Econ 4211
Answers to some review questions

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1. Consider a market with the following supply and demand:

$$Q_d = 10 - P_c$$
$$Q_s = \frac{P_p - 2}{3}$$

   (a) Calculate the initial equilibrium in this market.
   $10 - P = \frac{P-2}{3}$, Solution is: $P = 8$, $Q = 2$.

   (b) Suppose a unit tax of one dollar is imposed on consumers. Calculate the equilibrium quantity and the price paid by consumers and price received by producers.
   $10 - (P_p + 1) = \frac{P_p - 2}{3}$, Solution is: $P_p = \frac{29}{4}$, $P_c = \frac{33}{4}, Q = 10 - \frac{33}{4} = \frac{7}{4}$

   (c) Will your answer to the previous question change if the unit tax is levied on producers?
   No: Producer's price is his marginal cost of production:
   $3Q_s + 2 = P_p = MC = P_c - 1$
   So in the new equilibrium $10 - P_c = \frac{P_c - 1 - 2}{3}$, Solution is: $P_c = \frac{33}{4}, P_p = \frac{29}{4}, Q = 10 - \frac{33}{4} = \frac{7}{4}$.

   (d) Who bears higher burden of the tax? Why? Demonstrate your answer using (calculated) elasticity of supply and demand.
   Consumer's price rises by $\frac{1}{4}$, producer's price falls by $\frac{3}{4}$, indicating higher "burden".
   Indeed, elasticity of demand is $Q'_d(P)\frac{P}{Q} = -1 * \frac{8}{2} = -4$ which is higher in absolute value than the supply elasticity, $Q'_d(P)\frac{P}{Q} = \frac{1}{3} * \frac{8}{2} = \frac{4}{3}$, so demand is relatively more elastic, therefore more tax is 'shifted' onto the producer.

2. The demand for snorkels in Bergama is given by $Q_s = 500 - 8P_s$, the supply is perfectly elastic, with marginal cost of production 10. The demand for kayaks is $Q_k = 650 - 6P_k$, their supply is perfectly elastic, with

marginal cost of production 15. Both goods are currently untaxed but the government needs to raise \$500 in tax revenues. What tax should be levied on each of the two goods assuming no income effects in both markets.

By Ramsey rule,

$\frac{t_k}{t_s} = \frac{\varepsilon_s^D}{\varepsilon_k^D}$

Also the revenue requirement is

$500 = t_k P_k Q_k(t_k) + t_s P_s Q_s(t_s)$, where

$P_k = 15$, $P_s = 10$ (corresponding marginal cost)

$Q_k(t_k) = 650 - 6P_k(1+t_k) = 650 - 6(15)(1+t_k) = 560 - 90t_k$

$Q_s(t_s) = 500 - 8P_s(1+t_s) = 500 - 8(10)(1+t_s) = 420 - 80t_s$

$\varepsilon_s^D = -8 * \frac{Q_S^*}{P_S^*} = -8 * \frac{500-8*10}{10} = -336$

$\varepsilon_k^D = -6 * \frac{Q_k^*}{P_k^*} = -6 * \frac{650-6*15}{15} = -224$

To summarize, the two equations determining the tax rates are

$\frac{t_k}{t_s} = \frac{-336}{-224} = \frac{3}{2}$;

$500 = 15t_k(560 - 90t_k) + 10t_s(420 - 80t_s)$

From the first condition $t_k = \frac{3}{2}t_s$, use that in the second condition to get $500 = 15 * \frac{3}{2}t_s\left(560 - 90 * \frac{3}{2}t_s\right) + 10t_s(420 - 80t_s)$. There are two solutions, first one is not between zero and one: $t_s = \frac{672}{307} + \frac{2}{307}\sqrt{109\,826} = 4.347\,9$, second one is: $t_s = \frac{672}{307} - \frac{2}{307}\sqrt{109\,826} = 2.996\,7 \times 10^{-2}$. This corresponds to the tax rate of 2.997% on snorkels and, given $t_k = \frac{3}{2} * 2.996\,7 \times 10^{-2} = 4.495\,1 \times 10^{-2}$, the tax on kayaks should be 4.495%.