

## Econ 6808 Introduction to Quantitative Analysis

review questions -set 2.

### III. Economic Curvature and VI. Economic Applications of Duality Theory

#### Review questions.

1. What does it mean to say that the function  $y = g(m_1, m_2)$  is *concave*? Give an answer in both words and in mathematical notation.
2. Convince me that the cost function,  $c = c(y, \mathbf{w})$  where  $\mathbf{w} = [w_1, w_2, \dots, w_N]$ , is not strictly convex in input prices. **Assume** that this cost function is increasing in  $y$ , increasing in  $\mathbf{w}$ , and everywhere twice differentiable. As part of your answer define *strictly convex*.
3. Consider a cost function,  $c = c(y, \mathbf{w})$  where  $\mathbf{w} = [w_1, w_2, \dots, w_N]$ . **Assume** that this cost function is nondecreasing in  $y$ , nondecreasing in  $\mathbf{w}$ , quasiconcave in  $\mathbf{w}$ , and everywhere twice differentiable. What does it mean to say that this cost function is *nondecreasing in  $y$* ? Use the definition of a cost function to prove (to convince me) that a cost function must be nondecreasing in  $y$ .
4. Consider a cost function,  $c = c(y, \mathbf{w})$  where  $\mathbf{w} = [w_1, w_2, \dots, w_N]$ . **Assume** that this cost function is nondecreasing in  $y$ , nondecreasing in  $\mathbf{w}$ , quasiconcave in  $\mathbf{w}$ , and everywhere twice differentiable. What does it mean to say that this cost function is *nondecreasing in  $\mathbf{w}$* ? Use the definition of a cost function to prove that a cost function must be nondecreasing in  $\mathbf{w}$ .
5. Define *Shephard's Lemma* in terms of production theory. Prove it.
6. Define both in words, and in functional notation the competitive firm's cost function and its conditional input demand functions.
7. Convince me that an individual, Wilma, who is maximizing her utility,  $u(x, y)$ , subject to a budget constraint will behave **as if** her utility function is *quasiconcave* even if it is not. You might want to convince me using graphs of indifference curves and budget lines to identify utility maximizing bundles. As part of your answer, define quasiconcave for this utility function, and explain what it tells you about the shape of the indifference curves. Assume that Wilma's utility function is increasing in  $x$  and  $y$ . The point of this exercise is to demonstrate that it is not restrictive to assume that a utility function that is increasing in its arguments is also quasiconcave.
8. Assume that Wilma's utility function,  $u(x, y)$  is increasing in  $x$  and  $y$ . Given

this, what is gained by assuming that it is *strictly quasiconcave* rather than just *quasiconcave*?

9. Consider an individual's utility function,  $u(\mathbf{x})$  where  $\mathbf{x} = [x_1, x_2, \dots, x_N]$ , that has all the standard properties (increasing in  $\mathbf{x}$ , quasiconcave in  $\mathbf{x}$ , and twice differentiable. One can think of this utility function as a production function where utility is being produced using goods as inputs.

Given this, define in both words and functional notation the consumer theory analog of the competitive firm's *cost function*. This function is called the *expenditure function* where  $E$  is conventionally used as the dependent variable. Duality theory tells us that this expenditure function is dual to the *direct* utility function,  $u(\mathbf{x})$ , and also describes the individual's preferences.

Then define both in words and in functional notation the consumer theory analog of a firm's conditional demand functions for inputs. In consumer theory, these are called the *hicksian demand functions*.

Can one use the expenditure function to derive these hicksian demand function? How?

Assume a simple functional form for a two-good utility function and derive both the expenditure function and the hicksian demand functions.

10. Intuitively explain, using a graphical analysis, how one can derive the production function,  $y = f(k, l)$  from the cost function  $c = c(y, r, w)$ .
11. Choose some explicit cost function, and from it derive the corresponding production function.
12. Define an individual's compensating variation for a change from  $(m^0, p^0)$  to  $(m^1, p^1)$  in four ways: in words, in terms of an indifference relationship between two states of the world, in terms of the indirect utility function, and in terms of the expenditure function.
13. Explain how one can derive a competitive firm's cost function,  $c = c(y, \mathbf{w})$  where  $\mathbf{w} = [w_1, w_2, \dots, w_N]$  from its production function  $y = f(\mathbf{x})$  where  $\mathbf{x} = [x_1, x_2, \dots, x_N]$ . Now assume a simple functional form for a 2-input production function and derive the firm's cost function.