# How responsive are you?

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The more one reacts to an outside influence, the more *responsive* you are to that influence.

For example, if the addition of Sara Palin to the Republication presidential ticket caused you to switch from being a die-hard Democrat to stanch Republican, you were very responsive to that exogenous influence.

If you go from wanting nothing to do with Wanda but then marry her because one day she wore a blue dress, you are very responsive to blue dresses, particularly on Wanda.

Like to blue dresses, many people are responsive to price changes: when the price changes some respond by buying more or less of the product, those who do not buy more or less are unresponsive to the price change.

Most of us do not respond to most price changes. (For example, most of us would not change our behavior if the rental rates for commerical property in Paris increases by 1%.)

How much more or less likely one is to respond depends on how much the price changes, and how responsive one is to price changes.

# 1 How response are you to a change in the price of gasoline?

With recent dramatic rises and falls in the price of gasoline, the answer is important. For example, how much will a run-up in the price of gas decrease how many miles we drive, and/or what we drive.

If the price increase reduces demand a lot, much less gas will be burned and we won't pollute as much or have to import as much oil from countries that do not necessarily like us. But not so if a price increase has little effect on demand.

Before what's his name, the foreigner, got elected President, Hillary Clinton and John McCann both proposed a summer vacation from the Federal gas tax due to the high price of gasoline? What would that have done to the demand for gasoline? How much?

What factors influence the amount of gas one buys in a week

where one lives

where one works

type of car

number of kids and where they go to school

how much you enjoy driving

the availability of alternative forms of transportation (buses, trains, bikes, feet, etc.) and what they cost. Alternative forms of transportation are substitues.

level of physical fitness?

how you shop (number of trips, whether you buy on or off-line)

where your friends live

where your significant other lives. I know a number of couples who live thousands of miles from one another.

how much you like to ski, fish, party in Vegas

your income

etc.

changes in any of these will likely influence your demand for gas.

How could you reduce the amount of gas you consume?

list at least five things in your notes

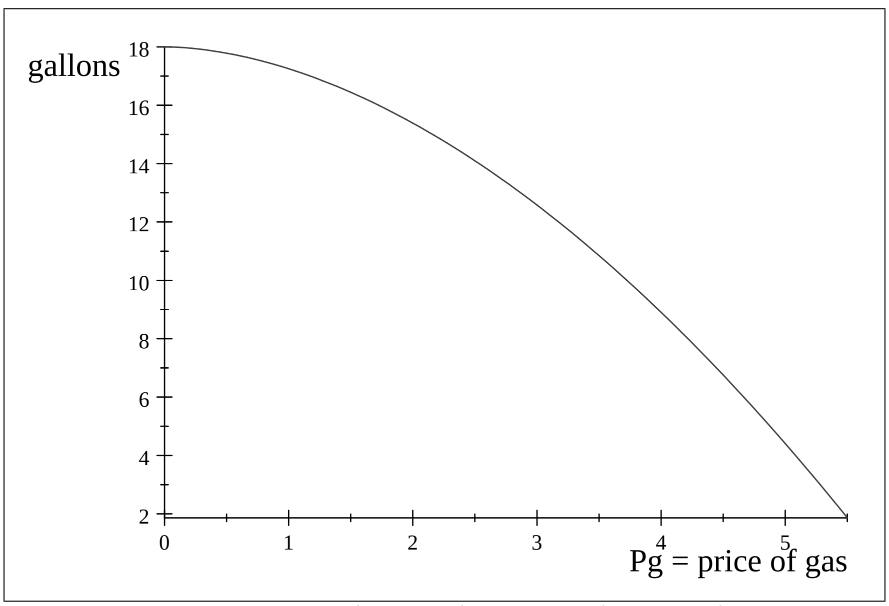
1. quit your job
2. dump the girlfriends in Kansas or boyfriend in Paris.
3. take the bus
4. buy a bike, and use it for trips that were previously made by car
5. shop online
6. lose weight so you can walk to work without dying
7. tune up the car
8. move closer to work

9. get a job closer to where you live

## 2 Consider Wilbur's per-week demand function for gasoline. He commutes 25 miles a day and likes to go to the mountains on weekends, a 100-mile round-trip. Assume his current car get 25 m.p.g.

His demand function might look as follows:

$G_D^W = 18 - .75p_G^{1.8}$ - at a zero price Wilbur would buy 18 gallons a week, and the quantity he buys decrease, at an increasing rate, at the price increases.




