Econ 7050
A model of the Final Exam

December 6, 2005

Problem 1 *Assume two individuals i and j are risk averse and their preferences over lotteries can be represented by a von Neumann—Morgenstern expected utility function. Pick a monetary prize x.*

1. *Show that it is possible to normalize Bernoulli function of individual j such that at point x the Bernoulli functions of both individuals are tangent:*

$$u_i'(x) = u_j'(x) \tag{1}$$
$$u_i(x) = u_j(x). \tag{2}$$

Hint: start with two arbitrary concave increasing real-valued functions, $v_j(y)$ and $u_i(y)$, and find an affine transformation of $v_j(y)$ into $u_j(y)$ that satisfied conditions $(1,\ 2)$.

Solution 2 *Assume $v_j'(x)$ is distinct from zero. Let real numbers a and b satisfy:*

$$\begin{aligned} av_j'(x) &= u_i'(x) \\ av_j(x)+b &= u_i(x), \end{aligned}$$

so if

$$\begin{aligned} a &= \frac{u_i'(x)}{v_j'(x)}, \\ b &= u_i(x) - \frac{u_i'(x)}{v_j'(x)} v_j(x) \end{aligned}$$

we have $u_j(y) = av_j(y) + b$ *with the desired properties. Bernoulli function is defined up to the affine transformation, therefore expected utility of* u_j *represents the same preferences over lotteries as* v_j.

2. *Assume also that* i *is more risk averse than* j. *Show that after the normalization* $u_i(y) \leq u_j(y)$ *for all* $y \in \mathbb{R}$.

   Solution 3 *As agent* i *is more risk averse than* j *there should be a concave function strictly increasing* f *such that* $f(u_j(y)) = u_i(y)$. *By normalization,*

$$\begin{aligned} f(u_j(x)) &= u_i(x) = u_j(x) \\ f'(u_j(x))\, u_j'(x) &= u_i'(x) = u_j'(x), \end{aligned} \tag{3}$$

   so $f'(u_j(x)) = 1$. *Therefore, at that point* f *is tangent to the 'diagonal' or identity function. As* f *is concave, it has to be below that tangent line, so for any* z, $f(z) \leq z$. *But this also implies*

$$u_i(x) = f(u_j(x)) \leq u_j(x)$$

   Done.

Problem 4 *19.D.2, p.726 MWG*

Solution 5 *2 goods, 2 states, 2 people. 4 A-D contingent commodities.*
$\omega_1 = (g, b, 0, 0)\,;\ \omega_2 = (0, 0, g, b)$
Find an AD equilibrium.

$$\begin{aligned} &\max_{x_i \in \mathbb{R}^4} \frac{1}{2}\left(u(x_{1i}) + u(x_{2i})\right) \\ s.t.px_i &= p\omega_i \end{aligned}$$

$\nabla u(x_{1si}, x_{2si}) = 2\lambda_i (p_{1s}, p_{2s})$
$px_i = p\omega_i \Rightarrow$
$p_{11}x_{111} + p_{21}x_{211} + p_{12}x_{121} + p_{22}x_{221} = p_{11}g + p_{21}b$
$p_{11}x_{112} + p_{21}x_{212} + p_{12}x_{122} + p_{22}x_{222} = p_{12}g + p_{22}b$
$x_{111} + x_{112} = g = x_{121} + x_{122}$
$x_{211} + x_{212} = b = x_{221} + x_{222}$

12 equations, 13 unknowns (including $\lambda :$ should be the same for both).

$$\frac{u_1(x_{11}, x_{21})}{u_2(x_{11}, x_{21})} = \frac{p_{11}}{p_{21}}$$
$$\cdots$$

Guess: if prices are all the same across states: $\hat{p}_1, \hat{p}_2$ then

$$\begin{aligned} x_{111} &= x_{112} = g/2 = x_{121} = x_{122} \\ x_{211} &= x_{212} = b/2 = x_{221} = x_{222} \end{aligned}$$

satisfies optimization conditions, and the demand is such that market clears. So, it should be that $\nabla u(g/2, b/2) = 2\lambda(\hat{p}_1, \hat{p}_2)$. Normalize u such that $\lambda = 1$.

Find Radner equlibrium. Keep the same spot prices. Find the prices for the first period contingent trade (in the first commodity): q_1, q_2. Following the construction in the proof,

$$q_1 = p_{11}; q_2 = p_{12}.$$

so, both are $\hat{p}_1$. Then to get the equivalence of the budget constraints it is left to calculate money holdings in the first period.

$$\begin{aligned} z_{si} &= \frac{\hat{p}_1(x_{1si} - \omega_{1si}) + \hat{p}_2(x_{2si} - \omega_{2si})}{\hat{p}_1} \\ z_{11} &= -g/2 - \frac{\hat{p}_2}{\hat{p}_1} b/2 \text{ (a promise to deliver)} \\ z_{21} &= g/2 + \frac{\hat{p}_2}{\hat{p}_1} b/2 \text{ (entitlement to get)} \end{aligned}$$

Problem 6 *Consider the monopolistic screening problem with a continuum of types from chapter 2 of BD. Solve for the full information allocation rule (for a monopolist who can perfectly observe the type of a buyer). Show that for all the types but the highest imposing the full information solution (the first best) will result in violating some incentive constraints.*

Solution 7 *In the full information case, the maximization problem of the monopolist is constrained only by the IR, which binds for all types: $\theta v(q) - T(\theta) = 0$ and the problem*

$$\max_{T,q} T(\theta) - cq(\theta)$$

then becomes

$$\max_q \theta v(q(\theta)) - cq(\theta)$$

so it has to satisfy the "first best"

$$\theta v'(q^*(\theta)) = c,$$

and the monopolist gets all the willingness to pay (consumer surplus), $\theta v(q^*(\theta)) = T^*(\theta)$. *Observe that* q^* *is increasing in* θ.

To show that this is not IC, let us say that for some type $\theta_0 < \bar{\theta}$ *this condition is satisfied. Consider type* $\theta_0 + \varepsilon$, $\varepsilon > 0$. *If this individual pick "her contract" then the payoff is zero,*

$$(\theta_0 + \varepsilon) v(q^*(\theta_0 + \varepsilon)) = T^*(\theta_0 + \varepsilon).$$

Can she do better? Assume she pretends to be θ_0. *Her utility then is*

$$(\theta_0 + \varepsilon) v(q^*(\theta_0)) - T^*(\theta_0) = \varepsilon v(q^*(\theta_0)) > 0.$$