Game Theory

January 28, 02

Readings

The Lost Years of a Nobel laureate, NYT, Nov 13, 1994.

Between Genius and Madness, Nyt, June 14, 1998.

Review of A Beautiful Mind, NYT, June 11, 1998.

Chapter 1 of A Beautiful Mind

Varian, Hal R., Intermediate Microeconomics: A Modern Approach. The chapter on game theory

Poundstone, Richard, Prisoner's Dilemma, pages 37-64 and 101 - 131.

I also recommend reading The Evolution of Cooperation, by Robert Axelrod, $8 or $9. It is nontechnical and well worth $9.

There are many good game theory texts. If you want to get into game theory in more detail, may I suggest Eric Rasmusen's Games and Information: An Introduction to Game Theory, published by Basil Blackwell

Economic Models

Before we start with Game Theory, let's spend a few minutes discussing economic models.

Economic models are designed to explain the behavior of economic agents (individuals, firms, governments, etc.).

How these agents react to their constraints and the outcomes of their interactions.

One of the driving assumptions in most economic models is the notion of equilibrium.

In single agent models, the agent is in equilibrium when the agent is doing the best it can given its constraints (since there is only one agent there is no strategic interaction between agents).

e.g. neoclassical models of a consumer or a firm (either competitive and monopolistic)

In multiple agent models, a system is in equilibrium when everyone is doing the best they can given their constraints, but now their constraints can include what other agents do.

One way to simplify multiple agents models is to make the number of agents very large.

In that case, the impact of the actions of a single agent can be effectively ignored.
when each agent decides what to do he takes the actions of other as given.

e.g. economic models of competitive equilibrium.

when N is large there is no strategic interactions amongst agents.

The other extreme is when N is one.

Many interesting cases occur when N is small but not one.

Consider modeling the outcome of an interaction amongst a small number of economic agents.

In which case, each agent's activities will, in general, have a significant influence on the other agents, and all the agents know this.

Therefore, what I do (my strategy) will be influenced by what I expect you to do (your strategy) and vice-versa.
In which case we will both behave strategically.

Therefore, to explain behavior the model needs to incorporate these interactions.
Game theory is designed to model the outcome(s) of any interaction involving a small number of agents.

Varian says,

Game Theory is concerned with the general analysis of strategic interactions.

Game Theory can be used to model interactions amongst

- firms
- consumers
- children
- countries at war (hot)
- countries at war (cold).

People play recreational games
such as chess, checkers, Go, Tic Tac Toe, Baseball, etc.

While not the first to consider the modeling of games, John von Neumann is generally credited as the founder of Game Theory. He was one of the inspirations for Dr. Strangelove.

In 1928 von Neumann published the first theoretical results in Game Theory. Game Theory was developed by John von Neumann and others at the Rand Corporation in the late 1940s and early 1950s.

A famous name in game theory is John Nash, a mathematician that recently won the

Noble Prize in Economics. See
The lost years of a Nobel laureate
Between Genius and Madness
Review of the book “A Beautiful Mind”
Chapter 1 of A Beautiful Mind
Russell Crowe stars as John Nash on the Big Screen

A few comments are in order before we proceed.

Varian states (p. 259), ”Game Theory emphasizes a study of cold -blooded rational decision making.” That is, game theory assumes everyone is rational, smart (invented by a genius) and wants to maximize their personal gain.

Interestingly, it turns out that John Nash has not always been rational.

The best way to start Game Theory is with some examples: Nash Equilibrium in pure strategies

Assume N players and a finite number of strategies.

Specifically assume

- 2 players (Wilbur and Fred)
- 2 strategies (each player has two options; strategy 1 and strategy 2)

Assume both players decide what to do (which option to play) at the same time (in contrast to a game where one agent moves first).
For now, further assume that the game is only played once.

