Econ 4808: review questions for basic algebra

1. Factor the expression $a^{x+2} = a^x a^2$
2. Expand and simplify $\frac{a^4 b^{-3}}{(a^2 b^{-3})^2} = \frac{a^4 b^{-3}}{a^4 b^{-6}} = \frac{b^6}{b^3} = b^3$
3. Expand the expression $(2t-1)(t^2-2t+1) = 2t^3 - 4t^2 + 2t - t^2 + 2t - 1 = 2t^3 - 5t^2 + 4t - 1$
4. $\frac{x+y}{x(x+1)} = \frac{(1+\frac{y}{x})}{x+1}$ is true or false? Prove it. It is true $\frac{x+y}{x(x+1)} = \frac{\frac{1}{x}(x+y)}{\frac{1}{x}x(x+1)} = \frac{(1+\frac{y}{x})}{x+1}$
5. What does $\frac{m+2}{m-1} < 0$ tell one about the value of m? $\frac{m+2}{m-1}$ will be negative if (a) $(m+2)$ is positive and $(m-1)$ is negative, or (b) if $(m+2)$ is negative and $(m-1)$ is positive. Condition a requires that $1 > m > -2$,and condition b requires that $1 < m < -2$. But this second inequality cannot be true: m cannot both be greater than 1 and less than -2. That is, $(m+2)$ negative and $(m-1)$ positive is impossible. So the answer is $1 > m > -2$. In words m is between one and negative two, not including 1 or negative 2.
6. My mother taught me that one should never date anyone that is less than half their age plus 7. Let A_o be the age of the older person and let A_y be the age of the younger person. Express this constraint algebraically. Then use this algebraic expression to determine at what age one can only date people of one's own age. $A_y \geq .5A_o + 7$. At what age will $A = .5A + 7$, Solution is: 14.0.
7. Factoring: algebraic expression can often be factored: that is written as the product of a number of terms. Factor the following expressions

$$a(1+m) - a(1+m)m$$

: $a\,(1-m)\,(m+1)$

$$K^{-4} - LK^{-5}$$

: $K^{-5}\,(K-L)$

8. Is the following simplification correct

$$\frac{m}{5m+m^2} = \frac{1}{5} + \frac{1}{m}$$

Yes or no and explain in words why. No it is not correct, the correct answer is $\frac{m}{5m+m^2} = \frac{1}{m+5}$. That is, one can factor the m out of both the numerator and denominator and then cancel. There is a rule that says the each additive expression in the numerator can be divided by the term in the denominator. In incorrectly concluding that $\frac{m}{5m+m^2} = \frac{1}{5} + \frac{1}{m}$,the misguided math wiz incorrectly thinks the inverse holds; that is, one can divide each additive expression in the denominator by the common expression in the numerator (e.g. $\frac{1}{2+3} \neq \frac{1}{2} + \frac{1}{3}$)

9. Simplify the following expression:

$$\frac{3}{2b} - \frac{5}{3b}$$

: $\frac{3}{2b} - \frac{5}{3b} = \frac{9}{6b} - \frac{10}{6b} = -\frac{1}{6b}$

10. Try to simplify the following expression so that it is a function of only one square root, and the square root appears in the numerator.

$$\frac{\sqrt{a}}{\sqrt{b}}$$

Assume a and b are positive. If a and b are positive, then $\frac{\sqrt{a}}{\sqrt{b}} \equiv \frac{a^{\cdot 5}}{b^{\cdot 5}}$ multiply the numerator and denominator by $b^{\cdot 5}$ to get $\frac{a^{\cdot 5}b^{\cdot 5}}{b^{\cdot 5}b^{\cdot 5}} = \frac{a^{\cdot 5}b^{\cdot 5}}{b} = \frac{(ab)^{\cdot 5}}{b} = \frac{\sqrt{ab}}{b}$

11. Solve, showing all of your steps,

$$(\frac{1}{27})^{-\frac{2}{3}}$$

: $(27)^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$