

1 Set theory: some additional examples

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before you read this read the Chapter 3 set theory notes which introduce sets, set notation, Euler and Venn diagrams, and the algebra of sets.

if you have a math-econ book or a book on probability theory, check it out for a section on sets and set theory

1.1 Consider the following set that might be important to a firm that uses only two inputs (l and k) to produce its output:

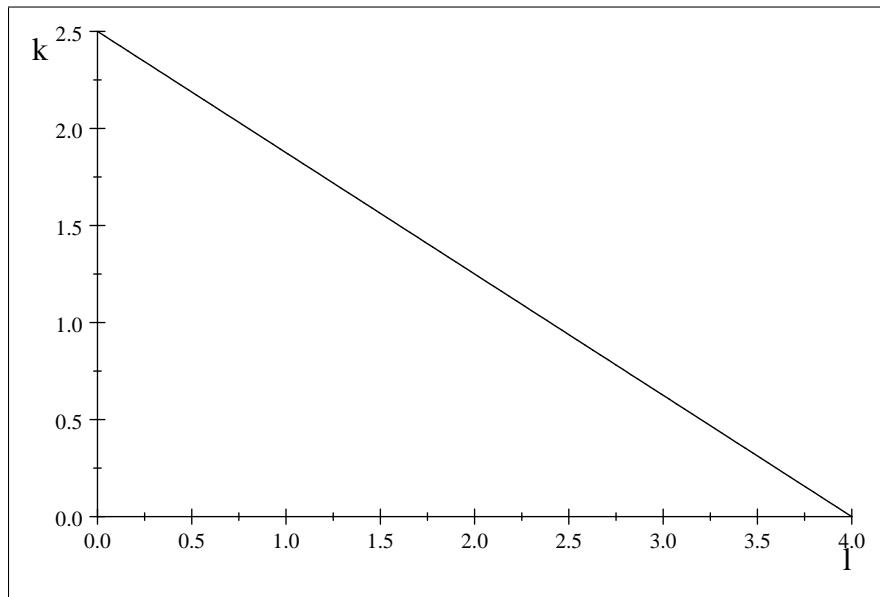
$$A \equiv \{(l, k) : wl + rk \leq m, l \geq 0, k \geq 0\}$$

where l is the quantity of labor, w , the wage rate (the price of labor), k is the quantity of capital, r is the rental price of capital, and m is some amount of money. Define this set in words. It is "all those bundles of labor and capital that the firm can purchase for $\$m$ or less". **Graph this set with k on the vertical axis and l on the horizontal axis. What do we call the set?**

Now consider a different set

$$B \equiv \{(l, k) : wl + rk = m, l \geq 0, k \geq 0\}$$

To picture set B , solve $wl + rk = m$ for k , Solution is: $k = \frac{1}{r}(m - lw)$ if $r \neq 0$. Graph this with k on the vertical axis and l on the horizontal axis. For example, if $m = 100$, $w = 25$, and $r = 40$, $k = \frac{1}{40}100 - l25$, and for $0 \leq l \leq 4$,



This set is the downward sloping line between $(0, 2.5)$ and $(4, 0)$: it is all those combinations of l and k that will cost 100 given that $w = 25$ and $r = 40$. Contrast set B with set A . If $m = 100$, $w = 25$ and $r = 40$, set A is the right triangle bounded by $(0, 0)$, $(4, 0)$ and $(0, 2.5)$: it is all those input combinations that cost 100 or less. If a firm were constrained to only spend $\$100$ on inputs, at these prices the firm would have to choose its input combination from set A .

1.1.1 Define and discuss the *input requirement set*.

Assume a firm produces product x using the inputs l and k . Further assume the production function

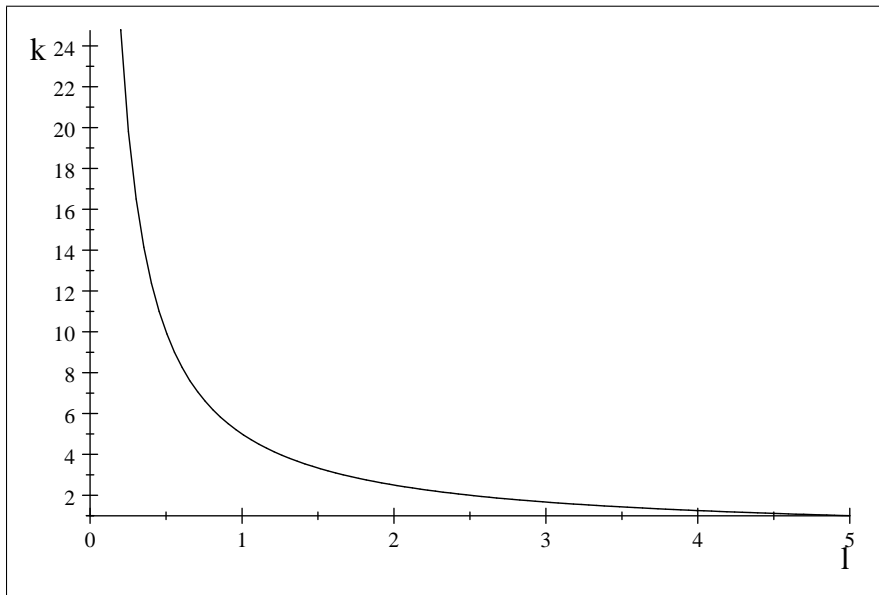
$$x = f(k, l)$$

which identifies the maximum number of units of output that can be produced using k units of capital and l units of labor.

