absolute.nb

■ This notebook addresses the absolute value estimation problem in min&mak.pdf

Minimizing Sum of Absolute Values of Residuals

Suppose a firm produces a single output Y. The quantity produced in period t is Y(t). Y(t) is produced using capital services K(t) and labor L(t) where t=1,2,3. The following data is available:

t	Y(t)	K(t)	L(t)
1	1	1	1
2	2	2	1
3	2	1	2

Let's assume that the firm's technology can be approximated with the linear production function

$$Y(t) = f(K(t),L(t)) = aL(t) + bK(t) + e(t)$$

where t=1,2,3; a and b are positive constants and e(t) is the approximateion error in year t.

Find the estimates of a and b that minimize the sum of the absolute values of the residuals

The following defines a function, f, that sums the absolute values of the residuals.

```
f[a_, b_] = Abs[1 - a - b] + Abs[2 - 2 a - b] + Abs[2 - a - 2 b]
```

```
Abs[2 - a - 2 b] + Abs[2 - 2 a - b] + Abs[1 - a - b]
```

```
FindMinimum[f[a, b], {a, b}]
```

```
FindMinimum::fmns : Starting value b in {a, b} is not a real number.
```

```
FindMinimum[f[a, b], {a, b}]
```

Not specifying a starting value resulted in an error message. Hence, the following command specifies starting points 0 and 1 for both a and b; in addition, the command also specifies that if the function gets outside the bounds of (0,99), the search algorithm should stop.

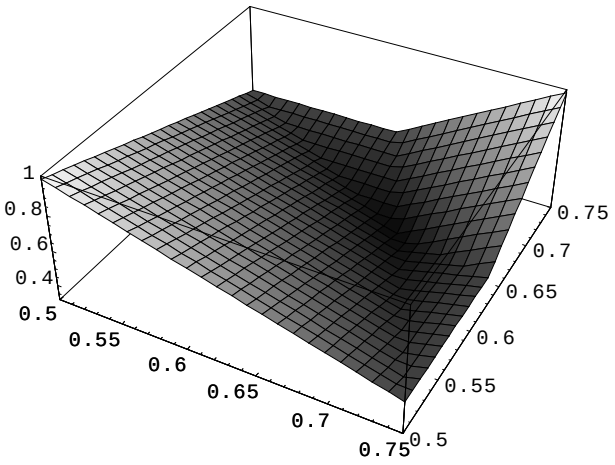
```
FindMinimum[f[a, b], {a, {0, 1}, 0, 99}, {b, {0, 1}, 0, 99}]
```

```
{0.333333, {a → 0.666667, b → 0.666667}}
```

The estimates of a and b that minimize the sum of the absolute values of the residuals are: a=2/3, b=2/3.

Graphical Result

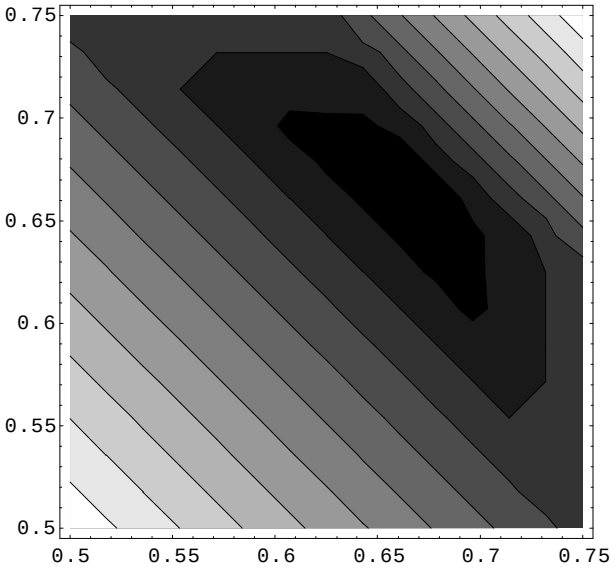
```
Plot3D[f[a, b], {a, 0.5, 0.75}, {b, 0.5, 0.75}, Lighting → False, PlotPoints → 25]
```




```
SurfaceGraphics
```

From the 3-dimensional graph, one can see that the minimum occurs between the range of 0.65 and 0.70. A contour plot can show where the minimum occurs more clearly. The darkest region is where the minimum occurs.

```
ContourPlot[f[a, b], {a, 0.5, 0.75}, {b, 0.5, 0.75}]
```




```
ContourGraphics
```

Let's compare the above results with the OLS estimation results. The OLS estimator minimizes the square of the residuals. The OLS estimates are: a=0.636364, b=0.636364. The absolute value estimates are greater than the OLS estimates.

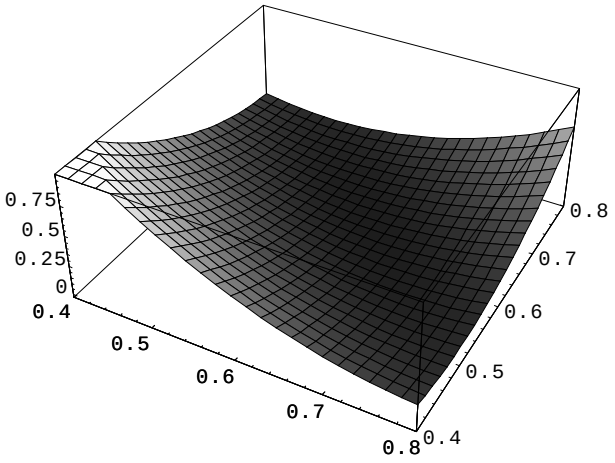
```
g[a_, b_] = (1 - a - b)^2 + (2 - 2 a - b)^2 + (2 - a - 2 b)^2
```

```
(2 - a - 2 b)^2 + (2 - 2 a - b)^2 + (1 - a - b)^2
```

```
FindMinimum[g[a, b], {a, {0, 1}, 0, 99}, {b, {0, 1}, 0, 99}]
```

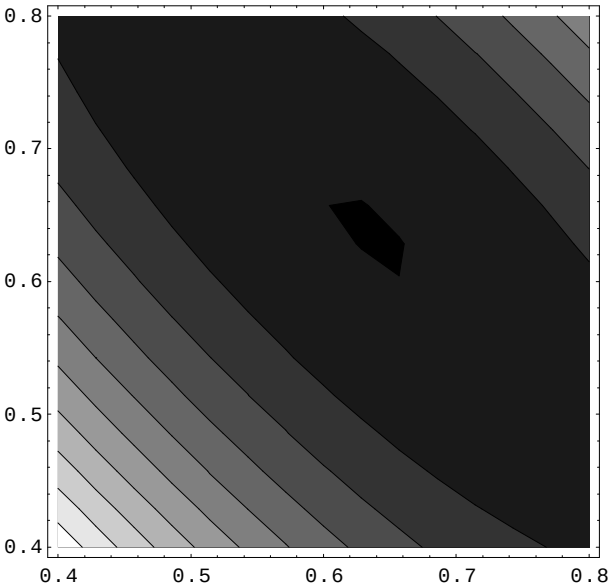
```
{0.0909091, {a → 0.636364, b → 0.636364}}
```

```
Plot3D[g[a, b], {a, 0.4, 0.8}, {b, 0.4, 0.8}, Lighting → False, PlotPoints → 25]
```




```
SurfaceGraphics
```

```
ContourPlot[g[a, b], {a, 0.4, 0.8}, {b, 0.4, 0.8}]
```




```
ContourGraphics
```