A convenient way to represent such a game is with a 2×2 payoff matrix.
For example

		Fred	
		strategy 1	strategy 2
Wilbur	strategy 1	a_{11}, b_{11}	a_{12}, b_{12}
	strategy 2	a_{21}, b_{21}	a_{22}, b_{22}

where a_{ij} is the payout to Wilbur if Wilbur plays strategy i and Fred plays strategy j,and b_{ij} is the payout to Fred if Wilbur plays strategy i and Fred plays strategy j. That is, Wilbur's possible outcomes are the a's and Fred's are the b's

Assume agents prefer larger payouts to smaller payouts and don't care about the payout to the other player(s). Consider

Game I

		Fred	
		strategy 1	strategy 2
Wilbur	strategy 1	$0, 1$	$2, 6$
	strategy 2	$2, 0$	$3, 1$

What should Wilbur and Fred do?

Write down your answer and explain why the answer you chose is an equilibrium in strategies.

Note that no matter what Fred does Wilbur should go with strategy 2.

Why? Because a payout of 2 or 3 is better than a payout of 0 or 2.

When agent A should pick strategy j no matter what the agent B does, we say that j is A's dominant strategy.

Strategy 2 is Wilbur's dominant strategy

By the same logic strategy 2 is Fred's dominant strategy

and the equilibrium outcome is strategy ($(2, 2)$ with payout $(3, 1)$.

Let's see if we can state a general result.

In a 2 person game where each person has 2 strategies, if both players have a dominant strategy, then both players playing his or her dominant strategy is the equilibrium.

Now consider

Game II

		Fred strategy 1	Fred strategy 2
Wilbur	strategy 1	0, 2	2, 1
	strategy 2	2, 0	3, 1

Note that Fred's two options don't have to be the same as Wilbur's two options.

In this case does Wilbur have a dominant strategy?
YES - play down
Does Fred have a dominant strategy?
NO

So we do not have an equilibrium in dominant strategies?

Is there equilibrium in some weaker sense?

How might we define equilibrium for a 2 person, 2 strategy game?

Suggestion:

The chosen strategies are an equilibrium if Wilbur is choosing his best strategy given Fred's choice and vice versa.

If this is true, no one wants to change their choice. Everyone is

doing the best they can given what everyone else is doing.

Such an equilibrium is called a Nash Equilibrium?

Nash Equilibrium is when each agent choice maximizes his expected payout given the strategy of the other agent.

Does our current game have a Nash Equilibrium?

Check each box

(3, 1) is the Nash Equilibrium.

Tedious process! There are software programs that can be used to find Nash Equilibrium (e.g., Mathematica can calculate Nash Equilibriums).

While the above is a correct description of the Nash Equilibrium concept, this definition is not as explanatory as it might be in that some of the requirements for the Nash Equilibrium are not explicitly stated.

Let me present an equivalent definition of 2-agent Nash Equilibrium.

a) each agent's expectations about which strategy the other agent will choose are correct

AND

b) each agent is choosing that strategy that maximizes his expected payout given his beliefs about which strategy the other agent will choose.

Nash Equilibrium has two components:

one having to do with beliefs,

one having to do with actions.

a & b together ⇔ each agent's choice maximizes his expected payout given the strategy of the other agent.

If the Nash Equilibrium is to be chosen, what thought process will each agent go through in making his or her choice?

S/he will think through the strategy of the other agent knowing that the other agent is, as he is, intelligent, rational and maximizing.

That is, he or she will play her best strategy given what she expects the other guy will do, and she assumes the other guy is doing the same.

In Game II, Wilbur says to himself stuff like the following:

If I expect Fred to play left then my best response is to play down and Fred knows this is my best response, but if I play down, Fred's best response is to play right (not left).

Therefore, Wilbur does not expect Fred to play left.

In addition,

Wilbur also says to himself,

If I expect Fred to play right then my best response is to play down and Fred knows this is my best response and if I play down Fred's best response is to play right.

Therefore, Wilbur expects that Fred will play right. By the same logic, Fred expects Wilbur to play down.

So, Wilbur plays down,

Fred plays right,

AND

a) each agents expectations about what the other agent will do are correct,

AND

b) each agent is choosing that strategy that maximizes his expected payout given his beliefs about what the other agent will do,

AND

(3, 1) will be chosen.

Now consider another game.

Game III

		Fred	
		left	right
Wilbur	up	$1,3$	$.5,1$
	down	$0,0$	$4,2$

Does anyone have a dominant strategy?

