ECON 3070
Spring 2007
Midterm: Suggested Solutions

February 26, 2009

Time: 70 min

20 percent of overall grade. Please, prove and interpret answers when necessary.

Honor code. On my honor, as a University of Colorado at Boulder student, I have neither given nor received unauthorized assistance on this work.

YOUR NAME:

Answer **three out of four** questions below in the space provided.

Problem 1 *Preferences. We have imposed 3 key assumptions on consumer preferences over consumption bundles: completeness, transitivity and 'more-is-better'.*

1. (2 pts) Explain each assumption, i.e., provide a definition for each.

   ANSWER:

   Preferences are complete if any two bundles can be compared, i.e., the consumer can say whether one of the two is better or (s)he is just indifferent between the two

   Preferences are transitive if they are consistent in a sense that for any three bundles A, B, C such that A is better than B and B is better than C it follows that A is better than C for this consumer.

Preferences are monotonic (satisfy ‘more-is-better’ assumption) if any given bundle is less prefferred to the one with bigger quantity of at least one good in it.

2. (2 pts) Give an example (describe a set of bundels) for the case in which ‘more-is-better’ is violated but completeness and transitivity hold. Illustrate your example graphically, providing a map of indifference curves.

   ANSWER.

   Examples: Perfect complements, Bads, Satiation, and others.

   As an example of the last one, take, say, bundles of water (in cups per day) and bread (pounds per day). The indifference map might look like this one:

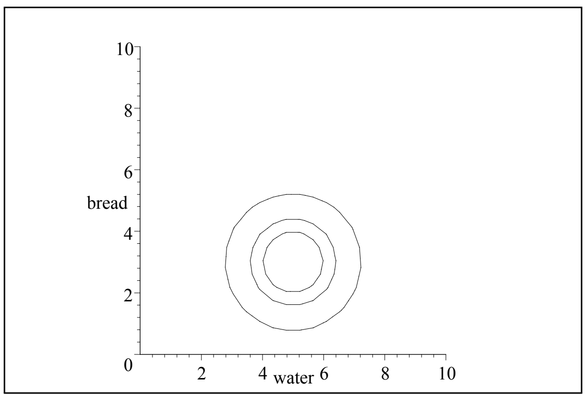

Figure 1: Here a bundle that has more than 5 cups of water and more than 3 pounds of bread is less preferred to $(5, 3)$, where utility is the highest.

Problem 2 *1. (3 pts) Janet likes to play golf and tennis. An hour of playing tennis gives her as much pleasure as two hours of playing golf. Are golf and tennis (playing) substitutes or complements? Explain. Draw a map of Janet's indifference curves.*

ANSWER: *For Janet two hours of golf and one hour of tennis are perfect substitutes, no matter how many hours she spends playing each. Thus the marginal rate of substitution is constant and is equal to two.*

The goods are not complements because she can consume (and enjoy) one without the other. Here is the map of the indifference curves:

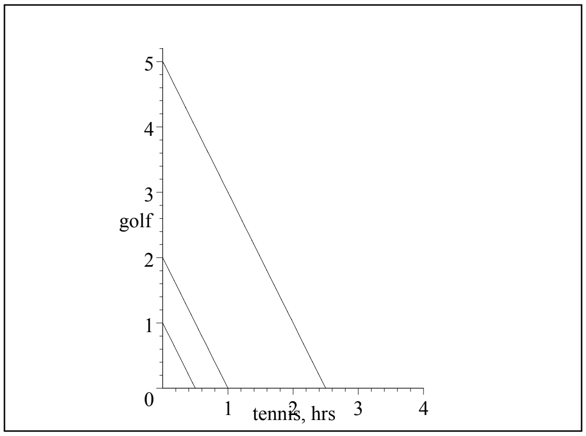

Figure 2:

Problem 3 *David is considering his purchases of food* (x) *and clothing* (y), *his utility function is* $u(x,y) = xy + 10x$. *His income is* $I = \$40$ *per week. David can purchase food at a price of* $p_x = \$1$ *per pound and clothing at* $p_y = \$2$ *per unit.*

