Econ 6808 Introduction to Quantitative Analysis

Quiz six- in class, groups of one

This quiz is worth 20 points.

III. Economic Curvature and VI. Economic Applications of Duality Theory

1. (10 points) Define an individual's compensating variation for a change from (m^0, p^0) to (m^1, p^1) in four ways: in words, in terms of an indifference relationship between two states of the world, in terms of the indirect utility function , and in terms of the expenditure function.

The individual's compensating variation for a change from (m^0, p^0) to (m^1, p^1) is the amount of money that has to be subtracted from m^1 to make him indifferent between (m^0, p^0) and $(m^1 - cv, p^1)$. That is $(m^0, p^0) \sim (m^1 - cv, p^1)$.

In terms of the indirect utility function, it is

$$V(m^0, p^0) = V(m^1 - cv, p^1)$$

and in terms of the expenditure function it is

$$cv = E(U^1, p^1) - E(U^0, p^1)$$

2. (10 points) Consider the profit function, $p(p)$, where p is the vector of input and output prices. Define in words the profit function. Define in words, and in functional notation, Hotelling's Lemma. Prove it.

Profit functions are defined for competitive firms. A competitive firm's profit function identifies its maximum profits as a function of the exogenous vector of output and input prices.

Hotelling's Lemma states that the partial derivative of the profit function wrt to the price of commodity i is the net supply of commodity i; that is,

$$x_i = \frac{\P p(p)}{\P p_i} \quad \forall i$$

where $x_j > 0$ implies x_j will be produced by the firm and $x_k < 0$ implies that the firm will use

$|x_j|$ as an input.

One proof is like the proof of Shepard's Lemma

Define a function $g(p)$, where $g(x) = \pi(p) - p' x^*$ and x^* is the input/output vector that maximizes profits when $p = p^*$.

> Note that $g(p^*) = \pi(p^*) - p^{*\prime} x^* = 0$ and $g(x) = \pi(p) - p' x^* > 0$ if $p \neq p^*$. Therefore $g(p)$ takes it minimum value when $p = p^*$.

Differentiate $g(x) = \pi(p) - p' x^*$ with respect to p_i to obtain

$$\frac{\partial g(p)}{\partial p_i} = \frac{\partial \pi(p)}{\partial p_i} - x_i^*$$

But since $g(p)$ is maximized when $p = p^*$

$$\frac{\partial g(p^*)}{\partial p_i} = \frac{\partial \pi(p^*)}{\partial p_i} - x_i^* = 0 \quad \Rightarrow x_i^* = \frac{\partial \pi(p^*)}{\partial p_i}$$

So $\frac{\partial \pi(p^*)}{\partial p_i}$ is the net supply of commodity i when $p = p^*$.

But, since there is nothing special about p^*

$$x_i = \frac{\partial \pi(p)}{\partial p_i} \quad \text{qed}$$

Alternatively, one could prove it as a special case of the Envelope theorem, which is in our notes. In that proof I distinguished between input and output prices.

Things to consider:

If you are going to define the profit functions as $\pi(p, w)$, where *p* is the price of the scalar ouput and *w* is the vector of input prices, then you need to notationally distinguish between the output and the input vector. You can't use *x* for both.

Memorizing stuff is a waste of your time.

Some of you never answered the easiest part of the questions. E.g., define in words Shephard's Lemma.