NO.

Find the Nash Equilibrium?

Up-Left YES
Up-Right NO
Down-Left NO
Down-Right YES

There are two Nash Equilibrium. In both equilibrium, each party will be doing the best they can given the actions of the other (beliefs fulfilled and each agent is maximizing her expected payout).

PROBLEM:

Come up with a game that has two players, two strategies for each player, no player has a dominant strategy, and there is only one equilibrium.

Prove that no such game exists

Consider the following theorem.

It is impossible to have a game with all of the following 4 properties:

1. two players
2. each player has two strategies
3. no player has a dominant strategy
4. there is one, and only one Nash Equilibrium in pure strategies.

Proof (by contradiction).

Assume a two-player, two strategy game (that is, a game that fulfills properties 1 & 2).

		Fred L	R
Wibur	U	a_{11}, b_{11}	a_{12}, b_{12}
	D	a_{21}, b_{21}	a_{22}, b_{22}

The names of the agents and strategies are immaterial

Now impose the condition that there is at least one equilibrium (part of condition 4).

Without loss of generality, assume Up-Left $\equiv UL$ is a Nash Equilibrium

I could have chosen any of the 4 boxes.

But if UL is an equilibrium,
then,

$U = B(L)$ read as U is Wilbur's best strategy if Fred plays Left.

and

$L = B(U)$ read as L is Fred's best strategy if Wilbur plays up.

That is

$$UL \text{ is a Nash equil } \Leftrightarrow \begin{cases} U = B(L) \\ L = B(U) \end{cases}$$

Two different ways to say the same thing.

But if, in addition,

Now impose condition 3 that no one has a dominant strategy.

So far we know that given UL is an equilibrium, Wilbur's best response when Fred plays left is to play up $\Leftrightarrow U = B(L)$.

Therefore, if Wilbur's best response when Fred plays right is to play up, then Wilbur has a dominant strategy.

Therefore, Wilbur will not have a dominant strategy only if Wilbur's best response when Fred plays right is to play down ⇔ $D = B(R)$.

So,
(UL equilibrium) and (Wilbur no dominant strategy) ⇒ $D = B(R)$.
Analogously,

given UL is an equilibrium,
Fred's best response when Wilbur plays Up is to play Left ⇔ $L = B(U)$.

If Fred's best response when Wilbur plays down is to play Left, then Fred has a dominant strategy.

Therefore, no dominant strategy for Fred requires that right is the best response to down ⇔ $R = B(D)$.

Resaid,
(UL equilibrium) and (no dominant strategy for Fred)
⇒ $R = B(D)$. So we have
$D = B(R)$
and
$R = B(D)$

but

$$DR \text{ is a Nash equil } \Leftrightarrow \begin{cases} D = B(R) \\ R = B(D) \end{cases}$$

which proves that

2 players, 2 strategies for each, and no dominant strategy for either player ⇒ if there is one Nash equilibrium in pure strategies there will be at least two.

qed

Note that we haven't yet talked about mixed strategies, but it is possible to have game with

- 2 players
- 2 strategies for each
- no dominant strategy
- and only one equilibrium if that equilibrium is a mixed strategy equilibrium.

In a moment, we will consider mixed strategy equilibrium

but first,

A few more thoughts about Nash Equilibrium.

Many of you might be still be bothered by this equilibrium concept.

If so, should you also be bothered by the concept of competitive equilibrium: The equilibrium concept you have learned so much about as an economics major.

In competitive equilibrium everyone maximizes utility or profits subject to prices, expectations are fulfilled, and S = D for every good.

There can be multiple competitive equilibrium.

In general, you haven't worried about adjustments to equilibrium, just that once there, there is a tendency to stay there. Now consider another game.
The Welfare Game.

Nash Equilibrium in Mixed Strategies

Game IV

		Poor person with kids	
		work for wages	goof off
The Government	provides childcare	$3,2$	$-1,3$
	provides no childcare	$-1,1$	$0,0$

Find the Nash Equilibrium.