Wilbur's weekly demand function for gas as a function of the price

Why did I pick (makeup) a mathematical function with a graph like this?

I wanted it to reflect reasonable behavior given Wilbur's commuting constraints, Wilbur's preference for trips to the mountains, and the current price of gas.

With gas at \$4 a gallon, Wilbur buys only $18 - .75(4)^{1.8} = 8.9057$, enough to get him to work and one trip to the mountains, but no errands or other driving to fun places.

What will happen if the price rises to \$5 gallon? Wilbur buys only $18 - .75(5)^{1.8} = 4.4104$, enough only to drive to work four days a week (will need to take the bus the other day) and no trips to the mountains.

Alternatively, if we drill for oil everywhere and the price drops to \$1 gallon, unlikely, Wilbur goes wild and buys $18 - .75(1)^{1.8} = 17.25$ gallons a week, enough to drive wherever he wants to go.

I chose a mathematical function that got steeper and as the price of gasoline rises.

Note that my chosen demand function implies that Wilbur will be taking the bus or walking to work when the price is around $6/*gallon*.

## 2.1 So, given his demand function for gasoline, how *responsive* is Wilbur to an increase in the price of gasoline?

It depends on the current price and on how one measures responsiveness.

We are interested in how much Wilbur's demand for gasoline will change in the price increases by, for example, $1.

### 2.1.1 The slope/steepness of his demand function:

One possible measure of responsiveness is $\frac{\triangle G_D^W}{\triangle p_G}$: the change in Wilbur's quantity demand divided by the change in price.

Steepness as a measure of responsiveness.
Would you expect this measure of responsiveness to be positive or negative?

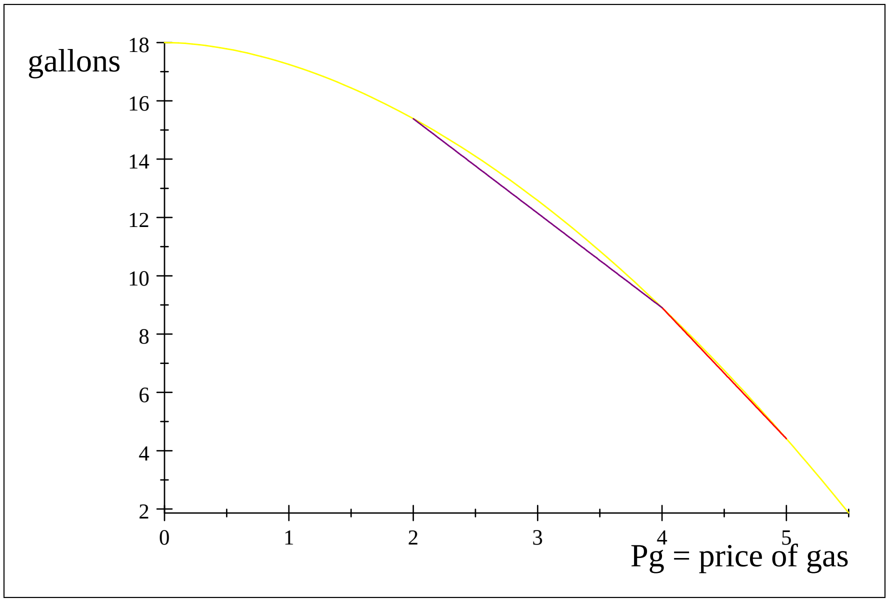
For example if the price increases from \$4 to \$5, Wilbur's demand for gasoline drops from $8.9057$ gallons to $4.4104$, a drop of $4.4953$ gallons.

In this case, $\frac{\triangle G_D^W}{\triangle p_G} = \frac{4.4104-8.9057}{5-4} = -4.4953$ gallons.

And if the price rises from \$2 to \$4 demand would increase decreases from $15.388$ gallons to $8.9057$ gallons.

In this case, $\frac{\triangle G_D^W}{\triangle p_G} = \frac{8.9057-15.388}{4-2} = -3.2412$ gallons

Note that $\frac{\triangle G_D^W}{\triangle p_G}$ is the slope of the Wilbur's demand function between the lower and higher price, it is a negative number because the demand curve slopes down.




