

Econ 7050 Fall 2007 Final Exam

Please, state clearly any additional assumptions you need.
Have fun!

Problem 1 (30%) *Assume an individual whose preferences over lotteries admit von Neumann-Morgenstern representation is risk averse and has a quadratic utility function over monetary prizes, $x \in \mathbb{R}$:*

$$u(x) = ax^2 + bx$$

Let F be a (cumulative) distribution with support $[0, \frac{-b}{2a}]$.

- 1. Show that for any lottery F the von Neumann Morgenstern utility $U(F)$ of this individual depends only on the mean, μ_F , and the variance, σ_F^2 , of F . Impose the necessary conditions on parameters, $a, b \in \mathbb{R}$.*
- 2. Assume that a financial market in ‘Beta-land’ offers securities (lotteries) at a fixed price, and that for any F*

$$\mu_F = \beta \sigma_F^2$$

for some strictly positive $\beta > 0$, so that securities with higher mean have also higher variance, and the ratio, β , of the mean to the variance is constant. Analyst Joe says individuals with quadratic utility should be indifferent between all the securities offered on the market. Prove him wrong.

Problem 2 (30%) *Two pessimists trade in Cocoa land. They sign contracts in period 0 and consume in period 1, when only one of them gets good crops, so period one endowments are $\omega_1 = (1, 0)$ and $\omega_2 = (0, 1)$, and each believes (stubbornly so) that the other will be lucky with probability $3/4$: $\pi_{11} = 1/4$, $\pi_{12} = 3/4$. Preferences over lotteries of each admit von Neumann-Morgenstern representation, both are risk averse with the same utility over consumption of cocoa beans, $u(x) = \sqrt{x}$, $x \in \mathbb{R}_+$.*

1. Formulate conditions for the Arrow-Debreu equilibrium for this economy.
2. Is there an equilibrium in which a pessimist consumes the same amount of cocoa beans in each state?
3. Calculate Arrow-Debreu equilibria.
4. How would you interpret your findings in the view of the empirical observation that, comparing across countries, belief that (pure) luck determines income is positively correlated with the social spending (as a % of GDP).¹

Problem 3 (40%) *An on-line bookstore "Inkheart" specializing in old rare books is one-of-a-kind (a monopoly). It sells to customers who differ by their desire to pay for older books, θ . The owner of the store knows that a buyer's utility over is quasilinear: $\theta\sqrt{q} - T$, where q is the age of the book and T is the payment. The seller's cost is $C(q) = q^2$.*

1. *Assume the seller can perfectly determine the type of the buyer (from the purchase history of a naive customer). What 'deals' (q, T) will she offer to each customer? What is the lowest type of the customer, θ , that will be served?*
2. *Assume that now there is a new regulation requiring all on-line stores to post all the deals on-line. Customers also found ways to delete their purchase histories. The seller (based on her previous experience) believes the realized value of θ for a customer is an independent draw from a uniform distribution with support $[0, 10]$.*
 - (a) *What deals (q, T) will be posted by the seller now?*
 - (b) *What is the lowest type of the customer, θ , that will be served?*
 - (c) *Will all the customers support the new regulation?*

¹See, for example, Roland Bénabou and Jean Tirole, "Belief in a Just World and Redistributive Politics," NBER, W11208.