0.1 Additional notes and problems on what is a theory (model)

Edward Morey, January 26, 2011 theory.pdf

These notes are intended to be read after one reads, and digests,

http://www.colorado.edu/economics/morey/2010/Lectures/2010_Lecture_IntroducToEconModels.pdf which are my Econ 2010 (micro principles) notes "An introduction to economic models"

Mathematics greatly helps with the process of building theories

What is a theory? It is a thing, specifically a logical construct, with the following three parts:

1. definitions
2. assumptions (more than one and have to be consistent with one another)
3. hypotheses ("if ..... then" statements; that is, predictions). These follow from the assumptions and definitions

Note that a prediction in one theory can be an assumption in another theory

Whoever develops a theory get to decide on what is assumed in the theory and how things are defined.

A theory must have all three parts and the hypotheses must follow logically (can be deduced) from the definitions and assumptions. A theory needs a least two assumptions. That said, one might imagine a set of assumptions and definitions where predictions follow from those assumptions and definitions but no one has yet figured out what those predictions are. This is definitely a potential theory.

Is the following statement a hypothesis (prediction in the scientific sense of the word)?
"The end of the world is coming"

Consider now the process of deriving predictions from the assumptions and definitions.

The process of deduction is often difficult, but can be made easier by describing the assumptions mathematically.

In which case, the body of mathematical logic can be applied to the problem.

1. mathematical symbolism is precise
2. things are expressed neatly and compactly, so its easier not to get confused
3. there are all these math theorems (inferences that have already been proven correct) to help us mathematically deduce the predictions

E.g. of an assumption in words
"the aggregate level of consumption increases as the level of aggregate income increases."

To express an assumption mathematically, we use mathematical symbols to define the variables in our model, and use functions (or other mathematical relationships) to specify the relationships between those variables

Our same assumption in mathematical notation:

$$C = f(Y)$$

and C increases when Y increases, where Y is defined as aggregate income and C is defined as aggregate consumption. That is, $C \equiv$ aggregate level of consumption measured in dollar units, and $Y \equiv$ aggregate level of national income measured in dollars

One could make the assumption more restrictive by assuming.

$$C = f(Y)$$
$$\frac{dC}{dY} > 0$$

The difference between $C = f(Y)$, and C is increasing in Y is a bit more general than $C(Y)$ where $\frac{dC}{dY} > 0$. Can you explain why?
Or one could make the assumption even more restrictive (specific) by assuming

$$C = a + bY, \text{ where } a,b > 0$$

or even more specific by assuming

$$C = 7.23 + .79Y$$

or

$$C = 4 + .5Y$$

Note that I have just used four examples of a function. What properties do all of these "functions" share. What makes a function a function? We will have a lecture on this.

The more specific are the assumptions in a theory, the more specific will be the predictions of the theory

Class/home exercise: Create an assumption tree with different assumptions about the relationship $C = f(Y)$, with most general, $C = f(Y)$, at the bottom (the trunk) and more restrictive assumptions as one proceeds out different branches. As you create your tree, keep in mind the terms neccesary and sufficient.

There is a dating question in the review questions.

The question is something like.

> Mothers teach that you should never date anyone less than half your age plus 7, and Fred, Helen and Margaret obey their mothers. Let A_F denote Fred's age in years, A_H Helen's age and A_M Margaret's age. Assume age is always expressed as an integer. Fred is the youngest in years. Helen dates Fred and Margaret dates Helen. Fred is 18. [The preceeding are all assumption.]Write down in terms of equations what we know about everyone's ages and deduce what can be predicted from this set of assumptions. [That is, what follows logically from these assumptions?]

- $A_F \geq .5A_H + 7$, $A_H \geq .5A_M + 7$ and $A_F = 18$, so putting these together $18 \geq .5A_H + 7$ and $A_H \geq .5A_M + 7$.
- What do we know about Margaret's age? Explain/show how you reached your conclusion. Margaret is at least 19 because Fred is 18 and Fred is the youngest.
- Helen's maximimum age is determined by solving $18 \geq .5A_H + 7$ for A_H. $18 \geq .5A_H + 7 \Rightarrow 11 \geq .5A_H \Rightarrow 22 \geq A_H$, so Helen is a maximum of 22, so between 19 and 22. (For example, if Helen were 23, her date has be an age that is greater than 18, 18.5, and Fred is only 18.)
- If Helen is a maximum of 22 how old can Margaret be? Solve $22 \geq .5A_M + 7$ for A_M. The solution is 30. Margaret is between 19 and 30.

So,

Definition:

1. $A_i \equiv$ age in individual i in years, $i = F, H, M$

Assumptions:

1. $A_F \geq .5A_H + 7$
2. $A_H \geq .5A_M + 7$
3. $A_F = 18$

Predictions that can be deduced from the above definitions and assumptions. For example,

Individual M is between 19 and 30 years of age

1. Other predictions? For example, do the definitions and assumption imply that individual M is older than individual H? Other predictions?

Note that you probably implicitly assumed that Fred was a male and Helen and Margaret were females, but this was not assumed so not something you can assume when you derive the predictions of this theory.

Break into a small group and make up a simple theory about *Gomers* and *Gubers*. Make sure to define both terms. You can define them however you please. Derive a prediction or predictions from your theory.

My Gomer/Guber theory

Definitions:

1. A gomer is an economist who studies game theory
2. A guber is a small squirrel-type animal.

Assumptions:

1. Each week, every gomer meets on the street one, and only one, guber.
2. Except for the street, gomers and gubers have no contact.
3. Gomers like gubers
4. Gomers, when they meet someone or something they like, kiss it
5. After every kiss, the kisser hits the kissee with a golf club.

Predictions?

Assumptions 3 and 4 imply that a gomer will kiss a guber if they meet a guber.

This and assumption 1 implies that each week a gomer kisses one, and only one guber

This and asssumption 5 imples that each guber each week gets hit once with a golf club by a gomer.

This and definitions 1 and 2 imply that every economist who studies game theory once a week hits a squirrel-type animal with a golf club.

Other predictions?

What doesn't my theory predict?

For example, it does not predict that squirrel-type animals get wacked with a golf club only once a week. For example, the theory predicts nothing about how often squirrel-type animals get wacked each week by their significant other.

Play with this theory. For example, eliminate one or more of the assumptions and see what is still predicted and what is no longer predicted.

Change an assumption and figure out how the predictions change.