Wilbur's weekly demand function for gas as a function of the price

The steepness of the red line indicates the slope of Wilbur's demand function between \$4 and \$5, and the purple line between \$2 and \$4.

Note that the slope is getting steeper as the price increases: it becomes a larger negative number.

### 2.1.2 But maybe $\frac{\triangle G_D^W}{\triangle p_G}$ is not the best measure of price responsiveness

Consider Italy and assume gasoline is priced in Euros and sold by the liter, so the price is Euros per liter.

Consider our earlier example where the price per gallon increases from \$4 to \$5, Wilbur's demand for gasoline drops from 8.9057 gallons to 4.4104, a drop of 4.4953 of gallons. In this case, $\frac{\triangle G_D^W}{\triangle p_G}$ was $\frac{4.4104-8.9057}{5-4} = -4.4953$ gallons.

What does one get if one expresses the exact same change in Euros and liters. For simplicity assume 4 liters per gallon and \$1.50 per Euro. The change in quantity is now $-4.4953(4) = -17.981$ liters, and the price change is $\frac{5}{1.5} - \frac{4}{1.5} = 0.66667$ Euro.

In which case, for the exact same change $\frac{\triangle G_D^W}{\triangle p_G} = \frac{-17.981}{0.66667} = -26.971$, which is a different number than one things were measured in \$ and gallons.

What is going on?

$\frac{\triangle G_D^W}{\triangle p_G}$ changes every time one changes the units in which quantity or price are measured. This is not a good thing.

If we measured gas in quarts rather than gallons the slope would quadruple in magnitude, though nothing has changed in terms of how Wilbur reacts to a price change.

A better measure of responsiveness would not do this.

## 2.2 Units will not matter if we measure responsiveness in terms of $\frac{\%\triangle G_D^W}{\%\triangle p_G}$: the percentage change in quantity demand divided by the percentage change in price.

Typically we ask how much quantity demanded will change in percentage terms when price increases by one percent.

Let's see if we can figure this out for Wilbur?

A price change from \$4 to \$5 is, in percentage terms, is $\frac{5-4}{(5+4)/2}(100) = 22.222\%$: the price change in dollars divided by the average of the two prices, multiplied by 100.

In general terms, the percentage change in the price is

$$\frac{p_G^1 - p_G^0}{(p_G^1 + p_G^0)/2}(100)$$

where $p_G^1$ is new price and $p_G^0$ the initial price

A quantity change from $8.9057$ gallons to $4.4104$ gallons is, in percentage terms, is $\frac{4.4104-8.9057}{(4.4104+8.9057)/2}(100) = -67.517\%$

In general terms, the percentage change in the quantity is

$$\frac{G_D^W(p_G^1) - G_D^W(p_G^0)}{(G_D^W(p_G^1) + G_D^W(p_G^0))/2}(100)$$

where $G_D^W(p_G^1)$ is Wilbur's demand for gasoline at the new price and $G_D^W(p_G^0)$ is his demand at the initial price.

### 2.2.1 so putting the two together

If a price rise from \$4 to \$5 causes Wilbur's demand for gasoline to drop from

8.9057 gallons to 4.4104 gallons

$$\frac{\% \triangle G_D^W}{\% \triangle p_G} = \frac{-67.517}{22.222} = -3.0383$$

This can be read as a one percent increase in the price of gasoline, from its current level, causes Wilbur's demand for gasoline to decrease 3.03%.

We would have gotten the same answer if things were expressed in Euro and liters, or dollars and quarts. Do the calculation, and see.

#### 2.2.2 The general formula is

$$\frac{\% \triangle G_D^W}{\% \triangle p_G} = \frac{\frac{G_D^W(p_G^1) - G_D^W(p_G^0)}{(G_D^W(p_G^1) + G_D^W(p_G^0))/2}}{\frac{p_G^1 - p_G^0}{(p_G^1 + p_G^0)/2}}$$

This measure of responsiveness is call an *elasticity*; in particular *a price elasticity of demand*: how responsive one is to a change in demand.

The word after the words *elasticity of* (in this case *demand*) is what is changing in response to the word before *elasticity* (in this case *price*).[1]

I am not sure why the word *elasticity*. Looking up the word elasticity, the dictionary says, "The condition or property of being elastic; flexibility." Looking up elastic one gets, "capable of being easily stretched or expanded and resuming former shape : flexible <an elastic bandage>, and capable of ready change or easy expansion or contraction : not rigid or constricted <an elastic concept>

[1] Price elasticity of demand is different from demand elasticity of price.

