

HW3

Econ 4211

Due Friday, Oct 3

September 19, 2008

1. Two villagers want to hire several Samurai days to protect their crops. It costs 4 bags of rice to hire a Samurai for one day. Assume (for simplicity) that the villagers' preferences are defined over bundles of rice, (consumed privately) and "safety" (number of Samurai days in the village). The utility of the two villagers, a, b are:

$$\begin{aligned}u_a(r_a, g) &= 4 \ln g + \ln r_a \\u_b(r_b, g) &= 16 \ln g + \ln r_b\end{aligned}$$

where r_a is the rice (in bags) consumed by villager a and r_b is the rice consumed by villager b , g is the number of Samurai days hired in the village. The first villager holds initially $R_a = 100$ bags of rice and the second one has $R_b = 120$.

- (a) Write down the resource constraint of the village (in terms of bags of rice).
- (b) Formulate the problem of the benevolent planner (the wise man) for this village.
- (c) Solve the problem. What is the optimal number of Samurai that the villagers should hire together? (Hint: in this case the wise man has to solve for both the rice consumption and the number of samurai – use all the optimality conditions from the wise-man problem.)

- (d) Assume now the villagers decide how much protection to hire without consulting each other or the wise man (each decides on own expense on the Samurai taking the decision of the other as fixed.) How many Samurai will be hired then?
- (e) Do the two answers (c,d) differ? Why? What problem does the Wise Man help the villagers to solve?

Problem 1 Problem 2 (Based on q 12, p196)

Andrew, Beth and Cathy live in Lindville. Bike paths (measured in miles, Q) are commonly used (public good). Andrew's willingness to pay (inverse demand) is $P_a(Q) = 6 - Q/2$, Beth's one is $P_b(Q) = 18 - Q$ and Cathy's is $P_c(Q) = 24 - 3Q$. $MC(Q) = 21$.

1. Assume the government can perfectly observe the willingness to pay for each. How long should be the bike path? How much should each contribute?
2. Will equal tax on all lead to an optimal provision of bike paths?

Problem 3 Consider the confectioner and the doctor example discussed in class (the value for doctor to operate is 60, the value for the confectioner is 40). Assume in addition doctor can install a soundproofing device that costs 18, in which case he can operate in the same building as the confectioner. This is known to the confectioner.

1. What is the socially optimal outcome (arrangement) in this case?
2. Describe the outcomes (who operates and what payments are made) in all the cases below and compare them to the optimal ones.
 - (a) Doctor has a right to operate in a quiet environment, negotiation is possible (costs of negotiation are negligible)
 - (b) Confectioner has a right to make noise, negotiation is possible.
 - (c) Negotiation is impossible (too expensive), government is unaware of the soundproofing technology available to the doctor and imposes a Pigouvian tax on the confectioner.

(d) Negotiation is impossible (too expensive), government is unaware of the soundproofing technology available to the doctor, a permit to make noise is offered for sale. (Only the holder of the permit has a right to make the noise.)