

Econ 6808 Introduction to Quantitative Analysis

Quiz 2

The quiz is due at the beginning of class on Tuesday September 8.

It is a group quiz, groups of three, no more, no less. The group will hand it one quiz.

Turn in a hard copy of your pertinent *Mathematica* results. I recommend that you use the Notebook front-end to do the quiz. That way, you can incorporate explanatory text in with your *Mathematica* input and output. As part of answers, explain each step in words.

I. An introduction to the *Mathematica* software

Do the whole quiz using *Mathematica*.

Assume a world of two goods x and y), and assume Wilbur's preferences can be represented by the simple Cobb-Douglas utility function, $u = x^8y^3$.

1. Generate a 3D plot of Wilbur's utility function. Give some thought to the domain of x and y . Maybe start with $75 \leq x \leq 0$ and $50 \leq y \leq 0$.

Now assume the budget constraint, $I = p_x x + p_y y$, where I is income, p_x is the price of good x and p_y is the price of good y .

2. Write down Wilbur's constrained optimization problem in terms of the choice variables x and y . Then, using *Mathematica*, convert it into a problem that is, from a mathematical perspective, an unconstrained optimization problem in x .

Now assume $p_x = 2$, $p_y = 5$ and $I = 100$.

3. Generate a plot of utility as a function of x . Start with $75 \leq x \leq 0$ and $50 \leq y \leq 0$. If any messages appear on the screen, explain what they mean? Then try it with $50 \leq x \leq 0$ and $20 \leq y \leq 0$.

4. Use the *Mathematica* command *FindMinimum* to find that x that maximizes utility. Is this Wilbur's demand for good x ? What is Wilbur's maximum utility? Note *FindMinimum* finds the minimum, not the maximum, and it will probably not work if you start the search at $x = 0$.

5. Now determine Wilbur's demand for good y .

Now assume someone invents a third good and that Wilbur's preferences are now represented by the utility function, $u = x^8y^3z^5$.

6. Assuming the budget constraint $100 = 2x + 5y + z$, write down Wilbur's constrained optimization problem in terms of the choice variables x , y and z . Then convert it into a problem that is, from a

mathematical perspective, an unconstrained optimization problem in x and y .

7. Generate a 3D plot of utility as a function of x and y . Again, choose the domains for x and y wisely. If messages appear, explain what they mean.

8. Use *FindMinimum* to find Wilbur's demand for x and demand for y . What is his demand for good z ? What is Wilbur's maximum utility?

9. How would your answers to the above questions change if $u = x^8y^3$ was replaced with $u = x^8y^3$ and $u = x^8y^3z^{.5}$ was replaced with $u = x^8y^3z^5$?