### 2.2.3 Let's use the general formula for price elasticity of demand to calculate Wilbur's price elasticity of demand for an increase in the price of gasoline from $0 to $1

$$\begin{aligned}
\frac{\% \triangle G_D^W}{\% \triangle p_G} &= \frac{\frac{G_D^W(p_G^1)-G_D^W(p_G^0)}{(G_D^W(p_G^1)+G_D^W(p_G^0))/2}}{\frac{p_G^1-p_G^0}{(p_G^1+p_G^0)/2}} \\
&= \frac{\frac{G_D^W(1)-G_D^W(0)}{(G_D^W(1)+G_D^W(0))/2}}{\frac{1-0}{(1+0)/2}} \\
&= \frac{\frac{G_D^W(1)-G_D^W(0)}{(G_D^W(1)+G_D^W(0))/2}}{2}
\end{aligned}$$

But Wilbur's demand for gas when the price is zero is
$G_D^W(0) = 18 - .75(0)^{1.8} = 18.0$ gallons

And his demand when the price is $1 is
$G_D^W(1) = 18 - .75(1)^{1.8} = 17.25$ gallons

So the price elasticity in this area Wilbur's demand function (a price increase from zero to $1) is

$$\begin{aligned}
\frac{\% \triangle G_D^W}{\% \triangle p_G} &= \frac{\frac{G_D^W(1)-G_D^W(0)}{(G_D^W(1)+G_D^W(0))/2}}{2} \\
& \frac{\frac{17.25-18}{(17.25+18)/2}}{2} \\
&= -.02
\end{aligned}$$

In this area of the demand function, a price increase of one percents leads to only a .02% decrease in the amount of gas Wilbur buys, not much of a decrease.

If you calculate the elasticity at higher prices, you will see that Wilbur's price elasticity of demand for gas increases in absolute value as the price of gas increases.

### 2.2.4 Price elasticities of demand below $1\%$ are considered low, unresponsive, and are referred to as *inelastic*.

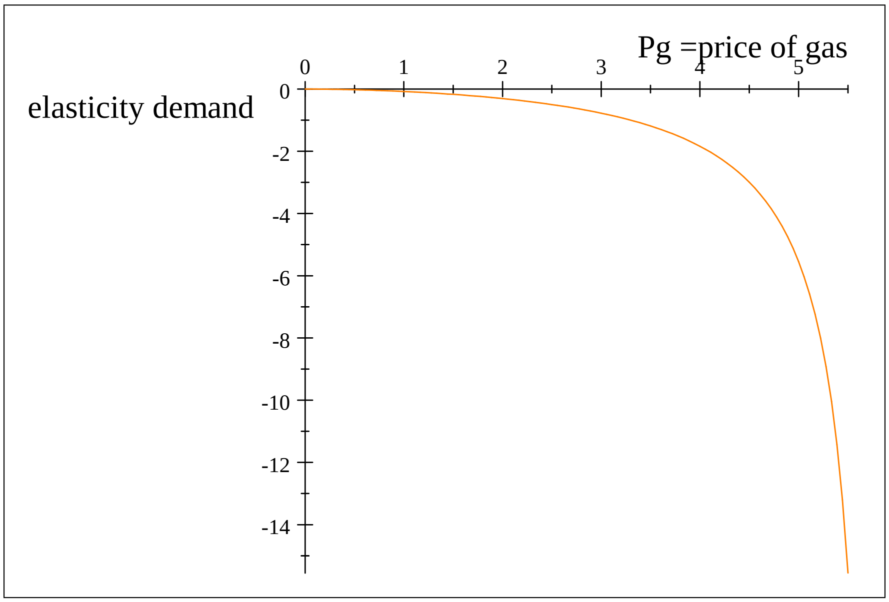
So, Wilbur demand for gasoline is very unresponsive to the price when the price is very low: his demand is inelastic.

Price elasticities of demand above $1\%$, in absolute terms, are considered responsive, and referred to as *elastic*.

So, Wilbur's demand for gasoline is very responsive to the price when the price is currently \$4 gallon: his demand is elastic. $-3.03\%$ is very elastic.

A price elasticity of $-1\%$ is called *unitary elastic*.

