HW2
Econ 4211
Suggested solutions

September 20, 2008

Consider an economy with 2 inputs (K, L), 2 outputs (C, F), 2 individuals. (A, E). There is a fixed amount of inputs in the economy, total capital amount $\bar{K}$ and total amount of labor (hours) $\bar{L}$.

Recall the four conditions determining **PE allocation**

1. Feasibility: inputs used in each type of production equal to total amount of inputs in the economy. Outputs consumed by all sum up to outputs produced.

2. Final goods allocation efficiency: $MRS_{C,F}$ marginal rates of substitution between clothing and food across individuals are equal:

$$MRS^E_{C,F}\left(C^E, F^E\right) = MRS^A_{C,F}\left(C^A, F^A\right)$$

3. Input efficiency: The marginal rates of technical substitution in production of the two goods should be equal:

$$MRTS^C_{L,K}\left(L^C, K^C\right) = MRTS^F_{L,K}\left(L^F, K^F\right)$$

4. Substitution efficiency.

$$MRS_{C,F}\left(C, F\right) = MRT_{C,F}\left(C, F\right)$$

In an **equilibrium** of a perfectly competitive economy

1. Consumers (A, E) choose the best affordable bundle (each maximizes own utility subject to the budget constraint) taking the prices of goods as given

2. Producers (firms) maximize their profits. They choose inputs to minimize costs and outputs to maximize final profit.

3. Prices (p_C, p_F, r, w) are such that all markets (for inputs and for final goods) – 4 of them – clear.

Claim 1 *Market allocations are Pareto efficient.*[1]

Proof.

1. Feasibility follows from market clearing.

2. Final goods allocation efficiency. Given output equilibrium prices, p_C, p_F, Adam is choosing the best affordable bundle, so

$$MRS^A_{C,F}\left(C^A, F^A\right) = \frac{p_c}{p_f}$$

   Given output equilibrium prices, p_C, p_F, Eve is choosing the best affordable bundle, so

$$MRS^E_{C,F}\left(C^E, F^E\right) = \frac{p_c}{p_f}$$

   Therefore,

$$MRS^A_{C,F}\left(C^A, F^A\right) = MRS^E_{C,F}\left(C^E, F^E\right)$$

   so final goods allocation efficiency is satisfied.

3. Input efficiency. Firms producing clothing are minimizing costs. Given prices for inputs in equilibrium, r, w, they choose the input combination, $\left(L^C, K^C\right)$ such that

$$MRTS^C_{L,K}\left(L^C, K^C\right) = \frac{w}{r}$$

[1] Some assumptions on preferences and technology are omitted here for simplicity, but they are not very restrictive. Ask your instructor about them if interested.

Firms producing food are minimizing costs. Given prices for inputs in equilibrium, r, w, they choose the input combination, $\left(L^F, K^F\right)$ such that

$$MRTS^F_{L,K}\left(L^F, K^F\right) = \frac{w}{r}$$

Therefore,

$$MRTS^C_{L,K}\left(L^C, K^C\right) = MRTS^F_{L,K}\left(L^F, K^F\right)$$

so input efficiency is satisfied.

4. Substitution efficiency. For both consumers,

$$MRS_{C,F}\left(C, F\right) = \frac{p_c}{p_f}$$

Firms producing clothing are maximizing profits, so they choose the output quantity such that

$$MC^C\left(C\right) = p_C$$

Firms producing food are maximizing profits, so they choose the output quantity such that

$$MC^F\left(F\right) = p_f$$

By definition,

$$MRT_{C,F}\left(C, F\right) = \frac{MC^C\left(C\right)}{MC^F\left(F\right)}$$

So,

$$MRS_{C,F}\left(C, F\right) = MRT_{C,F}\left(C, F\right)$$

Therefore, substitution efficiency is satisfied.

End of the sketch of the proof.

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Problem 2 *Assume Adam has 14 pheasants and 1 apple, Eve has 9 apples and 1 pheasant. Preferences are represented by the following utility functions:* $u^A\left(x^A, y^A\right) = x^A y^A, u^E\left(x^E, y^E\right) = x^E y^E$.

1. Find the set of all Pareto efficient allocations for this economy

2. Find competitive equilibria (prices and allocations).

3. Are equilibrium allocations Pareto efficient?

Solution 3 *1. (Interior) Pareto Efficient allocations satisfy*

$$\begin{aligned} MRS^A_{x,y}\left(x^A, y^A\right) &= MRS^B_{x,y}\left(x^E, y^E\right) \\ x^A + x^E &= 15 \\ y^A + y^E &= 10 \end{aligned}$$

The first equation is then $\frac{y^A}{x^A} = \frac{y^E}{x^E}$, *combining with the last two, we get*

$$\frac{y^A}{x^A} = \frac{10 - y^A}{15 - x^A}$$

Solution is: $y^A = \frac{2}{3}x^A$

2. Normalize $p_y = 1$. *Each consumer maximizes the utility subject to the budget constraint. Optimality conditions for Adam:*

$$\begin{aligned} MRS^A_{x,y}\left(x^A, y^A\right) &= p_x/p_y \\ p_x x^A + p_y y^A &= 14p_x + p_y \end{aligned}$$

so, as $u^A\left(x^A, y^A\right) = x^A y^A$,

$$\begin{aligned} \frac{y^A}{x^A} &= p_x \\ p_x x^A + y^A &= 14p_x + 1 \end{aligned}$$

implying $2p_x x^A = 14p_x + 1$, *therefore Adam's demand is*

$$\begin{aligned} x^A &= \frac{1}{2}\frac{14p_x + 1}{p_x} \\ y^A &= \frac{1}{2}\left(14p_x + 1\right) \end{aligned}$$