Consider the following sets

$$I(x) = \{(l, k) : f(k, l) \geq x, l, k \geq 0\}$$

This is called the *input requirement set*. It might look like everything on and to the right of the line in the next graph

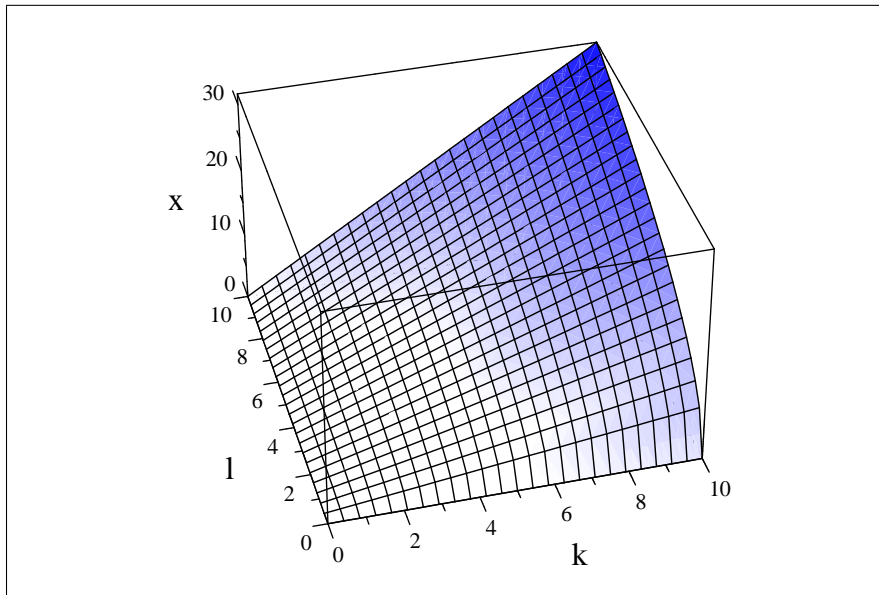


Contrast with

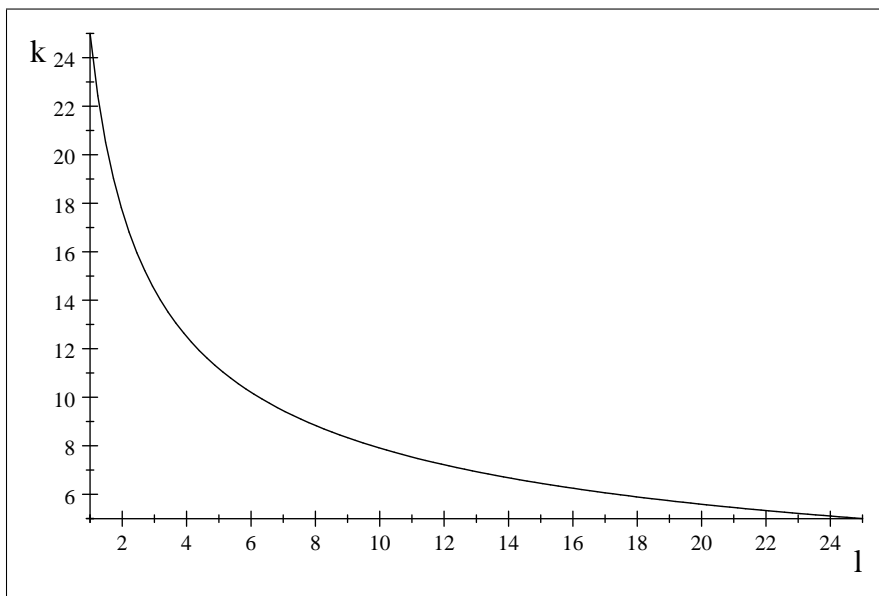
$$I_x(x) = \{(l, k) : f(k, l) = x, l, k \geq 0\}$$

This latter set is the *isoquant* for output level x . Define the isoquant in words. It is all those combinations of labor and capital that are just capable of producing x units of output

For example, if $x = f(k, l) = kl \cdot 5$



And the associated isoquant for $x = 25$ is



1.2 Imagine that we are having a quiz on deducing predictions from a set of definitions and assumptions

1.2.1

1.2.2 Definitions:

1. $y \equiv$ income
2. $p_i \equiv$ price of good i
3. $x_i \equiv$ quantity consumed of good i , $i = 1, 2$.
4. $x^m \equiv$ bundle m
5. $x^* \equiv$ is the chosen bundle. Note that x^* is a vector with two elements.

