Consumer Theory in a Nutshell

The intent is build a model to explain what an individual will purchase in the market place.

Start by defining some terms:

$x_i \equiv$ amount of good *i* consumed by the individual
$p_i \equiv$ price of good *i*
$y \equiv$ the individual's income

Assume two goods, x_1 and x_2, y exogenous, and p_1 and p_2 exogenous.

Further assume that the individual can rank bundles of goods and that the individual chooses the a highest ranked bundle from among those she can afford.

The above is consumer theory in a nutshell.

Elaboration:

The individual has a ranking over bundles of goods such that $\forall j$ and k either

$x^j \succ x^k$

$x^j \prec x^k$

or

$x^j \sim x^k$

> where "$\succ$" denotes preferred (ranked higher), and "$\geq$" denotes weakly preferred. Note that $x^j \geq x^k$ and $x^j \leq x^k \Rightarrow x^j \sim x^k$ (indifference).

Assume this ranking has the following properties:

$x^m \geq x^m \ \forall m$, and $x^k \geq x^l \geq x^m \ \Rightarrow \ x^k \geq x^m$.

The individual's ranking of bundles is based on her *preferences*.

The inequality $y \geq p_1 x_1 + p_2 x_2$ identifies the bundles that the individual can afford.

If more is always preferred to less, the chosen bundle, $x^* \equiv (x_1^*, x_2^*)$ will have the following properties:

$y = p_1 x_1^* + p_2 x_2^*$ and

$(x_1^*, x_2^*) \geq (x_1^j, x_2^j) \ \forall \ j : y = p_1 x_1^j + p_2 x_2^j$.

x_1^* and x_2^* will depend on y, p_1, and p_2, and the individual's ranking of bundles. Note that x^* is not necessarily unique.

In review, to solve the consumer's problem we need only the individual's budget set and ranking over bundles.

Note the we completely specified consumer theory without ever mentioning the terms *utility* or *utility function*.

What is a utility function?

Just a simple, but potentially misleading, way to represent an individual's ranking over bundles.

Call a function that correctly represents an individual's ranking a *snerd* function. What properties must a snerd function have to correctly represent an individual's ranking of bundles?

It must associated some number, *s*, with every possible bundle; that is $s = s(x_1, x_2)$

such that

$$x^k \succ x^m \Rightarrow s(x_1^k, x_2^k) > s(x_1^m, x_2^m)$$
$$x^k \prec x^m \Rightarrow s(x_1^k, x_2^k) < s(x_1^m, x_2^m)$$

and

$$x^k = x^m \Rightarrow s(x_1^k, x_2^k) = s(x_1^m, x_2^m)$$

If $s(x_1, x_2)$ has these properties, it will correctly rank the bundles.

Note that all of the information about how bundles are ranked can be inferred from the ordinal properties of $s(x_1, x_2)$. While the snerd function, like all functions, has both ordinal and cardinal properties, the cardinal properties of the snerd function provide no additional information about the ranking.

In general, we don't call a function that correctly represents the ranking a snerd function. Rather, we call it a *utility* function. $u = u(x_1, x_2)$ represents the individual's ranking of bundles if

$$x^k \succ x^m \Rightarrow u(x_1^k, x_2^k) > u(x_1^m, x_2^m)$$
$$x^k \prec x^m \Rightarrow u(x_1^k, x_2^k) < u(x_1^m, x_2^m)$$

and

$$x^k = x^m \Rightarrow u(x_1^k, x_2^k) = u(x_1^m, x_2^m).$$

If $x^k \succ x^m$ all that matters is that $u(x_1^k, x_2^k) > u(x_1^m, x_2^m)$, not how much more.

Given $u(x_1, x_2)$ the individual's choice problem can be represented as one of

$$\max_{\text{wrt } x_1, x_2} u(x_1, x_2)$$
$$s.t. \;\; y = p_1 x_1 + p_2 x_2$$

If the solution, x^*, is unique, it will be of the form

$x_1^* = f(y, p_1, p_2)$ and $x_2^* = g(y, p_1, p_2)$, where the form of *f(.)* And *g(.)* is determined by $u(x_1, x_2)$. *f(.)* and *g(.)* are the individual's demand functions for the two goods.

Note that if x^*

$$\max_{\text{wrt } x_1, x_2} u(x_1, x_2) \ s.t. \ \ y = p_1 x_1 + p_2 x_2 ,$$

then it also

$$\max_{\text{wrt } x_1, x_2} U(x_1, x_2) \ s.t. \ \ y = p_1 x_1 + p_2 x_2$$

where $\dfrac{\partial (U(x_1, x_2))}{\partial u(x_1, x_2)} > 0.$

If $U(x_1, x_2)$ has this property, $U(x_1, x_2)$ is said to be a increasing monotonic transformation of $u(x_1, x_2)$.

For example, if $x^* \max_{\text{wrt } x_1, x_2} u(x_1, x_2) \ s.t. \ \ y = p_1 x_1 + p_2 x_2$,

then x^* will also maximize any of the following $U(x_1, x_2)$ subject to the budget constraint

$$U(x_1, x_2) = (u(x_1, x_2))^2$$
$$U(x_1, x_2) = \ln(u(x_1, x_2))$$
$$U(x_1, x_2) = e^{u(x_1, x_2)} + 14$$

and

$$U(x_1, x_2) = \left(\frac{(12 + u(x_1, x_2))}{17}\right)^4 + 9$$

Why?

Because all of these utility functions represent the same ranking of bundles.

What then is the meaning of $\frac{\partial u(x_1, x_2)}{\partial x_j}$?

It has no meaning in terms of the individual's ranking of bundles. Note that the partial derivative of $u(x_1, x_2)$ with respect to x_j is not invariant to a monotonic transformation of $u(x_1, x_2)$. You might have been tempted to call $\frac{\partial u(x_1, x_2)}{\partial x_j}$ the marginal utility of x_j, but marginal utility has no meaning if an individual only has a ranking over bundles, and a ranking is all we need for consumer theory.