1 Rules for Differentiation

rules.tex and .pdf April 3, 2007

Deriving the derivative of a function using the basic definition of a derivative is revealing but is typically quite tedious when one wants to differentiate a complicated function. So typically when we take a derivative we use "rules" for differentiation that have been derived by other from the basic definition of a derivative.

We use them so frequently that they should be memorized

- The constant function rule

$$\text{if } f(x) = k, \ \frac{df(x)}{dx} = 0$$

- The power rule

$$\text{if } f(x) = x^r, \ \frac{df(x)}{dx} = rx^{r-1}$$

- The sum-difference rule

$$\text{if } f(x) = m(x) + h(x), \ \frac{df(x)}{dx} = \frac{dm(x)}{dx} + \frac{dh(x)}{dx}$$

- The ln rule and the exponential rule[1]

$$\text{if } f(x) = \ln x, \ \frac{df(x)}{dx} = \frac{1}{x} \text{ where } \ln \text{ denotes natural log}$$

$$\text{if } f(x) = e^x, \ \frac{df(x)}{dx} = e^x \text{ Wow}$$

- The product rule

$$\text{if } f(x) = m(x)h(x), \ \frac{df(x)}{dx} = \frac{dm(x)}{dx}h(x) + \frac{dh(x)}{dx}m(x)$$

- The chain rule (differentiation of a function of a function)

$$\text{if } f(x) = g(m(x)), \ \frac{df(x)}{dx} = \frac{dg(m(x))}{dm(x)}\frac{dm(x)}{dx}$$

[1] Note that $\ln x \equiv \log_e x$, where e is the base of the natural log. $e = 2.71828......$

- The log rule (in more general form)[2]

$$\text{if } f(x) = \log_b x,\ \frac{d}{dx}(\log_b(x)) = \frac{1}{x \ln b}$$

  Note that if $b = e$, $\ln(e) = 1$ and $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$ as was noted above. What if $b = 10$?

,

- the m^x rule (note that the expodential rule is a special of this rule where $m = e$, so $\ln m = 1$)

$$f(x) = m^x,\ \frac{d}{dx}(m^x) = m^x \ln m$$

- The quotient rule

$$\text{if } f(x) = \frac{m(x)}{h(x)},\ \frac{df(x)}{dx} = \frac{\frac{dm(x)}{dx}h(x) - \frac{dh(x)}{dx}m(x)}{(h(x))^2}$$

These rules, and some others, can all be derived from the basic definition of a derivative.

[2] Note that $\log_b x = (\log_b e)(\log_e x) = (\log_b e) \ln x$. In terms of notation, when some authors write just $\log x$ they mean $\log_{10} x$, which is sometimes called the common log.

Consider the *constant rule*

$$\text{if } f(x) = k, \ \frac{df(x)}{dx} = 0$$

If $f(x) = k$, $f(x_o) = k$ and $f(x_o + t) = k$, so

$$\frac{df(x_0)}{dx} = \lim_{t \to 0} \frac{f(x_0 + t) - f(x_0)}{t} = \lim_{t \to 0} \frac{k - k}{t} = 0$$

Now consider the *power rule* in the simple case where $r = 2$, if $f(x) = x^2$, $f(x_o) = x_0^2$ and $f(x_o + t) = (x_o + t)^2 = x_0^2 + 2tx_o + t^2$, so

$$\frac{df(x_0)}{dx} = \lim_{t \to 0} \frac{f(x_0 + t) - f(x_0)}{t} = \lim_{t \to 0} \frac{x_0^2 + 2tx_0 + t^2 - x_0^2}{t} = \lim_{t \to 0} \frac{2tx_0 + t^2}{t} = \lim_{t \to 0} 2x_0 + t = 2x_0$$

Do you think you can derive it for any value of r?

Try and derive the product rule? The quotient rule from the product rule and the other rules.

Can you derive a rule for the derivative of a special type of function. E.g. a special case of one of the functions in the above rules.