1.2.3 Assumptions:

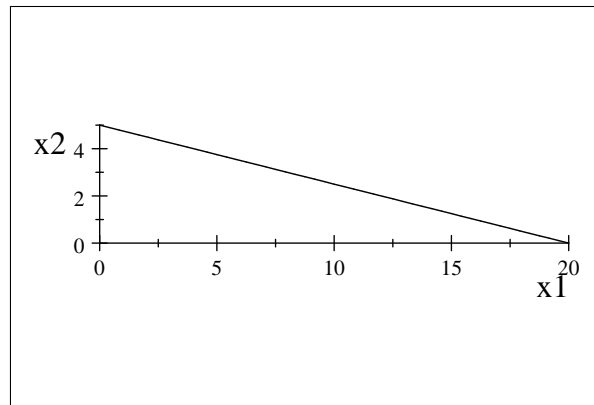
1. An individual is restricted to purchase a bundle that belongs to the set $S \equiv \{x_1, x_2 : p_1x_1 + p_2x_2 \leq y\}$. That is, the chosen bundle $x^* \in S$.
2. Assume the individual always prefers more to less.

What can we predict about x^* based on the definitions and these two assumptions? We deduce that

$$x^* \in B \equiv \{x_1, x_2 : p_1x_1 + p_2x_2 = y\}$$

That is, the individual will choose a bundle on the budget line. Be able to explain why this prediction follows from the definitions and assumptions.

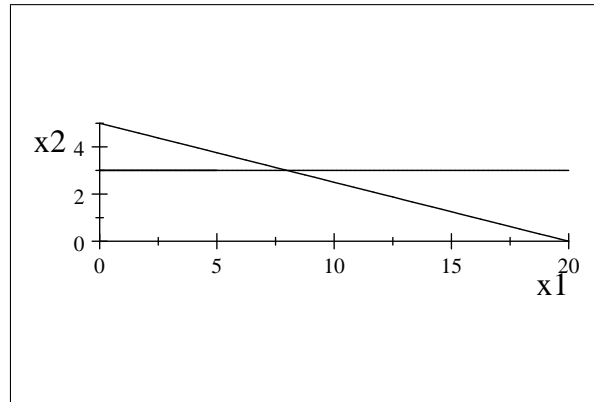
For example if $y = 100$, $p_1 = 5$ and $p_2 = 20$, $100 = 5x_1 + 20x_2$, which implies $x_2 = 5 - .25x_1$



Now add an additional assumption.

1. **The individual's mother makes him consume at least 3 units of x_2**

Can we now be more precise in our prediction about x^* ? Yes.



The individual will consume on the budget line and at least 3 units of x_2 .

Now add another assumption that makes the chosen bundle unique.

Make up a theory with definitions and assumptions, and then use set theory and Euler diagrams to derive at least one prediction from your theory.

1.3 The weakly preferred set

Before finish up with set theory, I want to introduce one more set, "the weakly preferred set which is fundamental to your understanding of consumer theory.

You have all read and studied the notes "consumer theory in a nutshell" and studied the notes about the utility function.

let $x^j \equiv (x_1^j, x_2^j)$ denote bundle j

Assume that an individual has preferences over bundles of goods. That is, an individual can rank bundles in the sense, for every two possible bundles, she either prefers bundle k to bundle m , prefers bundle m to bundle k , or is indifferent between these two bundles

Notationally

$x^k \succ x^m$ says bundle k is strictly preferred to bundle m

$x^k \prec x^m$ says bundle m is strictly preferred to bundle k

$x^k \succeq x^m$ says bundle k is weakly preferred to bundle m

$x^k \preceq x^m$ says bundle m is weakly preferred to bundle k

and

$x^k \sim x^m$ says the individual is indifferent to the two bundles

Imagine that I bring a 1000 bundles into the room. If you have preferences in the economic sense of the word, you can rank the 1000 bundles.

Now lets denote some specific bundle, $x^0 \equiv (x_1^0, x_2^0)$. Using set notation, identify the set of bundles that the individual ranks the same as this bundle. What do we call this bundle?

$$I(x^0) = \{x^j : x^j \sim x^0\}$$

This is the indifference curve associated with bundle x^0 . Draw it out on the board

What then is

$I_p(x^0) = \{x^j : x^j \succeq x^0\}$? Draw it out. It is called the "weakly preferred set".

What then is

$I_s(x^0) = \{x^j : x^j \succ x^0\}$? Draw it out. It is called the "strongly preferred set".