Econ 3070
Returns to scale and Marginal Cost

March 13, 2007

Example 1 $F(L,K) = \min\{L,K\}$. *Optimal input combination to produce Q units of output: ..L = K = Q. Cost function:* $wL + rK = Q(w+r) = C(Q)$. *Marginal cost is constant,* $MC(Q) = w + r > 0$.

Example 2 $F(L,K) = L^{1/3}K^{1/3}$, $w = 14, r = 7$ *Optimal input combination to produce Q units of output:* $L^* = \sqrt{Q^3/2}; K^* = \sqrt{2Q^3}$, *so* $C(Q) = w\sqrt{Q^3/2} + r\sqrt{2Q^3}|_{w=14,r=7} = 14\sqrt{2}\sqrt{Q^3}$.

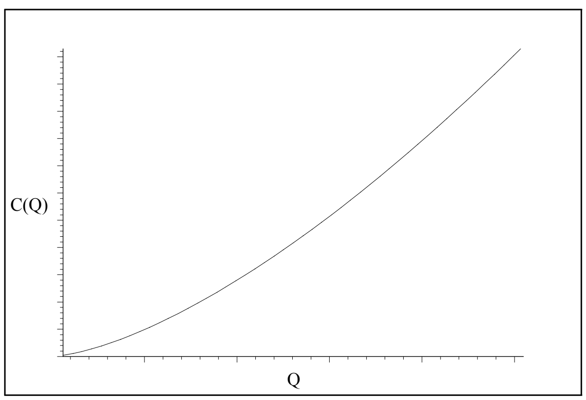
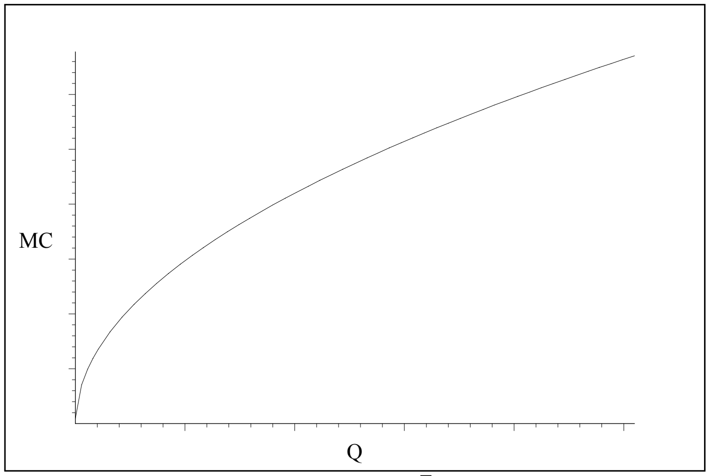


Figure 1: $C(Q) = 14\sqrt{2}\sqrt{Q^3}$

Marginal cost: $MC(Q) = \frac{\partial}{\partial Q}C(Q) = \frac{\partial}{\partial Q}14\sqrt{2}\sqrt{Q^3} = 21\frac{\sqrt{2}}{\sqrt{Q^3}}Q^2$



$MC(Q) = 21\frac{\sqrt{2}}{\sqrt{Q^3}}Q^2$

Example 3 *$F(L,K) = LK = Q$. Increasing returns. Optimal input combination satisfies: $L^*/K^* = r/w$, $L^*K^* = Q$. $L^* = rK^*/w$, and $(rK^*/w)K^* = Q$, Solution is: $K^* = \frac{1}{r}\sqrt{rQw}$,then $L^* = r\left(\frac{1}{r}\sqrt{rQw}\right)/w = \frac{\sqrt{rQw}}{w}$. Cost function: $C(Q) = w\frac{\sqrt{rQw}}{w} + r\frac{1}{r}\sqrt{rQw} = 2\sqrt{rQw}|_{w=14,r=7} = 2\sqrt{14}\sqrt{7}\sqrt{Q}$*

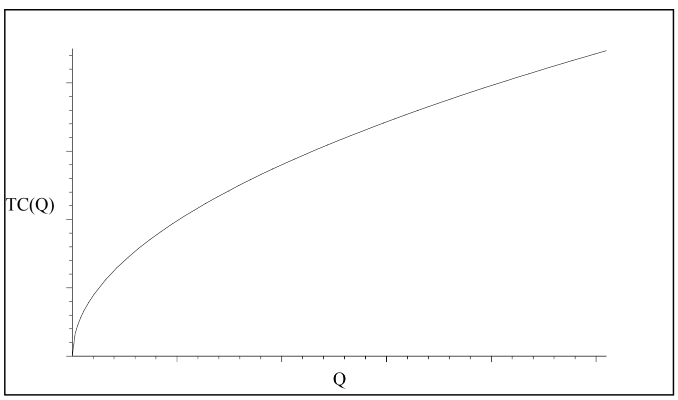
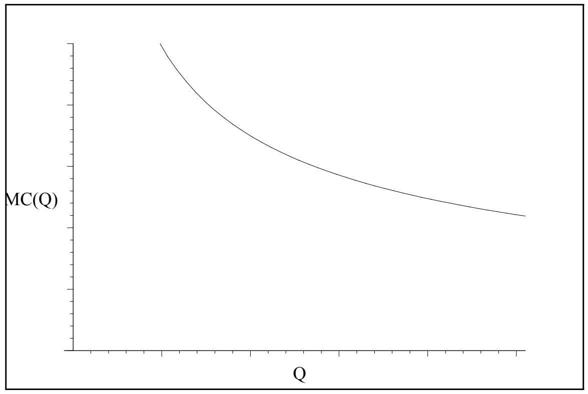


Figure 2: $TC(Q) = C(Q) = 2\sqrt{14}\sqrt{7}\sqrt{Q}$

$MC(Q) = \frac{\partial}{\partial Q}C(Q) = \frac{\partial}{\partial Q}2\sqrt{14}\sqrt{7}\sqrt{Q} = 7\frac{\sqrt{2}}{\sqrt{Q}}$



$MC\,(Q) = 7\sqrt{2}/\sqrt{Q}$

CONCLUSIONS:

Returns to scale	MC, AC
constant	constant
increasing	decreasing
decreasing	increasing

Explain why (in words).