Econ 6806, Final, December 17, 1996

Answer the questions to the best of your ability in the allotted time. Make sure you have all nine questions The exam consists of three pages and question two has five parts.

You can choose 20 points worth of questions for which you will be given full credit without answering the question . Specify clearly the questions for which you want automatic credit by writing "full credit" in the space provided for the answer.

Thank you for being in my class. I have enjoyed the experience immensely.

Please use the provided blue books for your answers.

1. (10 points) Prove that Nash Equilibria are not always efficient. As part of your answer define efficient.

2. (20 points) Consider the game **Dueling Saabs**. This game is played by adolescent females who attend Boulder High School. They all own Saab convertibles with which they like to play **Chicken**.[1] Every time two females encounter one another they are compelled to play **Chicken**. They drive their Saabs at each other at breakneck speed and then either swerve, or don't swerve, at the last second. If one swerves she is a wimp, if one does not swerve she is considered aggressive. Ceteris paribus, aggressive is considered a desirable trait (it attracts adolescent boys), but aggressiveness can be costly if one encounters another aggressive driver. Assume the payout matrix for this game, in number of boys attracted (or repelled)[2], is

	Female B		
		Swerve	Don't Swerve
Female A	Swerve	1, 1	0, 5
	Don't Swerve	5, 0	-3, -3

While wandering through adolescence each female will encounter many other Saabs. Suppose that she can not tell in advance whether the other Saab driving adolescent will act aggressively or wimp

[1]Car insurance is too costly for adolescent males so they remain car-less.

[2]The blood and guts resulting from accidents grosses out the boys)

out. The payoff to adopting either strategy oneself depends on the proportion of the other females that act aggressively and the proportion that are chicken.

a. (5 pts) If strategies which attract more boys get chosen more often than strategies which attract fewer boys, convince me that there cannot be an equilibrium in which all adolescent female Saab drivers are wimps.

b. (5 pts) If strategies which attract more boys get chosen more often than strategies which attract fewer boys, convince me that there cannot be an equilibrium in which all adolescent female Saab drives are aggressive.

c. (5 pts) Since there is not an equilibrium where every female chooses the same strategy, determine whether there is an equilibrium where some proportion of the adolescent females choose to act aggressively, and the rest chose to wimp out. What is the equilibrium proportion of aggressive drivers?

d. (3 pts) If the proportion of aggressive drivers is greater than the equilibrium proportion of aggressive drivers, which strategy does better? If the proportion of aggressive drivers is less than the equilibrium proportion of aggressive drivers, which strategy does better?

e. (2 pts) If the proportion of aggressive drivers is not the equilibrium proportion, will the proportion move back toward the equilibrium proportion as either wimps get more aggresive or previously straight drivers chicken out?

3. (10 points) Convince me that an individual, Wilma, who is maximizing her utility, u(x, y), subject to a budget constraint will behave **as if** her utility function is *quasiconcave* even if it is not. You might want to convince me using graphs of indifference curves and budget lines to identify utility maximizing bundles. As part of your answer, define quasiconcave for this utility function, and explain what it tells you about the shape of indifference curves. Assume that Wilma's utility function is increasing in x and y. The point of this exercise is to demonstrate that it is not restrictive to assume that a utility function that is increasing in its arguments is also quasiconcave.

4. (10 points) Defince *Shephard's Lemma* in terms of consumer theory. What is derived and what does is mean?

5. (10 points) Consider the following utility maximization problem.

$$Max_{wrt\ x,\ y}\ u(x,y)\ s.t.\ I\ '\ p_x x\ \%\ p_y y$$

where I is income p_x is the price of good x and p_y is the price of good y. Assume that the utility function is strictly quasi-concave. Now assume that you have found a *critical* point in terms of x, y and the lagrangian multiplier ?; i.e. a point where all the derivatives of the Lagrangian are zero. Convince the reader, however you can, that this critical point maximizes utility subject to the budget constraint.

6. (10 points) Demonstrate, using graphs or whatever, that the production function $X = L^{.5}K^{.5}$, is strictly quasi-concave in terms of L and K. Hint: think about the shape of the isoquants for this production function and whether this production function is increasing in its arguments. As part of your answer define quasi-concavity of the production function in terms of L and K

7. (10 points) What is a random variable? Start your answer with the statement, " x is a random variable if ". Define the joint density function for the random variables x and y. Also define in both words and mathematical symbols the marginal distribution of x.

8. (10 points) Assume a profit maximizing monopolist whose demand function is

$x = x(p) = b-ap \qquad b>0$ and $a>0$

and whose cost function is

$c(x,w,r) = x^2 g(w,r)$

where

$g(w,r)>0$

Derive the profit maximizing level of output, x^s, as a function of w and r

i.e.,

$x^s = x^s(w,r)$

Don't forget to check the second order conditions.

9. (10 points) Assume an indirect utility function $U = V(y, p_1, p_2)$, where y is income and p_i is the price of good i. Assume the system of Hicksian demand functions are

$$x_1^h = x_1^h(U,\ p_1,\ p_2) = U(\alpha + \gamma\, p_1^{-1/2}\, p_2^{1/2})$$

$$x_1^h = x_1^h(U,\ p_1,\ p_2) = U(\beta + \gamma\, p_1^{1/2}\, p_2^{-1/2})$$

Derive the functional form of the indirect utility function.