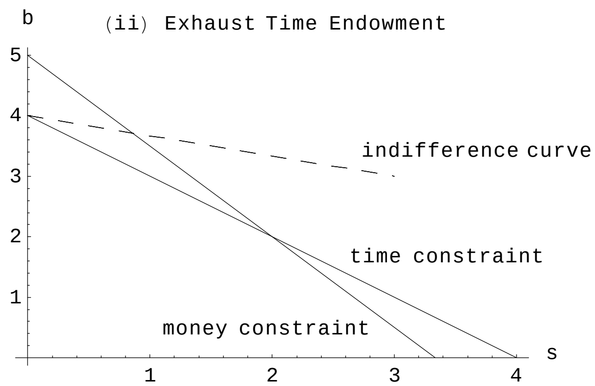
Graphics

```
timebinds2 =
 Show[Graphics[Text["money constraint", {2.8, 0.4}, {1, 0}]], timebinds, utilitybline,
  Axes → True, AxesLabel → {"s", "b"}, PlotLabel → "(ii) Exhaust Time Endowment"]
```



Graphics

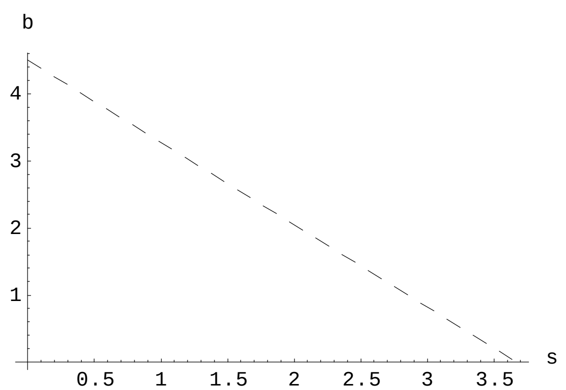
```
timebinds3 = Show[Graphics[Text["time constraint", {2.6, 1.6}, {-1, 0}]],
  timebinds2, utilitybline, Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "(ii) Exhaust Time Endowment"]
```



Graphics

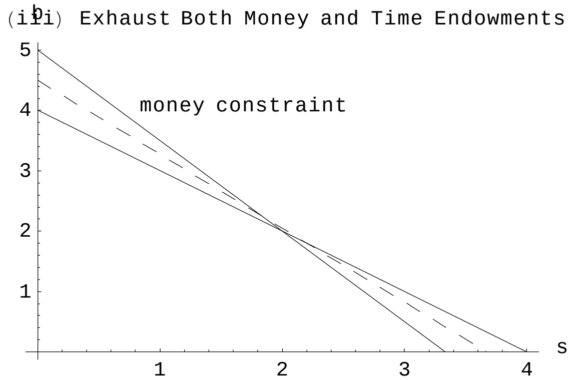
In case (iii), Wilbur spends all his money (m=100) and time (t=24) to consume 2 ski trips and 2 bowling games.

```
utilitybline = Show[Graphics[{Dashing[{0.03, 0.03}], Line[{{0, 4.5}, {3.67, 0}}]}],
  Axes → True, AxesLabel → {"s", "b"}]
```



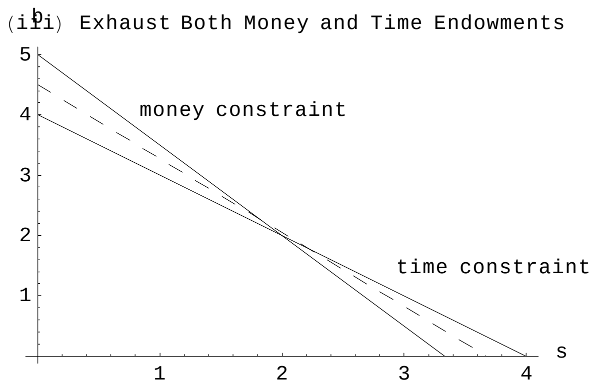
Graphics

```
both = Show[Graphics[Text["money constraint", {0.8, 4}, {-1, 0}]],
  moneygraph, timegraph, utilitybline, Axes → True, AxesLabel → {"s", "b"},
  PlotLabel → "(iii) Exhaust Both Money and Time Endowments"]
```




```
Graphics
```

```
both2 = Show[Graphics[Text["time constraint", {2.9, 1.4}, {-1, 0}]], both, Axes → True,
  AxesLabel → {"s", "b"}, PlotLabel → "(iii) Exhaust Both Money and Time Endowments"]
```




```
Graphics
```

3. Use the ConstrainedMax command in Mathematica to solve this problem for specific values of a, b, m, t, p_s, p_b, t_s, and t_b.

Let's start this question with the following "benchmark" case where $\alpha=4$, $\beta=4$, $m=100$, $t=24$, $p_s=30$, $p_b=20$, $t_s=1$, and $t_b=3$.

In this particular case, only money constraint binds; the time constraint plays no role.

```
ConstrainedMax[4 s + 4 b, {30 s + 20 b ≤ 100, s + 3 b ≤ 24}, {s, b}]

{20, {s → 0, b → 5}}
```

Now let's increase the level of income to 1000 (m=1000) in order to find a solution where the money constraint does not bind anymore but the time constraint does.

```
ConstrainedMax[4 s + 4 b, {30 s + 20 b ≤ 1000, s + 3 b ≤ 24}, {s, b}]
```

{96, {s → 24, b → 0}}

Now, let's reduce the income to 200 (m=200) to find an internal solution where both constraints are binding.

```
ConstrainedMax[4 s + 4 b, {30 s + 20 b ≤ 200, s + 3 b ≤ 24}, {s, b}]
```

$\left\{\frac{256}{7}, \left\{s \to \frac{12}{7}, b \to \frac{52}{7}\right\}\right\}$

Finally let's try to change the preference parameters. Let's take the benchmark case and change the preference parameters. Let α=6 and β=2. Now Wilbur has his preferences biased toward the ski trips. Since we haven't changed the amount of income or total time endowment, we should expect to see again the income constraint binding and the time constraint playing no role.

```
ConstrainedMax[6 s + 2 b, {30 s + 20 b ≤ 100, s + 3 b ≤ 24}, {s, b}]
```

$\left\{20, \left\{s \to \frac{10}{3}, b \to 0\right\}\right\}$

Extra Credit: The following is another example of a linear programming problem.

A horticulturist wishes to mix fertilizer that will provide a minimum of 15 units of potash, 20 units of nitrates and 24 units of phosphates. Brand 1 provides 3 units of potash, 1 unit of nitrates, and 3 units of phosphates; the unit price of brand 1 fertilizer is $120. Brand 2 provides 1 unit of potash, 5 units of nitrates, and 2 units of phosphates; the unit price of brand 2 fertilizer is $60. What is the least-cost combination of fertilizers that the horticulturist will purchase?

```
ConstrainedMin[120 x + 60 y, {3 x + y ≥ 15, x + 5 y ≥ 20, 3 x + 2 y ≥ 24}, {x, y}]
```

{780, {x → 2, y → 9}}

The horticulturist will purchase 2 units of brand 1 and 9 units of brand 2.