I am going to try and graph the Wilbur's price elasticity of demand for gasoline as a function of the price of gasoline




Wilbur's price elasticity of demand for gasoline

The elasticities describe what we saw and are seeing in the gasoline market.

When the of price of gas is low, we do not respond much to a 1% increase in its price, but with much higher prices, we are more responsive to further price hikes.

My curve predicts, maybe incorrectly, drastic drops in Wilbur's demand if gas keeps going up in price.

## 3 Would you expect the price elasticity of demand for gasoline to increase or decrease as the customer has more time to react?

The more time you give Wilbur to respond to an increase in the price of gas, the more he will respond.

Wilbur, woke up this morning and gas is a buck higher than yesterday. He and the kids need to be to work in a hour. What can he do? Not much. No time to move, switch schools or change jobs. No time to buy bikes and force the kids to bike to school. Wilbur and the family are committed to a trip to Vail in two days with the inlaws, and there is no time to bail.

However, the more time you give him the more he will adjust to the new price (higher or lower).

The elasticity of demand for a product typically depends on how much time consumers have to react. Wilbur's demand function for gas per week will likely change as one looks more weeks into the future.

## 4 We started our discussion of responsiveness by assuming Wilbur's demand for gasoline depended only on the price of gasoline.

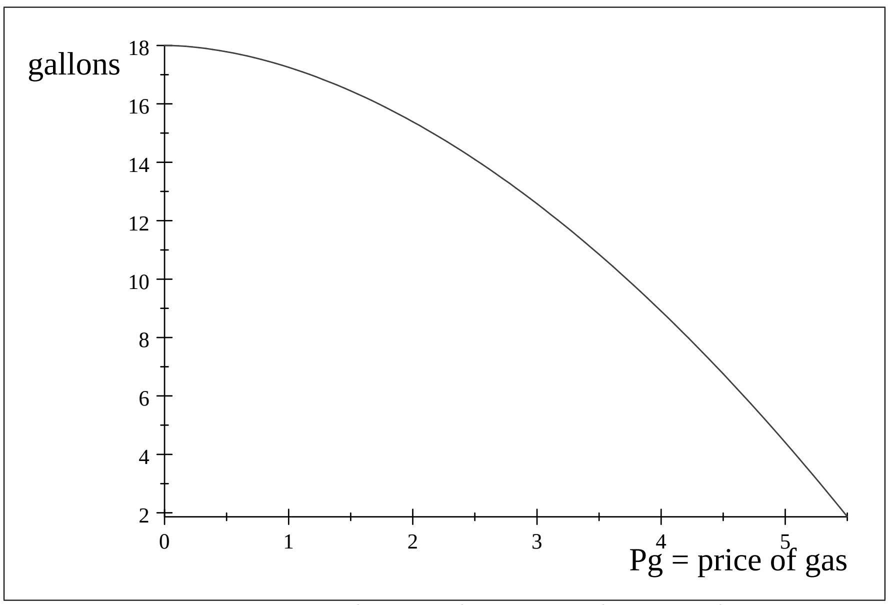
In terms of an equation

$$G_D^w = g(p_G)$$

Read this as how much gas Wilbur will buy per week is a function of (determined by) the price of gasoline. (make sure you understand functional notation.)

We further assume that $\frac{\triangle G_D^W}{\triangle p_G} < 0$, his demand function is downward sloping

In terms of a graph




Wilbur's weekly demand function for gas as a function of the price

## 4.1 But is price of gas the only thing that determines how much gas Wilbur buys?

## 4.2

**NOT**

It depends on a lot of things: his income, bus fare, how far he lives from work, etc.

Let' take the first one into account and make Wilbur's demand for gasoline a function of both the price of gas and Wilbur's income, $y_W$.