Child-care - work NO
Childcare - goof off NO
No childcare- work NO
No childcare -goof off NO

There is no Nash Equilibrium if we require that each agent has to choose a strategy rather than the probability of a strategy. But maybe there is an equilibrium if each agent can choose the probability of a strategy.

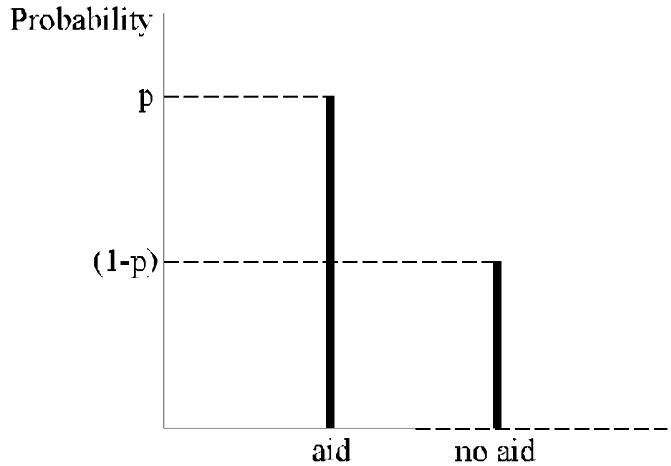
Let me begin by defining what I mean by the probability of a strategy.

Assume the government choice of strategy (whether to provide aid) is a random variable from the governments perspective (the gov rolls the dice)
and
the pauper's choice of strategy (whether to work for wages) is a random variable from the pauper's perspective (the pauper also rolls the dice) where

$p \equiv$ probability that government will provide childcare.
$(1-p) \equiv$ probability that government will not provide childcare.
$q \equiv$ probability that pauper will work for wages.
$(1-q) \equiv$ probability that pauper will stay home and goof off.

What does it mean that the government chooses p and the pauper chooses q?

Government chooses p, not whether to provide childcare to this individual. Whether childcare is provided is determined by randomly drawing from a distribution where the chance of choosing childcare is p.



Can you think of examples where the government does this? Income tax audits are random by category. Firms that sell by phone often listen in on some percentage of sales calls.

Why might the government do this?

To make what they will do uncertain to the pauper.

Earlier we were implicitly forcing the government to choose a p of either one or zero, and forcing the pauper to choose a q (the probability of working for wages) of either one or zero

In this new game the pauper chooses the probability of working, not whether to work, and the government chooses the probability of providing childcare, not whether to provide childcare.

Strategies where agents choose p and q are called mixed strategies.

If p is restricted to 0 or 1 and q is restricted to 0 or 1 we will call it a pure strategy. Up to now, we have restricted ourselves to Nash Equilibrium in pure strategies.

In the welfare game, is there a Nash Equilibrium in mixed strategies?

Definition of Nash Equilibrium in mixed strategies

in equilibrium, given the probability of the government providing aid (p), the pauper does not want to change her probability of working.

and

simultaneously, given the probability that the pauper will work (q), the government does not want to change its probability of providing aid.

Or said more explicitly,
in equilibrium
a) beliefs are correct
that is, $p = \pi_p$ and $q = \pi_q$
where π_p is the pauper's subjective probability that the government will provide aid.
and π_q is the government's subjective probability that the pauper will work.
and
b) The government is choosing p to maximize its expected benefits given its subjective probability of whether the pauper will work (π_q)
and
the pauper is choosing q to maximize its expected benefits given her subjective probability that the government will provide aid (π_p).

To determine whether the welfare game has a mixed strategy equilibrium, we need to first determine the expected benefits to both players.
Recollect that

Game IV

		Poor person with kids	
		work for wages	goof off
The Government	provide childcare	$3, 2$	$-1, 3$
	provide no childcare	$-1, 1$	$0, 0$

Therefore, in this game

$$EB_{gov} = p\pi_q 3 + p(1-\pi_q)(-1) + (1-p)\pi_q(-1) + (1-p)(1-\pi_q)0$$
$$= 5p\pi_q - p - \pi_q$$

and

$$EB_{pauper} = q\pi_p 2 + (1-q)\pi_p 3 + +q(1-\pi_p)1 + (1-q)(1-\pi_p)0$$
$$= -2q\pi_p + q + 3\pi_p$$

The government gets to choose *p* and the pauper *q*.

