0.1 Mathematics is a language of symbols, grammar and logic. It greatly facilitates the process of deduction

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While reviewing the syllabus, I said mathematics is a language.

Languages are ways of conveying information and meaning. Mathematics is a language whose elements consist of symbols (letters numbers, etc.).

There are rules, a "grammar", for arranging those symbols into meaningfull statements.

Much of the intent of mathematics is to facilitate the process of deduction (seeing what follows from what has been assumed). It is also good for determing the total of your monthly bills.

With that in mind, consider the symbol “3”

3 is a symbol in what language? It is an Arabic numeral

What does it mean? Write out what the symbol 3 means and be prepared to explain to me with a definition and examples.

"3" is difficult to define in words. 3-ness is a possible property of a set of elements; that is, some sets have it and some sets don't

For example, Create five sets of elements (books, students, phone etc.) – two with three elements, two with two elements and one with one element.

The two sets with three elements share the common property of 3, "three-ness"

Compare the symbol "3" with the symbol π. They are both "numbers", why represent one with an arabic numeral and one with a greek letter?

Now consider the symbol x as a way of representing one of more numbers.

For example, x might represent the weight of a flower. We call x a variable since it can vary; that is, it can take different numerical values. The variable x has some *domain of variation*, that is, some range of variation. Something that can vary in value is a *variable*.

Variables can be assumed to take only discrete values (e.g. $x = 1, 2, 3, ...$) or continuous values (e.g. $0 \leq x \leq 5$)

A great thing about symbols such as x is that they can represent different things at different times and in different contexts.

If x always had to denote the weight of flowers, one would quickly run out of symbols to represent things. So, make sure you always define x and if you change what it means, tell the reader.

The grammar of mathematics includes well know statements such as $3+5 = 10$, a/b, and $y = f(x) = 3x^2$. We all know -I hope - what these sorts of expressions mean. There is a whole set of notes on defining *functions* and *functional notation*.

One can use mathematics and its rules (often rules of logic) to derive results such as the sum of 4 and 2 is 6, and if $a > 5$ and $b > a+2$, then $b > 7$. Using the language of mathematics to deriving results will be a main goal of this course. Adding up numbers is not.

Our logical manipulations of a and b depend not at all on what a and b represent, so are very general manipulations