His demand function is then $G_D^w = g(y_W, p_G)$

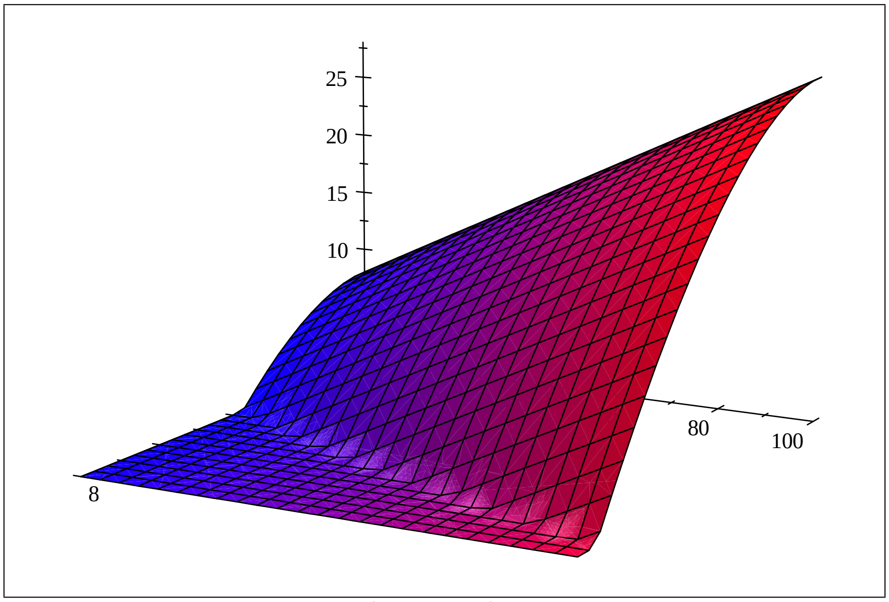
What would you surmise about the relationship between Wilbur's income and his gas consumption?

I don't know for sure but I am going to assume that $\frac{\triangle G_D^W}{\triangle y} > 0$, his demand function is upward sloping wrt to income.

## 4.3 I am going to try and produce a graph where Wilbur's demand for gas is decreasing in $p_G$ and increasing in $y_W$.

Specifically, I will assume $G_D^w = g(y, p_G) = (8 + .2y_W) - .75p_G^{1.8}$ .

where $y_W$ is Wilbur's income, measured in thousands
Look at this function and discuss its properties.




Wilbur's demand as a function of his income and the price

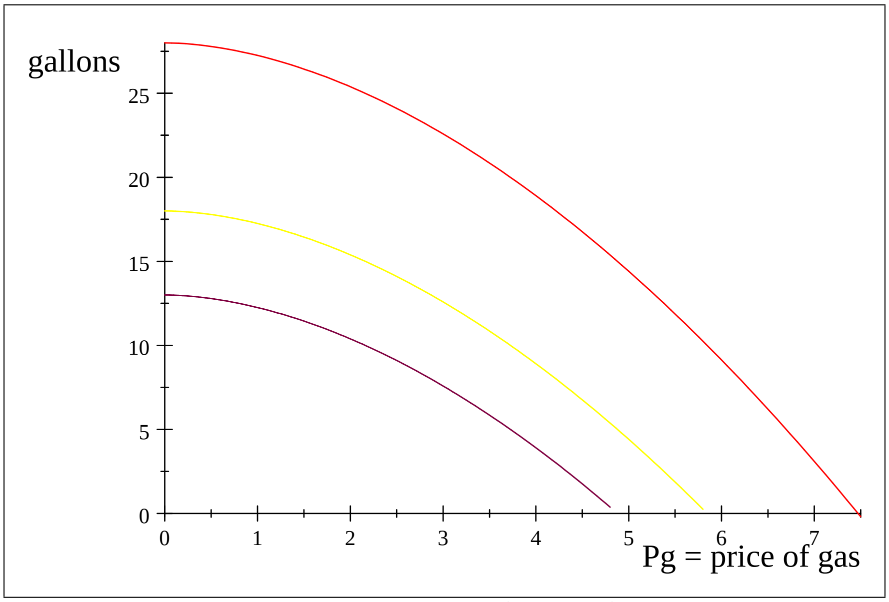
Note how, for a given income, demand falls as price increases, but as income increases, holding price constant, demand increases

Holding the price of gas constant, every time Wilbur's income goes up by $\$10K$ his consumption of gas increases by two gallons.

Represent the function in the room.

My earlier demand function for Wilbur (price only) was $18 - .75p^{1.8}$, which is $(8 + .2y_W) - .75p_G^{1.8}$ assuming Wilbur's income was $\$50K$.