That is the government wants to choose p to maximize EB_{gov} and the pauper wants to choose q to maximize EB_{pauper}

Start with the government's maximization problem

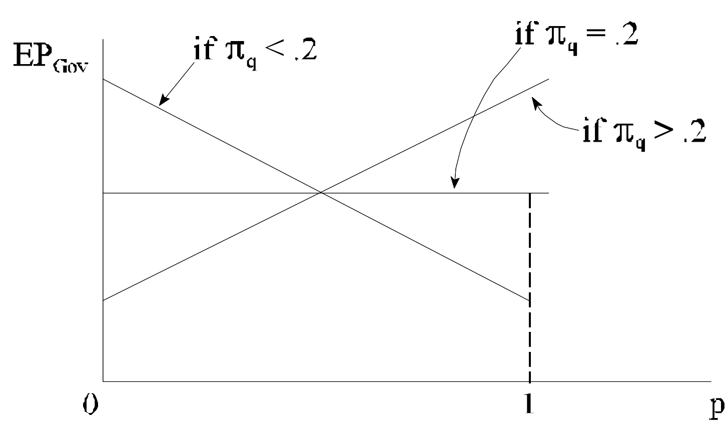
Look for interior maximum (a corner solution is a pure strategy) by finding the critical point

$$\frac{\partial EB_{gov}}{\partial p} = 5\pi_q - 1 \ \begin{matrix} > \\ = \\ < \end{matrix} \ 0 \text{ depending on the value of } \pi_q$$

Note that the derivative of the government's objective function with respect to its choice variable p is not a function of p. So, the government can't choose p to make this equal zero. There in no interior solution for the government unless $\pi_q = .2$

However, if $\pi_q = .2$, the government won't care what p it chooses.

Graphically



Now consider the pauper's choice

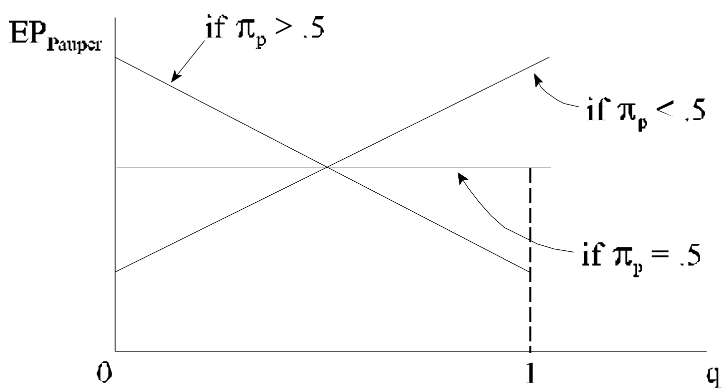
Choose q to maximize $EB_{pauper} = -2q\pi_p + q + 3\pi_p$

Look for an interior maximum.

$$\frac{\partial EB_{pauper}}{\partial q} = -2\pi_p + 1 \;\begin{matrix} > \\ = \\ < \end{matrix}\; 0$$ depending on the value of π_p

There is no interior solution to the pauper's problem unless $\pi_p = .5$. But in that case the pauper is indifferent to his choice of q.

Graphically



So what have we discovered?

If $p = .5$

pauper won't care what q he chooses,

and if $q = .2$

government won't care what p it chooses.

So,

$p = .5,\ q = .2$

is a mixed strategy equilibrium to

Game IV

		Poor person with kids	
		work for wages	goof off
The Government	provide childcare	$3,2$	$-1,3$
	provide no childcare	$-1,1$	$0,0$

In words, if government chooses childcare with a 50 percent probability and the poor person chooses works 20 percent of the time, neither agent will have an incentive to change its probability

AND

$p = \pi_p = .5$

$q = \pi_q = .2$

Beliefs are fulfilled

Note the observed equilibrium outcome could be any of the 4 boxes and depends on the two random draws

What is the probability associated with each outcome?

$$pq = .5.2) \qquad p(1-q) = .5(.8)$$
$$= .1 \qquad = .4$$

$$(1-p)q = .5(.2) \qquad (1-p)(1-q) = .5(.8)$$
$$= .1 \qquad .4$$

There is 40% chance of observing the government providing childcare and the pauper staying home, but only a 10% chance of observing the government providing childcare and the pauper working.

