

## **0.1 Mathematics is a language of symbols, grammar and logic. It greatly facilitates the process of deduction**

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While reviewing the syllabus, I said mathematics is a language.

Languages are ways of conveying information and meaning. Mathematics is a language whose elements consist of symbols (letters numbers, etc.).

There are rules, a "grammar", for arranging those symbols into meaningful statements.

Much of the intent of mathematics is to facilitate the process of deduction (seeing what follows from what has been assumed).

With that in mind, consider the symbol “3”

3 is a symbol in what language? It is an Arabic numeral

What does it mean? Write out, in groups of three what the symbol 3 means and be prepared to explain to me with a definition and examples.

"3" is difficult to define in words. 3-ness is a property of some sets of elements; that is, some sets have it and some sets don't

For example, Create five different sets of elements (combinations of books, students, phones, etc.) – two with three elements, two with two elements and one with one element.

The two sets with three elements share the common property of 3, "three-ness"

Compare the symbol "3" with the symbol  $\pi$ . They are both "numbers", why represent one with an arabic numeral and one with a greek letter?

Now consider the symbol  $x$  as a way of representing one of more numbers.

For example,  $x$  might represent the weight of the first customer of the day at IHOP. If  $x$  can take different values, vary, we call  $x$  it a variable: the weight of the first customer can take different values. The variable  $x$  has some *domain of variation*, that is, some range of variation. (*Domain* as in *domicile*: where the variable lives.) Something that can vary in value is a *variable*.

There are different assumptions one can make about how much a variable can vary. Variables can be assumed to take only discrete values (e.g.  $x = 1, 2, 3, \dots$ ) or continuous values (e.g.  $0 \leq x \leq 5$ ), and instead of  $0 \leq x \leq 5$ , one could assume  $0 \leq x < 5$  or  $-3 \leq x < -1$ .

A great thing about symbols such as  $x$  is that they can represent different things at different times and in different contexts.

If  $x$  always had to denote the weight of a IHOP customer, one would quickly run out of symbols to represent things. So, make sure you always define your  $x$  and if you change what  $x$  means in midstream (generally a bad idea), tell the reader.

The grammar of mathematics includes well know statements such as  $3 + 5 = 10$ ,  $a/b$ , and  $y = f(x) = 3x^2$ . We all know -I hope - what these sorts of expressions mean. There is a whole set of notes on defining *functions* and *functional notation*.

One can use mathematics and its rules, all either rules of logic or definitions, to derive results such as the sum of 4 and 2 is 6, and if  $a > 5$  and  $b > a + 2$ , then  $b > 7$ . Using the language of mathematics to deduce results will be a main goal of this course. Adding up numbers is not.

Our logical manipulations of the symbols such as  $a$  and  $b$  will not depend at all on what  $a$  and  $b$  represent, so are very general manipulations