ECON 7050
HW1
Due Wed, Sept 17

September 9, 2008

Problem 1 *Consider the Edgeworth box economy discussed in class. There are 10 pheasants and 10 buckets of apples, Adam and Eve have preferences over their own consumption bundles* $(x_{1i}, x_{2i}) \in \mathbb{R}^2_+$.

Characterize the set of Pareto efficient allocations (PEA) for the economy, where preferences are represented by

1. $u^i(x_{1i}, x_{2i}) = \sqrt{x_{1i}x_{2i}},\ i \in \{A, E\}$.
2. $u^A(x_{1A}, x_{2A}) = x_{1A}^{2/3}x_{2A}^{1/3}$ *and* $u^E(x_{1E}, x_{2E}) = x_{1E}^{1/3}x_{2E}^{2/3}$

   Hint: use the maximization problem of a fictitious benevolent planner: maximize utility of one agent keeping the utility of the other at least as high as some fixed number $\bar{u}$ *and subject to the resource (feasibility) constraint.*

   Depict this set in an Edgeworth box for each of the cases.

   Can you use the above algorithm for the following specifications of preferences:

3. $u^E(x_{1E}, x_{2E}) = ax_{1E} + x_{2E},\ u^A(x_{1A}, x_{2A}) = bx_{1A} + x_{2A}$ *with* $a, b > 0$
4. $u^E(x_{1E}, x_{2E}) = \min\{ax_{1E}, x_{2E}\},\ u^A(x_{1A}, x_{2A}) = \min\{bx_{1A}, x_{2A}\}$ *with* $a, b > 0$

If yes, then use to derive the set of PEA, if not, suggest a different way and use it.

Problem 2 *Characterize (in terms of initial endowments) competitive equilibria, if exist, of the Edgeworth box economy where Adam initially has* $\omega_{1A} > 0$ *pheasants (and no apples), and Eve has* $\omega_{2E} > 0$ *apples (and no pheasants) and where*

1. $u^i (x_{1i}, x_{2i}) = \sqrt{x_{1i} x_{2i}}$, $i \in \{A, E\}$
2. $u^A (x_{1A}, x_{2A}) = x_{1A}$, $u^E (x_{1E}, x_{2E}) = x_{1E} x_{2E}$.

Problem 3 *Write down a statement and a complete proof of the First welfare theorem for an exchange economy with I agents and L goods. Make sure that you indicate how each of the assumptions you make is used.*

Problem 4 *Find a mistake in the following ('fake') proof the statement that "a Walrasian equilibrium exists in any pure exchange economy in which* $\sum_i \omega_i >> 0$ *and every consumer has continuous, strictly convex, and strongly monotone preferences".*

"Proof": use propositions 17.B.2 and 17.C.2.