General pt

Each agent's expected benefits are always linear in its probabilities.

For example:

$EB_{gov} = 5p\pi_q - \pi_q - p$ linear in p

and

$EB_{pauper} = -2q\pi_p + q + 3\pi_p$ linear in q

SO,

in a mixed strategy equilibrium it will always be the case that each agent will be indifferent to its choice of probability given the other agent's equilibrium choice of probability.

In our example,

government was indifferent to choice of p given $q = .2$

and

poor person was indifferent to choice of q given $p = .5$.

More general pt

For all N-Person, N-strategy games, there will be at least one Nash Equilibrium if one allows mixed strategies.

Does our earlier pure-strategy games have, in addition to pure strategy equilibriums, mixed-strategy equilibrium?

Consider an aside to our earlier theorem that stated there is no game with

1) 2 players
2) 2 strategies for each player
3) neither party has a dominant strategy
3) one, but only one, Nash Equilibrium in pure strategies.

It is possible to have a game with

1) 2 players
2) 2 strategies for each player
3) no dominant strategies
4) one, and only one, Nash Equilibrium which is a Nash Equilibrium in mixed strategies.

Proof by example

E.g.

Game V

		Wilbur	
		heads	tails
Fred	heads	$1,-1$	$-1,1$
	tails	$-1,1$	$1,-1$

This is an example of a zero-sum game because the payouts in each box sum to zero.

This game has no dominant strategies, no Nash Equilibrium in pure strategies, and only one equilibrium in mixed strategies, which is each player playing heads with probability .5. Have the class work in groups to demonstrate this.

How should we interpret mixed strategy equilibrium?

They are not nearly as intuitive as pure strategy equilibrium.

One objection to mixed strategy equilibrium is the claim that agents do not take random actions in the real world.

But sometimes they do, e.g. probability of a tax audit.

A more troubling objection to mixed strategy equilibrium is that in equilibrium each agent is indifferent as to what he or she does.

In fact, in our example, in equilibrium the government does not care about whether it provides childcare or, for that matter, its choice of p. And in equilibrium, the poor person is indifferent to his or her choice of q.

That is, the government doesn't care what probability is chooses as long as $q = .2$, and the poor person doesn't care what probability he chooses as long as $p = .5$.

But if either agent decides to change its probability, the equilibrium would collapse.

e.g. if poor person changed his q to $.201$ from $.200$, which he is indifferent to doing given $p = .5$, the equilibrium will collapse. Or if the government changed its p to $.4999$ from $.5$, which its indifferent to doing given $q = .2$, the equilibrium will collapse.

In this sense, mixed strategy equilibrium are quite weak.

However, It is possible to reinterpret our welfare game in a way that makes the mixed strategy equilibrium more intuitive.

Imagine a world with many poor individuals where 20% are workers and 80% goof off. The government encounters a poor person and must decide whether to provide childcare before it can determine whether the individual is a goof off. What will the government do in equilibrium?

Provide childcare with probability $p = .5$.

In this variation on the welfare game the government plays a mixed strategy; that is, chooses p. Whereas, each poor person plays a pure strategy.

Alternatively, imagine a world with 20 different states (Colorado, New Jersey, California, etc.), that each either pay childcare to all poor people or to none, and 100 different poor people who each either always works or always goofs off independent of whether they get childcare.

In this game no one plays a mixed strategy.
Poor people wander from state to state not knowing or remembering which states pay for childcare, and states can't tell if an individual will works or not before they have to make the childcare decision. In equilibrium, how many states will pay aid?

10. That is 50%

In equilibrium, how many poor people will be workers?

20. That is 20%

Why?
In what sense is this an equilibrium?

This a pure strategy equilibrium.

Now that we have characterized the equilibrium concept let's examine the efficiency property of equilibrium(s).

Will the Nash Equilibrium necessarily be efficient?

