

1 Second Welfare Theorem

Proposition 1 Consider an Economy $(\{X_i, \succeq_i\}, Y_j, \bar{\omega})$. Assume Y_j is convex, \succeq_i is convex and LNS. Then for every POA (x^*, y^*) there is a price vector p non zero such that (x^*, y^*, p) is a quasiequilibrium with transfers, i. e., there is an assignment of wealth levels $w : \sum_i w_i = p\bar{\omega} + \sum py_j^*$ such that

1. Every firm maximizes profits
2. For every consumer i if $x_i \succ_i x_i^*$ then $px_i \geq w_i$.
3. Markets clear.

Proof. (The very best) $V_i = \{x_i : x_i \succ_i x_i^*\}$, $V = \sum_i V_i$
 (The feasible) $Y = \sum Y_j$, $Y + \{\bar{\omega}\}$ ■

1. V is convex
2. $Y + \{\bar{\omega}\}$ is convex
3. Their intersection is empty
4. There is a p and $r : pz \geq r$ for any z in V and $pz \leq r$ in any z in $Y + \{\bar{\omega}\}$.
5. If z is in the closure of V then $pz \geq r$
6. If z is on both boundaries, $pz = r$
7. Firms maximize profits, as for any $z = \bar{\omega} + y_j + \sum_{k \neq j} y_k^* \in Y + \{\bar{\omega}\}$

$$pz \leq r = p \left(\bar{\omega} + \sum_j y_j^* \right)$$

8. Consumers max: if $x_i \succ_i x_i^*$ then $px_i \geq px_i^*$
9. Wealth levels are

$$w_i = px_i^*.$$