What is a utility function?

Additions to the notes "Consumer Theory in a Nutshell"

WhatisaUtilityFunction.tex February 5, 2012

These notes are to be read after you have read, and digested, my Econ 2010 (micro principles) notes "Consumer Theory in a Nutshell"

at
http://www.colorado.edu/economics/morey/2010/Lectures/ct-nutshell_PrinciplesMicro.pdf

You might want to read up on consumer theory in your intermediate micro book

1 What is a utility function? Just a simple, but potentially misleading, way to represent an individual's ranking over bundles.[1]

Name a function that correctly represents an individual's ranking a Snerd function. What properties must a Snerd function have to correctly represent an individual's ranking of bundles?

It must associated some number, s, with every possible bundle; that is $s = s(x_1, x_2, ..., x_n)$ such that

$$\mathbf{x}^j \succ \mathbf{x}^k \Rightarrow s(x_1^j, x_2^j, .., x_n^j) > s(x_1^k, x_2^k, ...s_n^k)$$
$$\mathbf{x}^j \prec \mathbf{x}^k \Rightarrow s(x_1^j, x_2^j, ..., s_n^j) < s(x_1^k, x_2^k, ..., x_n^k)$$

and

$$\mathbf{x}^j \sim \mathbf{x}^k \Rightarrow s(x_1^j, x_2^j, ..., x_n^j) = s(x_1^k, x_2^k, ..., x_n^k)$$

If $s(x_1, x_2, ...s_n)$ has these properties it will correctly rank the bundles. If one functions correctly ranks there are an infinite number of other functions that will also rank correctly.

Note that all of the information about how bundles are ranked can be inferred from the ordinal properties of $s(x_1, x_2, ...s_n)$. While the Snerd function, like all

[1] Iin the aforementioned notes, I introducted *States of the World* and allowed states of the world to include non-market commodities such as your number of friends and pollution levels. In which case, as in the real world, one's rankings depends on those things. In these notes, to keep the notation simple I am ignoring non-market commodities.

functions, has both ordinal and cardinal properties, the cardinal properties of the Snerd function provide no additional information about the ranking. What does ordinal and cardinal mean? More later on ordinal and cardinal.

Note that we could economize on our notation letting $\mathbf{x} = [x_1, x_2, ..., x_n]$. In which case $s(x_1, x_2, ...s_n)$ can be written as $s(\mathbf{x})$

2 In general, we don't call a function that correctly represents the ranking a Snerd function. Rather, we call it a utility function. $u(\mathbf{x})$ represents the individual's ranking of bundles if (using the more compact notation)

$$\mathbf{x}^j \succ \mathbf{x}^k \Rightarrow u(\mathbf{x}^j) > u(\mathbf{x}^k)$$

$$\mathbf{x}^j \prec \mathbf{x}^k \Rightarrow u(\mathbf{x}^j) < u(\mathbf{x}^k)$$

and

$$\mathbf{x}^j \sim \mathbf{x}^k \Rightarrow u(\mathbf{x}^j) = u(\mathbf{x}^k)$$

If $\mathbf{x}^k \succ \mathbf{x}^m$ all that matters is that $u(\mathbf{x}^k) > u(\mathbf{x}^m)$, not how much more.

3 Given $u(\mathbf{x})$, the individual's choice problem can be represented as one of

$$\max_{wrt\ \mathbf{x}} u(\mathbf{x})$$

s.t.

$$y = p_1 x_1 + p_2 x_2 + ... + p_n x_n$$

if one assumes that more is preferred to less.

If the solution, $\mathbf{x}^*$, is unique, it will be of the form

$$x_i^* = x_i(y, \mathbf{p}) \ i = 1, 2, ...n.$$

where $\mathbf{p} = [p_1, p_2, ..., p_n]$. This is a system of n demand functions.

Consider the simple case of a world with only two goods. The consumer's choice problem can then be described as one of

$$\max_{wrt\ x_1, x_2} u(x_1, x_2)$$

s.t.

$$y = p_1 x_1 + p_2 x_2$$

if one assumes that more is preferred to less.

If the solution, $\mathbf{x}^*$, is unique, it will be of the form

$$x_1^* = x_1(y, \mathbf{p})$$

and

$$x_2^* = x_2(y, \mathbf{p})$$

For example, what are the demand functions if $u = u(x_1, x_2) = x_1 x_2$, or if $u = u(x_1, x_2) = x_1^2 x_2$. We will be solving explicit utility maximizing problems like this later in the term. Should we do one now?

Look at and make sure you can do the consumer theory problems in the first set of review questions.