Define efficient and inefficient.
Answer: NO
We can get stuck in an inefficient allocation
e.g.

Game VI

		Ernie	
		cooperate	defect
Burt	cooperate	$3,3$	$0,4$
	defect	$4,0$	$1,1$

What is the Nash Equilibrium?
Defect - Defect

WHY?

Compare DD with CC
DD is obviously inefficient.

Burt and Ernie would be better off at CC, but are stuck at DD.

In economics you have often learned equilibrium has desirable properties
e.g. competitive equilibrium is often efficient.

But this is not always the case as Game VI shows. Let me change the names of the players in Game VI.

Note that all four outcomes are efficient. E.g. $0,4$ and $4,0$ are also efficient.

Game VI with new names

		Soviet Union	
		disarm	arm
U.S	disarm	$3,3$	$0,4$
	arm	$4,0$	$1,1$

What's the Nash Equilibrium?
What happened during the Cold War?

Change the names again:
Game VI with new names

		U.S. environmental policy	
		strict	lax
Canadian environmental policy	strict	3,3	0,4
	lax	4,0	1,1

Have them read nyt article on lawyers.

Now consider a simple sequential game

I will first assume the players play simaltaneously, then change it to a sequential game.

Assume there are two not hot restaurants in Boulder, Giorgios and Ernies) and both specialize in DFC (deep fried cats), hereafter, product x. These are the only two restaruants servicing this specialty.

The inverse demand function for these meals is

$$p = f(x) = f(x_G + x_E)$$

where x is total meals served, p is the price of a meal, x_G is meals served at Giorgios, etc. The customers don't care where they eat their DFC. Ther respective cost functions are $c_G(x_G)$ and $c_E(x_E)$.

Further assume each restaurant takes the output (number of meals served) of the other restaurant as given. Probably a foolish assumption.

Is there a Nash equilibrium in terms of x_G and x_E?

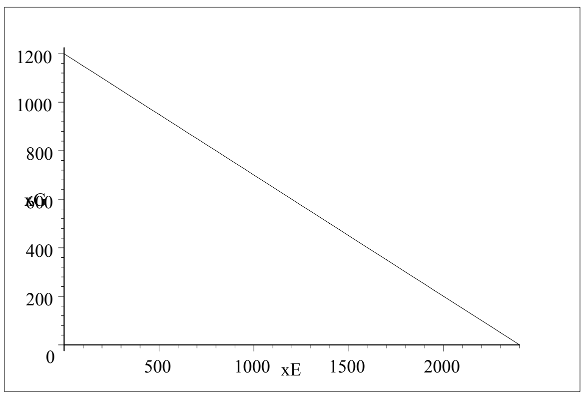
To make the problem more explicit assume $x = g(p) = 3200 - 1600p$, Solution is: $\left\{p = -\frac{1}{1600}x + 2 = -\frac{x_G+x_E}{1600} + 2\right\}$. Further assume $c_G(x_G) = 100 + .5x_G$ and $c_E(x_E) = 100 + x_E$.

Therefore, the profit maximizing decision for Giorgio is find the x_G that maximizes

$$\pi_G = \left[-\frac{x_G + x_E}{1600} + 2\right]x_G - 100 - .5x_G$$

Look for an interior solution

$\frac{d}{dx_G}\left[\left[-\frac{x_G+x_E}{1600} + 2\right]x_G - 100 - .5x_G\right] =$
$[[-6.25 \times 10^{-4}]x_G + [-6.25 \times 10^{-4}x_G - 6.25 \times 10^{-4}x_E + 2.0] - 0.5] = 0$, Solution is:
$\{x_G = -0.5x_E + 1200.0\}$



Giorgio’s reaction function

The reaction function tells us how many meals Giorgio will serve as a function of the number of meals Ernie serves. Now do the same for Ernie. herefore, the profit maximizing decision for Giorgio is find the x_G that maximizes

$$\pi_E = \left[-\frac{x_G + x_E}{1600} + 2\right]x_E - 100 - x_G$$

Look for an interior solution

$\frac{d}{dx_E}\left[\left[-\frac{x_G+x_E}{1600} + 2\right]x_E - 100 - x_E\right] = \left[\left[-\frac{1}{1600}\right]x_E + \left[-\frac{1}{1600}x_G - \frac{1}{1600}x_E + 2\right] - 1\right] = 0$,
Solution is: $\left\{x_E = -\frac{1}{2}x_G + 800\right\}$

