ECON 3070
HW 5
Solutions

April 10, 2009

Problem 1 *9.3.* $P = 150$

output, Q	*TC*	*TVC*	*AFC*	*AC*	*MC*	*AVC*
0	*120*	*NA*	*NA*			
1	*200*	*200-120*	*5*24*	*200*	*80*	*80*
2	*100+120*	*100*	*60*	*110*	*20*	*50*
3	*220+20*	*240-120*	*40*	*80*	*20*	*40*
4	*240+120*	*240*	*30*	*90*	*120*	*60*
5	*660-160*	*500-120*	*24*	*100*	*500-360*	*76*
6	*660*	*540*	*20*	*110*	*160*	*90*

*first step: 5*24=AFC of one unit=FC=120, then we get one unit TVC=80. The row for Q=1 follows.*

Optimal production is between 4 and 5 units (P=MC and MC is increasing.)Compare the profits: at 4 the profit is $4 * 150 - 360 = 240$

At $Q = 5$ *the profit is* $5 * 150 - 500 = 250$. *So, 5 units is the best quantity to produce.*

Problem 2 *In a soap-making industry in country Clean Land each firm can produce soap with the associated (long-run) cost*

$$TC(q) = 10q^3 - 5q^2 + 20q$$

Market demand is

$$D(P) = 39000 - 2000P$$

1. *What is the output of each firm in the long-run equilibrium?*

   Answer. *The output of a firm in the long-run is determined by the two conditions:*

   (1) Profit maximization (as in the short run) $P = MC(q)$

   (2) Zero profit (free entry) $P = AC(q)$

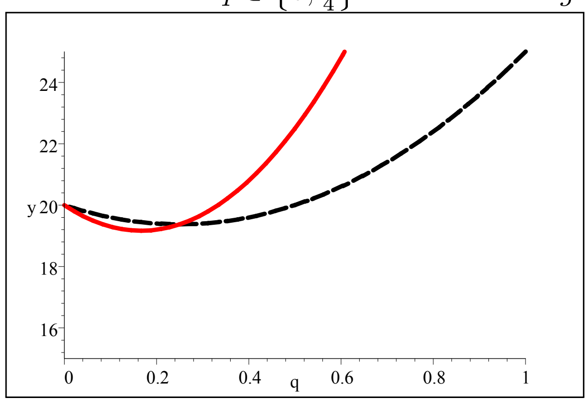
   Here

$$\begin{aligned} MC(q) &= 30q^2 - 10q + 20 \\ AC(q) &= 10q^2 - 5q + 20 \end{aligned}$$

   The two conditions imply

$$30q^2 - 10q + 20 = 10q^2 - 5q + 20$$

   This equation has two solutions $q \in \left\{0, \frac{1}{4}\right\}$. *Below is the graph,*



   $MC(q)$ *is depicted in red and* $AC(q)$ *is depicted in black.*

2. *What is the market equilibrium price?*

   Answer. *If* $q = 0$, *then by the first condition,* $P = MC(q)$, *the price should be* $P = 20$.

   If $q = \frac{1}{4}$, *then by the same condition* $P = MC\left(\frac{1}{4}\right) = 19.375$

3. *What is the total market demand?*

   If $P = 20$, *the total market demand is* $D(20) = 39000 - 2000 * 20 = -1000 < 0$, *thus,* $P = 20$ *can not be an equilibrium.*

   If $P = 19.375$, *the total market demand is* $D(19.375) = 39000 - 2000 * 19.375 = 250 > 0$, *a good candidate for an equilibrium.*

4. *What is the equilibrium number of firms in the long-run?*

   To equate demand and supply on the market the total number of firms (each producing $q = 1/4$) should satisfy

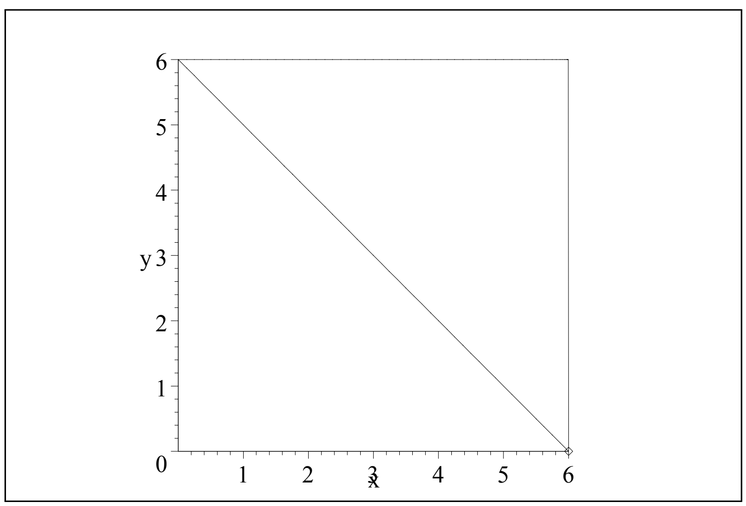
$$D(19.375) = 250 = Nq = \frac{N}{4}$$

   which gives $N = 1000$.

Problem 3 *Suppose Adam and Eve are the only individuals in the economy (a small island). Adam brings 6 pheasants (x) and Eve brings 6 buckets of apples (y) to the evening fire. Their preferences are represented by the same utility function,*

$$U^E(x, y) = U^A(x, y) = x + y$$

1. *Depict the initial allocation in an Edgeworth box. Draw the indifference curves of Adam and Eve through the initial allocation.*



2. *Is the initial allocation Pareto optimal?*

   ANSWER: *Yes, as the marginal rates of transformation between pheasants and apples is equal across consumers (Adam and Eve) at this initial allocation: $MRS^E_{x,y}(0, 6) = MRS^A_{x,y}(6, 0) = \frac{MU_x}{MU_y} = \frac{1}{1} = 1$*

3. *Describe the set of all Pareto efficient allocations, use the graph to illustrate your answer.*

   ANSWER: *The set of Pareto efficient allocations is just the set of all feasible allocations (all points in the Edgeworth Box). Indeed, the marginal rate of substitution between pheasants and apples is constant for both individuals and is always equal to* -1 *for both, therefore there is no re-allocation that can make either of them strictly better off without hurting the other. Therefore, if we pick any feasible allocation (so that the sum of pheasants given to Adam and Eve is six, which is also the sum of buckets of apples that Adam and Eve are given), then it is Pareto optimal.*