1. *(1 pt) Formulate David's optimization problem.*

$$\max_{x,y\geq 0} u(x,y)$$
$$s.t.$$
$$p_x x + p_y y \leq I$$

which implies

$$\max_{x,y\geq 0} xy + 10x$$
$$s.t.$$
$$x + 2y \leq 40$$

2. *(1 pt) Formulate optimality conditions.*

If the optimum is in the interior, the conditions are

$$\begin{aligned} \frac{u_x(x^*, y^*)}{u_y(x^*, y^*)} &= \frac{p_x}{p_y} \\ p_x x^* + p_y y^* &= I \end{aligned}$$

which implies in this case

$$\begin{aligned} \frac{y^* + 10}{x^*} &= \frac{1}{2} \\ x^* + 2y^* &= I = 40 \end{aligned}$$

If the solution is a corner one, then it should be either $(I, 0)$ *or* $(0, I/2)$ *which in this case is* $(40, 0)$ *or* $(0, 20)$.

3. *(3 pts) Solve for David's optimal consumption of food and clothing (per week).*

   ANSWER: *Solving the conditions for interior optimum, we get:* $x^* = 2(y^* + 10)$, *so* $2(y^* + 10) + 2y^* = I$, *which implies* $2y^* + 10 = I/2$, *so* $y^* = I/4 - 5 = 5$. *So* $x^* = 2(5 + 10) = 30$. *The optimal bundle is, indeed, interior* $(30, 5)$ *and it satisfies both optimality conditions.*

4. *(2 pts) Now his income fell to* $I = \$10$. *What his optimal bundle after this decrease?*

   ANSWER: *SEE LEARNING BY DOING EXERCISE 4.3, p.109.*

Problem 4 *James has convex preferences for high-speed Internet service (H) and cable television (T), and exhausts all his income (I) on the consumption of these goods. Let p_H be the price (per-minute) of Internet service and p_T the price (per-channel) of cable television. Assume high-speed Internet and cable television are normal goods.*

1. *(2 pts) Using indifference curves and budget lines, carefully illustrate and explain how the demand for cable television changes following a decrease in the price of cable television. Clearly identify the substitution and income effects.* *See part (a) next page*

2. *(2 pts) In words, carefully describe how a decrease in the price of cable television would impact the demand for cable when cable is an inferior good? Should you expect the demand to increase or decrease?* *See part (b) next page.*

3. *(3 pts) Illustrate your answer to the previous question. For each of the two points on the price consumption curve depict the corresponding points on the demand curve for cable television. Explain how quantity demanded should change. SEE PP SLIDES FOR CH.5 on our webpage.*

Problem 5 *Suppose the production of airframes is characterized by the production function* $Q = F(L, K) = \left(L^{1/2} + K^{1/2}\right)^2$. *Hint:* $MP_L(L, K) = \left(L^{1/2} + K^{1/2}\right) L^{-1/2}$.

1. *(1 pt) Consider the production function. Does it exhibit increasing, decreasing or constant returns to scale?*

   ANSWER. *Constant returns: for any positive* k

   $$F(kL, kK) = \left(k^{1/2}L^{1/2} + k^{1/2}K^{1/2}\right)^2 = k\left(L^{1/2} + K^{1/2}\right)^2 = kF(L, K).$$

2. *(1 pt) The producer wants to minimize the cost of producing 121,000 airframes. Suppose that the price for labor is \$10 per unit and the price of capital is \$1 per unit. State the problem that the producer faces.*

   ANSWER:

   $$\min_{L,K\geq 0} K + 10L$$
   $$s.t.$$
   $$\left(L^{1/2} + K^{1/2}\right)^2 \geq 121,000$$

3. *(1 pt) Formulate optimality conditions for the problem.*

   See problem 7.6,p.254 and its solution on p.692.

4. *(2 pts) Solve the problem. What is the optimal combination of L and K that minimizes the cost?*

   See problem 7.6,p.254 and its solution on p.692.

5. *(2 pts) What is the cost of producing 121,000 airframes? Can the cost of producing 121,000 airframes be reduced if the firm hires less labor than the amount you found in part 4.?*

   The cost is $C(121000) = K^* + 10L^* = 100000 + 10 * 1000 = 110\,000$.

   This is the smallest cost of producing 121000 airframes. If the firm hires less labor than 1000 units, it will have to hire more capital to produce this amount. But then it will deviate from the optimal input combination, thus the cost of production should increase. Hiring less labor will not reduce the costs therefore.