Solving for the equilibrium

$$x_G = -0.5x_E + 1200.0$$
$$x_E = -\frac{1}{2}x_G + 800$$

Solution is: $\{x_G = 1066.7, x_E = 266.67\}$. The point at which the two reaction functions cross. This is the Nash equilibrum to the game when both players play simaltaneously.

Now let Giorgio play first. Giorgio buys his cats at 4 a.m. every morning and stores them in his backyard until the breakfast rush. Ernie always sleeps in (he can't help it). Ernie goes to the market at 6 a.m. and on the way drives by Giorgio's house and counts the number of cats in Giorgio's backyard. In this game, Ernie's problem is the same as in the simaltanous game, but for Giorgio the game has changed. Since Ernie decides what to do based on what Giorgio did, Giorgio how has an added advantage. Giorgio knows that Ernie's demand function for cats is now

$$x_E = -\frac{1}{2}x_G + 800$$

so

$$\pi_G = \left[-\frac{x_G + (-\frac{1}{2}x_G + 800)}{1600} + 2 \right] x_G - 100 - .5x_G$$

Looking for Giorgio's profit maximizing level of output

$\frac{d}{dx_G}\left[\left[-\frac{x_G+(-\frac{1}{2}x_G+800)}{1600} + 2 \right] x_G - 100 - .5x_G \right] =$

$[[-3.\,125 \times 10^{-4}]x_G + [-3.\,125 \times 10^{-4}x_G + 1.\,5] - 0.\,5] = 0$, Solution is: $\{x_G = 1600.\,0\}$.

Giorgio serves all the meals. The Nash equilibrium to this sequential game is $x_G = 1600$ and $x_E = 0$. Confirm that Ernie will choose to serve no meals.

Now let's extend our notion of a game and let the players play the game more than once.

This is called a repeated game.

Assume the game is repeated N times where N is a finite number.

Consider Game VII,

which is Game VI (the Burt and Ernie game) repeated twice.

What is the Nash Equilibrium?

	Round 1	Round 2
Burt	D	D
Ernie	D	D

WHY?

Dominant strategy in Round 2 is to defect because nothing is gained by cooperation in the last round, but since both players know the other guy will defect in Round 2, the

best thing to do in Round 1 is to defect.
What if N = 3?
What if N = 100?
What if N = 1,000,000?
What if N = ∞?
What if N is unknown?

As long as N is a finite number, the Nash Equilibrium is to always defect, but this is not what is typically observed in the real world.

This is not what people do in the real world when N get large.

What would you do?

This is no optimal strategy but there is one strategy that does well most of the time against most other strategies. It's called Tit for Tat.

Start by cooperating and then doing what the other guy did on the previous move.

A number of years ago Robert Axelrod invited a number of game theorists, and other noted scientists, to submit strategies for playing a N period prisoner's dilemma game where N was not known to the players.

He played all the strategies against one another. Tit for Tat got the highest score.

Soldiers played Tit for Tat in the trenches in WWI (live and let live - shoot if they shoot first - if they stop shooting, stop as well).

What are Tit for Tat's properties?

1) The strategy cooperates; that is, it's never first to defect
2) Immediately punishes if the other guy defects, but then is ready to forgive, that is it immediately reciprocate both cooperation and defect.
3) The strategy is not too clever, so easy for the other guy to figure out what you are doing.

Think about other possible strategies

Think about how Tit for Tat would do against

a) always cooperating (Tit for Tat get maximum benefits each round)
b) cooperate until other guy defects then always defect
c) Tit for Tat
d) Tit for Two Tats - cooperate until other guy defects twice, then defect but cooperate again as soon as he does.

We have only looked at a few simple games. Game theory is much broader.

For example, our games assumed perfect information about the payout matrix. This assumption can be dropped. There are games with incomplete and/or asymmetric information.