Econ 4999 Math and Logic

Economic and statistical applications of integration

1. Assume that that the market rate of interest is r. Further assume that you own a bottle of Laffite 1961, which you purchased for in 1961 for \$35. Its value at time t is $V(t) = 35e^{gt}$. You would never drink it (you don't like good wine). When should you sell it?
2. Assume that that the market rate of interest is 04. Further assume that you own a bottle of Laffite 1961, which you purchased in 1961 for \$35. Its value at time t is $V(t) = 35e^{6t^{.8}}$. You would never drink it (you don't like good wine). When should you sell it?
3. Make-up a density function, prove that it is a density function, and derive its CDF. Graph both the density function and the corresponding CDF. I will grade this question on the basis of correctness, completeness, and originality.
4. For the density function you reported in the previous question, find its mean, median and mode. As part of your answer, define, in both words and mathematical notation, mean, median and mode.
5. For the density function you have made up, find its variance. As part of your answer, define, both in words and mathematical notation, variance.
6. Derive the specific indirect utility function, $u = v(m, p_1, p_2)$, assuming the following Hicksian demand functions: $x_1^h = (\alpha + \frac{1}{2}\gamma p_1^{-1/2} p_2^{1/2})u$ and $x_2^h = (\beta + \frac{1}{2}\gamma p_1^{1/2} p_2^{-1/2})u$. Do it using integration. Can you do it without using integration? Yes or No and how.
7. Derive the specific indirect utility function, $u = v(m, p_1, p_2)$, assuming the following Hicksian demand functions: $x_1^h = \alpha u p_1^{(\alpha-1)} p_2^{\beta}$ and $x_1^h = \beta u p_1^{\alpha} p_2^{(\beta-1)}$. Do it using integration.
8. For any continuous density function $f(x)$, $-\infty < x < +\infty$. What is the probability that x takes the specific value b?
9. Make up an example of an economic application of multiple integration. I will grade this on originality, complexity, the concept "taught" and correctness.
10. Assume you are the sole owner of a stock of sole. Denote the stock at time t as $s(t)$. Let $s(0) = s_0$. The stock grows at the biological rate $\frac{ds(t)}{dt} = g(s(t))$. Assume the price of sole, p_s, is constant over time. Denote the harvest rate at time t with $h(t)$. Obviously, $0 \le h(t) < s(t)$. Further assume it costs nothing to harvest fish. Write down an equation for the present value of this stock as a function of $h(t)$. Assume an interest rate r. Then write down the optimization problem you would want to solve find the path of harvest rates over time that would maximize the present value of the stock. You don't need to solve this problem.
11. Assume that

$$\frac{\partial c}{\partial y} = c'(y) = .8 + .1y^{-.5}$$

where y is aggregate level of national income and c is the aggregate level of consumption. Further assume that savings is zero when $y = 100$. Find the consumption function.

12. The demand for haute couture sunglass is growing at the rate of k percent a year. Assume $k > 0$. Assume that demand is currently 100 pair. How long will it take for demand to double? To double again? What's going on.

answer: Solve $200 = 100e^{kt}$, Solution is: $\left\{t = \frac{\ln 2}{k}\right\}$. This is how long it wil take for demand to grow from 100 to 200. For example, if $k = .50$, $t = \frac{\ln 2}{.5} = 1.3863$ years. To determine how long it will take for demand to increase from200 to 400, solve $400 = 200e^{kt}$. The solution is $t = \frac{1n2}{k}$ which is the same amount of time it takes to go from 100 to 200. The demand doubles every $\frac{1n2}{k}$ years.

13. A computer salesman sells his product within R miles of his store. He charges for shipping at the rate s dollars per mile. If m is the price at the store, then the delivered price, p, to a customer x miles form the store is

$$p = m + sx$$

where $0 \leq x \leq R$. The demand function for the computers is $c = 60 - p$. How many computers does he sell? What is the average price (including shipping).