

Cats, lions and zebras: assumptions, predictions, the utility function, and, sufficient but not necessary

catfood.tex February 20, 2007

Consider the following theory about "What's for dinner?":

Definitions:

1. $C \equiv$ a cat is a furry animal that meows or roars.
2. $L \equiv$ Lions are large, tan animals that are indigenous to Africa
3. $Z \equiv$ Zebra are hoofed animals with stripes

Assumptions:

1. All cats eat every type of available meat ($(C \equiv \text{large cats}) \Rightarrow (EAM \equiv \text{eat all available meat})$)
2. All cats are large
3. All hoofed animals are made of meat ($(H \equiv \text{hoofed animal}) \Rightarrow (M \equiv \text{meat})$)
4. Zebras are available.
5. Lions are the only type of large cat: lions and large cats are one and the same.

From this simple theory can we drive the prediction that *if you are a lion, you eat zebra?* Yes

Proof:

- a $L \Leftrightarrow C$ by definitions 1, 2 and assumption 2 and 4.

- b** $C \Rightarrow EAM$ by assumption 1, so by transitivity $L \Rightarrow EAM$. In words, *lions eat every type of available meat*.
- c** Continuing with our proof, $L \Rightarrow EAM$ and $(H \Rightarrow M)$ together imply that lions would eat hoofed animals if they were available ($L \Rightarrow EAH$), where *EAH* is eating available hoofed animals.
- d** So, $L \Rightarrow EAZ$ by the definition of zebra
- e** And $L \Rightarrow EZ$ by assumption 4. q.e.d.

Consider the role of Assumption 5 in my theory. What if I dropped it? My theory

would still predict that *lions eat zebra* but would not require every large cat to be a lion, so would lead to the more general prediction *All large cats eat zebra*.

Play with the above theory: remove assumptions one at a time and figure out what the theory predicts, if anything, with each reduced set of assumptions.

So, what does any of this have to do with what one assumes about preferences and utility functions?

The goal of consumer theory is to predict the bundle that an individual will consume. In class, we assumed an individual is constrained to consume a bundle that is feasible (affordable).

Given this, the assumption that she has **at least** a ranking over bundles, and the assumption that she will choose the highest ranked bundle she can afford, one can predict her chosen bundle if one knows her constraints and her ranking.

That is, all we need to assume about preferences for consumer theory is that the individual has at least a ranking of bundles - if an individual can't rank bundles (does not know their own preferences), the theory does not apply to her. This was independently deduced by Vilfredo Pareto and Irving Fisher.

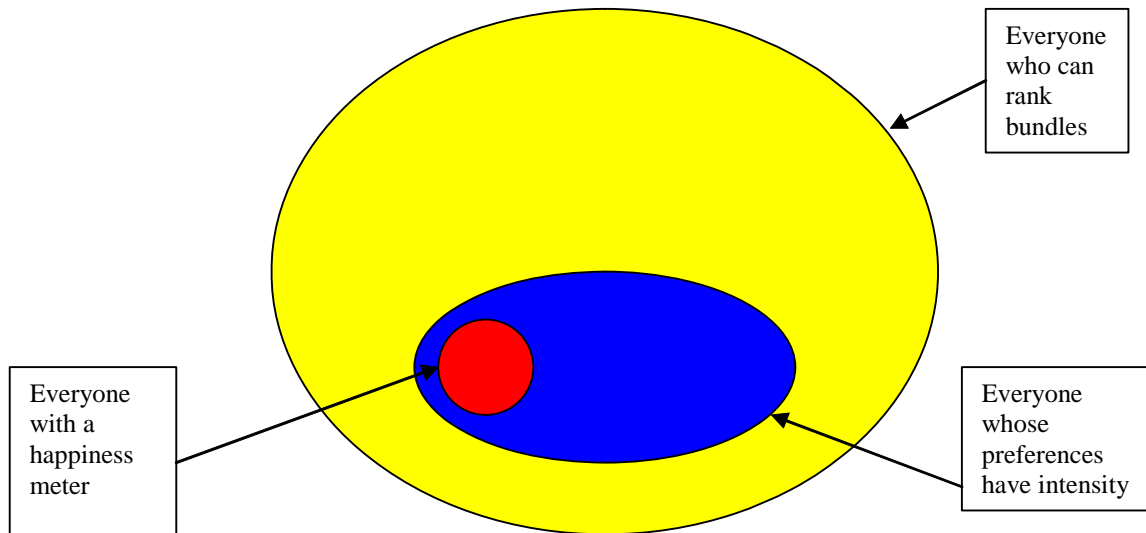
But, maybe individuals can do a lot more than just rank bundles (e.g. say how much more they like one bundle than another (preference with intensity) or, even more amazingly, have a happiness meter such that each bundle provides a quantifiable amount of happiness).

That individuals had such cardinal preferences was assumed by economists in the late 19th century and early 20th century.

These abilities, while possibly useful for something, are not needed to choose a bundle to consume: only the ranking property of preferences is used when we make choices.

So, when we specify consumer theory we typically don't assume individuals' preferences have these additional properties. If we required utility to represent happiness, rather than just a ranking, the applicability of our theory would be drastically diminished for no good reason.

Different types of preferences



Individuals not in the yellow circle are unable to rank bundles.

Assuming that everyone with preferences has a happiness meter is like assuming every large cat is a lion. Neither is necessary to make the prediction

Two last things about definitions and assumptions

If it is your theory you can define things however you want, along as you are consistent.

You can assume men are from Mars.

Or, if you were developing a theory to explain which bundle a individual will consume, you might assume that individuals have a happiness meter that associates a cardinal amount of happiness with each possible bundle. You could further assume that utility and happiness are the same thing.

But, after I examine your theory, I will likely tell you , "You can significantly weaken your preference assumption without changing what your theory predicts about consumer behavior, and generalizing your assumptions in this way is a good thing" (Occam's razor - why assume x things when fewer than x assumptions will generate the same set of predictions).

However, if I ask you to define, on a quiz or exam, terms such as *preferences* or *utility functions*, give the definitions presented in class. Then, if you so desire, tell me you feel that the class definition is not restrictive enough for the real world, e.g. you believe all people do have happiness meters.