Similarly, optimality conditions for Eve are

$$\begin{aligned} MRS^E_{x,y}\left(x^E, y^E\right) &= p_x/p_y \\ p_x x^E + p_y y^E &= p_x + 9p_y \end{aligned}$$

so, as $u^E\left(x^E, y^E\right) = x^E y^E$,

$$\begin{aligned} \frac{y^E}{x^E} &= p_x \\ p_x x^E + y^E &= p_x + 9 \end{aligned}$$

implying

$$\begin{aligned} x^E &= \frac{1}{2}\frac{p_x + 9}{p_x} \\ y^E &= \frac{1}{2}\left(p_x + 9\right) \end{aligned}$$

Market clearing for pheasants is

$$\frac{1}{2}\frac{14p_x + 1}{p_x} + \frac{1}{2}\frac{p_x + 9}{p_x} = 15$$

Solution is: $p_x = \frac{2}{3}$. *This implies that the equilibrium allocation is*

$$\begin{aligned} x^A &= \frac{1}{2}\frac{14\left(\frac{2}{3}\right) + 1}{\frac{2}{3}} = \frac{31}{4} \\ y^A &= \frac{1}{2}\left(14\left(\frac{2}{3}\right) + 1\right) = \frac{31}{6} \end{aligned}$$

and

$$\begin{aligned} x^E &= \frac{1}{2}\frac{\left(\frac{2}{3}\right) + 9}{(2/3)} = \frac{29}{4} \\ y^E &= \frac{1}{2}\left(\frac{2}{3} + 9\right) = \frac{29}{6} \end{aligned}$$

3. The allocation is Pareto, as the condition we got in part 1,$y^A = \frac{2}{3}x^A$, *is satisfied:* $\frac{31}{6} = \frac{2}{3}\frac{31}{4}$.

Problem 4 *Answer the questions above for a different economy: Adam has 10 pheasants, Eve has 10 apples. Preferences are represented by the following utility functions:* $u^A\left(x^A, y^A\right) = \left(x^A\right)^{1/2} y^A$, $u^E\left(x^E, y^A\right) = x^E y^E$.

Solution 5 *1. (Interior) Pareto Efficient allocations satisfy*

$$\begin{aligned} MRS^A_{x,y}\left(x^A, y^A\right) &= MRS^B_{x,y}\left(x^E, y^E\right) \\ x^A + x^E &= 10 \\ y^A + y^E &= 10 \end{aligned}$$

The first equation is then $\frac{y^A}{2x^A} = \frac{y^E}{x^E}$*, combining with the last two, we get*

$$\frac{y^A}{2x^A} = \frac{10 - y^A}{10 - x^A}$$

Solution is: $y^A = 20\frac{x^A}{10+x^A}$

2. Normalize $p_y = 1$*. Each consumer maximizes the utility subject to the budget constraint. Optimality conditions for Adam:*

$$\begin{aligned} MRS^A_{x,y}\left(x^A, y^A\right) &= p_x/p_y \\ p_x x^A + p_y y^A &= 10p_x \end{aligned}$$

so, as $u^A\left(x^A, y^A\right) = \left(x^A\right)^{1/2} y^A$,

$$\begin{aligned} \frac{y^A}{2x^A} &= p_x \\ p_x x^A + y^A &= 10p_x \end{aligned}$$

implying $3p_x x^A = 10p_x$*, therefore Adam's demand is*

$$\begin{aligned} x^A &= \frac{10}{3} \\ y^A &= \frac{20}{3}p_x \end{aligned}$$

Similarly, optimality conditions for Eve are

$$\begin{aligned} MRS^E_{x,y}\left(x^E, y^E\right) &= p_x/p_y \\ p_x x^E + p_y y^E &= 10p_y \end{aligned}$$

so, as $u^E\left(x^E, y^E\right) = x^E y^E$,

$$\begin{aligned} \frac{y^E}{x^E} &= p_x \\ p_x x^E + y^E &= 10 \end{aligned}$$

implying

$$\begin{aligned} x^E &= 5/p_x \\ y^E &= 5 \end{aligned}$$

Market clearing for pheasants is

$$\frac{10}{3} + 5/p_x = 10$$

Solution is: $p_x = \frac{3}{4}$*. This implies that the equilibrium allocation is*

$$\begin{aligned} x^A &= \frac{10}{3} \\ y^A &= \frac{20}{3}\frac{3}{4} = 5 \end{aligned}$$

and

$$\begin{aligned} x^E &= 4 * 5/3 = \frac{20}{3} \\ y^E &= 5 \end{aligned}$$

3. The allocation is Pareto, as the condition we got in part 1,$y^A = 20\frac{x^A}{10+x^A}$*, is satisfied:* $5 = 20\frac{\frac{10}{3}}{10+\frac{10}{3}}$.