Econ 4808: review questions for basic calculus quiz

January 17, 2007

1. Find $\frac{dy}{dx}$ where $y = f(x) = 3 + 2x + 3x^2 + 3\ln x$. $\frac{d(3+2x+3x^2+3\ln x)}{dx} = 6x + \frac{3}{x} + 2$

2. Assume that $h(w) = (w^3 - w)(5w^4 + w^2)$. The the derivative of $h(w)$ and simplify it as much as you can. Explain you steps in words.

   answer: One can take the derivative using the product rule. In this case the product rule says $h'(w) = f'(w)g(w) + f(w)g'(w)$ where $f(w) = w^3 - w$ and $g(w) = 5w^4 + w^2$. Therefore $h'(w) = (3w^2 - 1)(5w^4 + w^2) + (w^3 - w)(20w^3 + 2w) = 15w^6 - 2w^4 - w^2 + 20w^6 - 18w^4 - 2w^2 = 35w^6 - 20w^4 - 3w^2$.

3. Find $\frac{dy}{dm}$ where $y = f(m) = am^{1/2} + m^{1/3}$. $\frac{d(am^{1/2}+m^{1/3})}{dm} = \frac{1}{3m^{\frac{2}{3}}} + \frac{1}{2}\frac{a}{\sqrt{m}} = (1/2)am^{-1/2} + (1/3)m^{-2/3}$

4. Find $\frac{dy}{dx}$ where $y = f(x) = g(x)h(x)$. $\frac{d(g(x)h(x))}{dx} = \frac{dg(x)}{dx} \equiv \frac{dg(x)}{dx}h(x) + g(x)\frac{dh(x)}{dx} \equiv g'(x)h(x) + g(x)h'(x)$

5. Find the partial derivative $\frac{\partial y}{\partial x_1}$ where $y = f(x_1, x_2) = 2x_1x_2^2 + x_1^3x_2$. $\frac{\partial(2x_1x_2^2+x_1^3x_2)}{\partial x_1} = 2x_2^2 + 3x_1^2x_2$

6. Define in words and matematical notation the concept of a derivative.Mathematically, the derivative of $f(x)$ at $x = x_0$ is $\frac{df(x_o)}{dx} = \lim_{t\to 0}\frac{f(x_0+t)-f(x_o)}{t}$. In words, the derivative of $f(x)$ at $x = x_0$ is the slope of a tangent line to $f(x)$ at $x = x_0$, or said another way, it is the limit of slope of $f(x)$ between $x = x_0$ and $x = x_0 + t$

7. Define in words and mathematical notation the concept of a partial derivative. Consider the function $y = f(x_1, x_2)$. Mathematically the partial derivative of y wrt x_1 evaluated at x_1^0, x_2^0 is denoted $\frac{\partial f(x_1^0,x_2^0)}{\partial x_1}$ or $f_{x_1}(x_1^0, x_2^0)$ and is defined as $\lim_{t\to 0}\frac{f(x_1^0+t,x_2^0)-f(x_1^0,x_2^0)}{t}$ In words, it is the slope of a tangent line to $f(x_1, x_2)$ at x_1^0, x_2^0 in the x_1 direction, or, said another way, it is the limit of the slope of $y = f(x_1, x_2)$ between x_1^0, x_2^0 and $x_1^0 + t, x_2^0$.

8. Find $\frac{\partial m}{\partial a}$ where $m = m(a, z) = 4a^bz$. $\frac{\partial(4a^bz)}{\partial a} = 4bza^{b-1}$

9. Find the partial derivative $\frac{dx}{dm}$ where $x = x(m) = \frac{am^b}{\ln m}$. Using the quotient rule$\frac{d(\frac{am^b}{\ln m})}{dm} = \frac{bam^{b-1}\ln m - m^{-1}am^b}{(\ln m)^2} : \frac{1}{\ln^2 m}\left(ab(\ln m)m^{b-1} - am^{b-1}\right)$. Another way to do it: $x(m) = am^b(\ln m)^{-1}$. So, using the product rule $\frac{d(am^b(\ln m)^{-1})}{dm} = a\left[bm^{b-1}(\ln m)^{-1} - am^b(\ln m)^{-2}m^{-1}\right] = am^{b-1}(\frac{b}{\ln m} - \frac{1}{\ln^2 m})$. The two answers are the same, they just look different.

10. Thinking about derivatives, what is the product rule. State the rule in general terms and then give an example. If $y = f(x) = g(x)h(x)$, $\frac{d(g(x)h(x))}{dx} = \frac{dg(x)}{dx}h(x) + g(x)\frac{dh(x)}{dx}$. For example if $g(x) = e^x$ and $h(x) = qx^{.5}$ then $\frac{d(g(x)h(x))}{dx} = (e^x)qx^{.5} + e^x(.5qx^{-.5}) = e^x q\left[x^{.5} + (.5x^{-.5})\right]$

11. Consider the power rule. Define the power rule for the function $g = g(m) = sm^h$. Now provide a numerical example. The power rule is $\frac{d(sm^h)}{dm} = hsm^{h-1}$. For example if $s = 4$ and $h = 3$, $\frac{d(4m^3)}{dm} = (3)4m^2 = 12m^2$.