

Wilbur's Intertemporal Consumption and Substitution

Two periods ⁰ and ¹, then the world ends.

w^0 / Wilbur's wealth at the beginning of the 1st period (assume Wilbur can't/won't work).

Wealth, w , can be invested and earn the market rate of interest, r_m

c^0 / amount Wilbur consumes 1st period.

c^1 / amount Wilbur consumes 2nd period.

So,

$(w^0 - c^0)$ = amount invested and

$$(1 + r_m)(w^0 - c^0) = c^1.$$

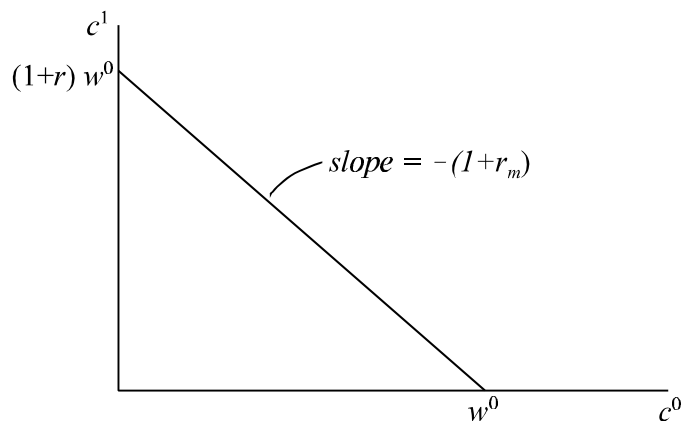
Therefore, his budget constraint is

$$w^0 = c^0 + \left(\frac{1}{1 + r_m} \right) c^1.$$

Or, rearranging,

$$c^1 = (1 + r) w^0 - (1 + r_m) c^0 \quad \text{and} \quad \frac{\partial c^1}{\partial c^0} = - (1 + r_m).$$

Graphing the budget line



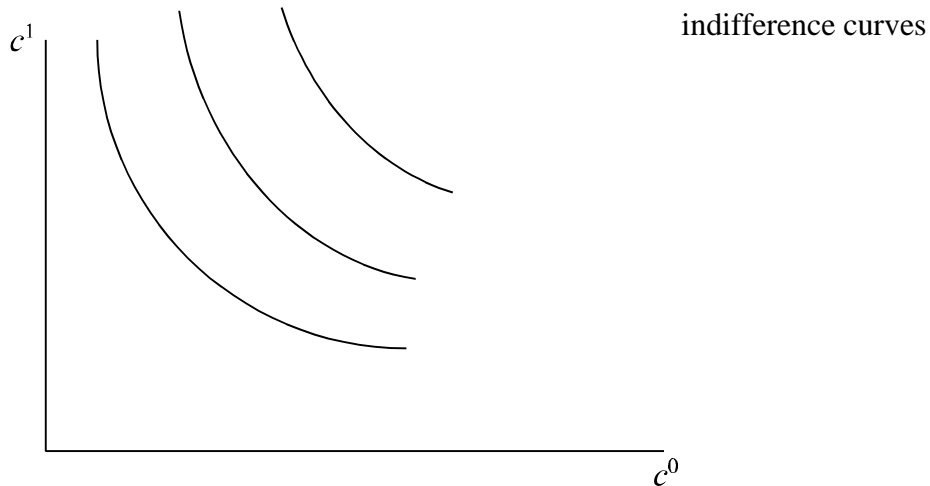
Therefore $(1 + r_m)$ is the rate at which Wilbur can substitute future for current consumption.

If he given up an additional unit of current consumption, future consumption will \uparrow by $(1 + r_m)$ units.

$1 + r_m = \frac{(1 + r_m)}{1}$ is the relative price of current consumption in terms of future consumption.

Wilbur also has preferences over future vs. current consumption.

$$u = u(c^0, c^1)$$



The slope of an indifference curve is

$$\left. \frac{\Delta c^1}{\Delta c^0} \right|_{\Delta u=0} \equiv \left. \frac{\partial c^1}{\partial c^0} \right|_{\Delta u=0} \equiv \text{the rate Wilbur is willing to trade current for future consumption.}$$

$$\text{E.g. if } \left. \frac{\partial c^1}{\partial c^0} \right|_{\Delta u=0} = -2$$

if c^0 by one unit, c^1 must 8 by 2 units for Wilbur to be indifferent.

We call the negative of the slope of the indifference curve

$$MRS_{c^0, c^1}$$

It is completely determined by Wilbur's preference for the present over the future.

If $MRS_{c^0 c^1} > 1$ Wilbur prefers current to future consumption. That is, Wilbur discounts the future.

If $MRS_{c^0 c^1} = 1$ Wilbur is indifferent between future and current consumption.

If $0 < MRS_{c^0 c^1} < 1$ Wilbur prefers future to current consumption. He would give up more than 1 unit of c^0 to get an additional unit of c^1 . *Wilbur lives for the future.*

A useful way to define $MRS_{c^0 c^1}$ is

$$MRS_{c^0 c^1}^w = \left(1 + r^w(c^0, c^1)\right) \text{ where } r^w \text{ is}$$

Wilbur's personal rate of discount given his current intertemporal consumption bundle

if $r^w > 0$, Wilbur discounts the future.

if $r^w = 0$, Wilbur is indifferent between current and future consumption.

if $r^w < 0$, Wilbur prefers the future over the present.

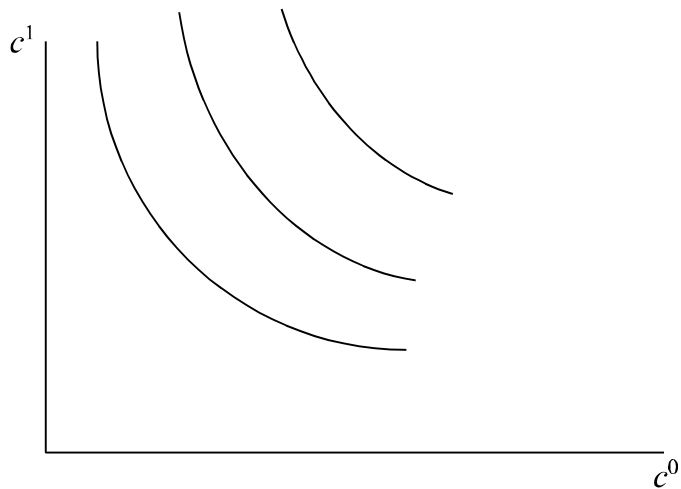
Note that $r^w = r^w(c^0, c^1)$.

Is your personal rate of discount positive?

What would you do right now if your discount rate were 4?

Using Wilbur's indifference curves and budget lines, identify his utility maximizing allocation of c^0 and c^1 .

Assuming smooth indifference curves that look like



In equilibrium, what is the relationship between r_m and r^w ?

Now draw 2 sets of indifference curves: one for George and one for Polly and the same budget constraint for each.

Such that,

George will save nothing for future consumption.

Polly will save everything for future consumption.

It more conservation always better than less?

What do George's indifference curves look like if George has a constant discount rate? Is this likely?

What do George's indifference curves look like if current and future consumption are compliments rather than substitutes?