In explanation, $(8 + .2(50)) - .75p_G^{1.8} = 18.0 - 0.75p_G^{1.8}$

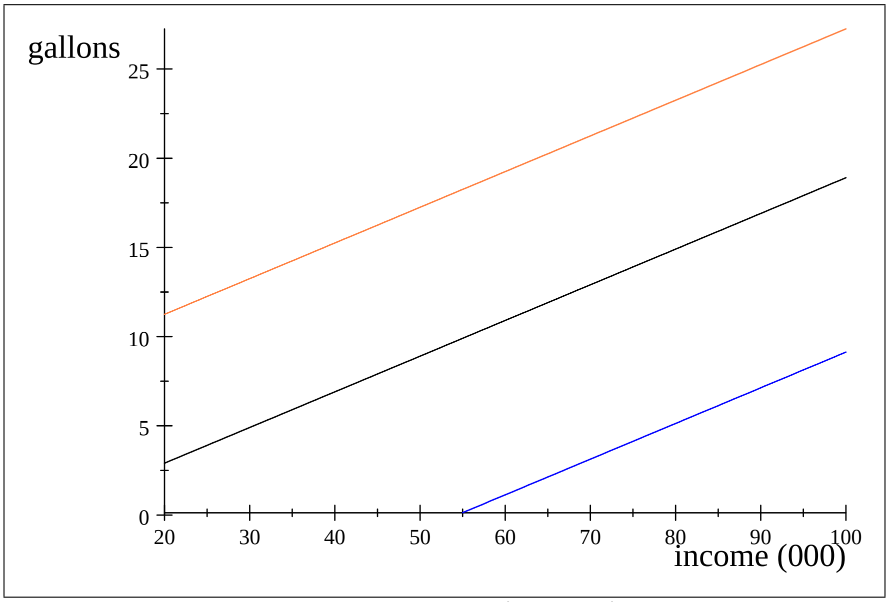



Wilbur's weekly demand function for gas

Yellow is when his income is $\$50K$, purple is when his income is $\$25K$ and red is when his income is $\$100K$.

Represent these functions in the room as slices: price on on horizontal axis, income on the other, and demand for gas on the vertical axis.

Now let's graph Wilbur's demand for gas as a function of his income holding the price of gas constant




Wilbur's weekly demand function for gas

Black is for $p_G = \$4$, orange is for $p_G = \$1$ and blue is for $p_G = \$6$

Note that if $p_G = \$6$ Wilbur would not buy any gas if his income was less than $\$55K$ a year.

### 4.4 A question for you

Assume the $p_G = \$4$

What is Wilbur's **income** elasticity of demand for gasoline?
Define it in words, and in terms of an equation

Figure out, and interpret, the specific number if his income increases from $\$50K$ to $\$55K$.

When $p_G = \$4$ and Wilbur's income is $\$50K$, he buys

$G_D^w = g(50, 4) = (8 + .2(50)) - .75(4)^{1.8} = 8.9057$ gallons

When $p_G = \$4$ and Wilbur's income is $\$55K$, he buys
$G_D^w = g(55, 4) = (8 + .2(55)) - .75(4)^{1.8} = 9.9057$ gallons.

So, increasing his income from $\$50K$ to $\$55K$ cause him to increase his gasoline consumption by $9.9057 - 8.9057 = 1.0$ gallon.

## 4.5 How would you define the income elasticity of demand for gasoline?

I might define it as the percentage change in the demand for gasoline divided by the percentage change in income, holding the price of gas constant.

$$\frac{\% \triangle G_D^W}{\% \triangle y_W} = \frac{\frac{G_D^W(y_W^1)-G_D^W(y_W^0)}{(G_D^W(y_W^1)+G_D^W(y_W^0))/2}}{\frac{y_W^1-y_W^0}{(y_W^1+y_W^0)/2}}$$

For $p_G = \$4$ and an increase in Wilbur's income for $\$50K$ to $\$55K$ it is

$$\begin{aligned}\frac{\% \triangle G_D^W}{\% \triangle y_W} &= \frac{\frac{G_D^W(y_W^1)-G_D^W(y_W^0)}{(G_D^W(y_W^1)+G_D^W(y_W^0))/2}}{\frac{y_W^1-y_W^0}{(y_W^1+y_W^0)/2}} \\ &= \frac{\frac{9.9057-8.9057}{(9.9057+8.9057)/2}}{\frac{55-50}{(55+50)/2}} \\ &= 1.11\%\end{aligned}$$

In words, if $p_G = \$4$ a 1% increase in income causes Wilbur to increase his consumption of gas by 1.11%, elastic, but not very elastic.

Would his income elasticity of demand by the same if the price was, instead, 1$?

No, Figure it out.